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Cherenkov radiation from relativistic electrons in inclined transparent radiator

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Based on the polarization currents model, a numerical calculation of the Cherenkov radiation photon yield in the wavelength range $400 < \lambda < 850$ nm from a silica aerogel radiator with a refractive index $n = 1.05$ and a thickness of 1 mm, located perpendicular to the electron velocity with the Lorentz factor $\gamma = 50$, was carried out. It was shown that the number of Cherenkov radiation photons propagating in a vacuum near a conical surface with the opening angle $\theta = 18.6^\circ$ deg coincides with the theoretical estimation from the Tamm–Frank formula. The same method was used to calculate the spectral-angular characteristics of Cherenkov radiation from an inclined quartz radiator ($n = 1.76$) of the same thickness. It was shown that for the radiator inclination angle $\psi = 24.25^\circ$ deg, part of the Cherenkov cone is extracted into vacuum at an angle $\theta_{\text{vac}} \approx 90^\circ$ deg relative to the electron momentum. The number of Cherenkov radiation photons in the same spectral range reaches the value of $\Delta N \approx 5.4$ photons/electron, which is 3 orders of magnitude higher than the yield of optical transition radiation, which is used to diagnose beams at modern accelerators.

Keywords: Cherenkov radiation, diagnostics, backward transition radiation.

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Introduction

Detectors of Cherenkov radiation have long been used in the high energy physics [1], in fusion plants to register „runaway" electrons [2], in radiation therapy for dose measurements [3], in diagnostics of accelerator beams [4].

Recently, as accelerator and laser technology are developing, Cherenkov–radiation (ChR) is used to measure the length of picosecond/subpicosecond electron bunches [5,6]. If the optical axis of the ChR detector (for example, a streak camera) is oriented perpendicularly to the radiation wave front, the duration of the ChR burst does not depend on thickness of the radiator and is determined only by the length of the bunch.

Often, for the purposes of diagnostics, back transition radiation (BTR) is used, which is generated by the electron bunch in thin metal targets. The number of BTR photons in this case is considerably lower than ChR intensity, however, the optical system of ChR emission and recording is much simpler and more reliable compared to ChR detection (with account of the requirement of ChR front perpendicularity relative to the axis of the detector). Thus, for example, the authors of the paper [6], in order to produce the parallel ChR beam from a special radiator with a refraction index $n = 1.05$, used a complex optical scheme with axicon lenses, which converts the ChR cone into „a hollow" cylindrical beam, coaxial with the electron pulse. The similar optical scheme focuses radiation on the detector into the ring, the diameter of which is considerably higher than the transverse size of the electron bunch. The BTR mechanism makes it possible to eject radiation at large angles relative to the electron bunch and focus the radiation in „the spot", matching in dimensions with the diameter of the electron bunch [7].

Further the paper will show that ChR of electrons crossing the inclined dielectric plate (radiator) may also be emitted into vacuum at large angles (for example, at angle $\theta = 90^\circ$) with the corresponding selection of the inclination angle. Despite the fact that in such geometry only a part of the Cherenkov cone is emitted into the vacuum, the number of ChR photons exceeds the output of BTR photons by 2–3 orders depending on the thickness of the radiator. The use of such method of diagnostics (including with the streak camera) will make it possible to considerably simplify the experimental equipment.

Model

The theory of radiation of the charge crossing the plate with dielectric permittivity ε and thickness L, was developed by V.E. Pafomov in the paper [8]. This model produced the expressions for two components of polarization of the spectral-angular distribution of the radiation intensity in the system of coordinates shown in Fig. 1, from the plate infinite in the transverse direction (see formulas (18.25), (18.26) of the cited paper). Based on these expressions, one may produce the spectral and angular distribution of number of

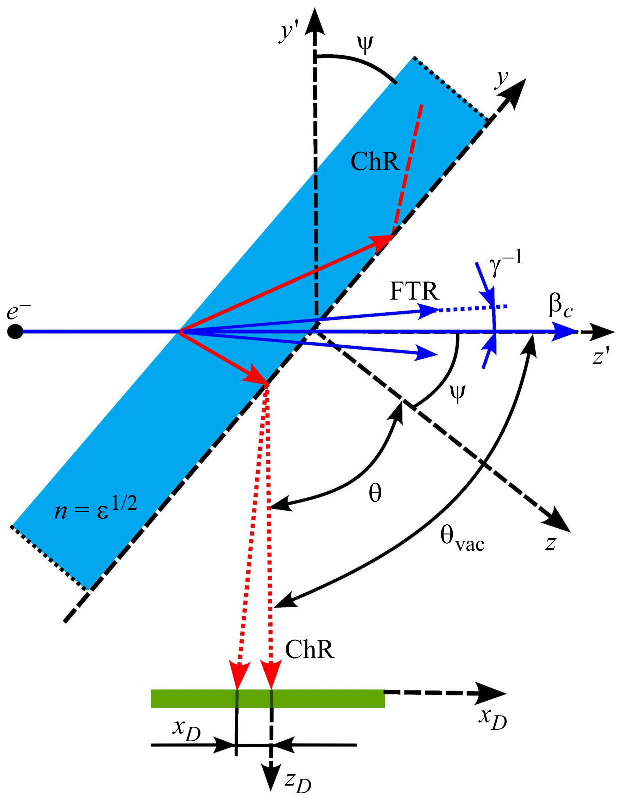

Figure 1. Angular variables describing the ChR process and systems of coordinates used for this purpose. In the lower part of the figure, the system of coordinates $\{x_D, y_D, z_D\}$ is used, the so called „detector system", where the axis z_D is turned by angle θ_{vac} relative to the electron speed. FTR — „forward" transition radiation, ChR — Cherenkov radiation.

photons emitted by the charge, $d^2N/d\Omega d\lambda$:

$$\frac{d^2N_{\parallel}}{d\Omega d\lambda} = \frac{\alpha}{\pi^2\lambda}\,\frac{\beta_y^2\beta_z^4 n_x^2 n_z^2|\varepsilon-1|^2}{\sin^2\theta|\big((1-n_y\beta_y)^2 - n_z^2\beta_z^2\big)|^2}$$
$$\times\,\frac{|(1-n_y\beta_y)^2 - Z^2\beta_z^2|^{-2}}{\left|e^{-\frac{i\omega LZ}{e}}(Z+\varepsilon\cos\theta)^2 - e^{\frac{i\omega LZ}{e}}(Z-\varepsilon\cos\theta)^2\right|^2}$$
$$\times\left|e^{-\frac{i\omega LZ}{e}}(1-\beta_y n_y+\beta_z Z)(Z+\varepsilon\cos\theta)\right.$$
$$\times\big(\sin^2\theta(1-\beta_z^2-\beta_y n_y-\beta_z Z)+\beta_y\beta_z n_y Z\big)$$
$$+\,e^{-\frac{i\omega LZ}{e}}(1-\beta_y n_y-\beta_z Z)(Z-\varepsilon\cos\theta)$$
$$\times\big(\sin^2\theta(1-\beta_z^2-\beta_y n_y+\beta_z Z)-\beta_y\beta_z n_y Z\big) - 2Ze^{-\frac{i\omega LZ}{\beta_z c}}$$
$$\times\big[\beta_z(\beta_y n_y-\sin^2\theta)(\beta_z\varepsilon-\beta_z\sin^2\theta+\varepsilon n_z-\beta_y\varepsilon n_y n_z)$$
$$\left.+\sin^2\theta(1-\beta_z^2-\beta_y n_y)(1-\beta_y n_y+\beta_z\varepsilon n_z)\big]\right|^2, \tag{1}$$

$$\frac{d^2N_{\perp}}{d\Omega d\lambda} = \frac{\alpha}{\pi^2\lambda}\,\frac{\beta_z^2 n_z^2|\varepsilon-1|^2}{\sin^2\theta|\big((1-n_y\beta_y)^2 - n_z^2\beta_z^2\big)|^2}$$
$$\times\,\frac{|(1-n_y\beta_y)^2 - Z^2\beta_z^2|^{-2}}{\left|e^{-\frac{i\omega LZ}{e}}(Z+\cos\theta)^2 - e^{-\frac{i\omega LZ}{e}}(Z-\cos\theta)^2\right|^2}$$
$$\times\left|e^{-\frac{i\omega LZ}{e}}(1-\beta_y n_y+\beta_z Z)(Z+\cos\theta)\right.$$
$$+\,e^{-\frac{i\omega LZ}{e}}(1-\beta_y n_y-\beta_z Z)(Z-\cos\theta)$$
$$\left.-\,2Ze^{-\frac{i\omega L(1-\beta_y n_y)}{\beta_z c}}(1-\beta_y n_y+\beta_z n_z)\right|^2, \tag{2}$$

$$\frac{d^2N}{d\Omega d\lambda} = \frac{d^2N_{\parallel}}{d\Omega d\lambda} + \frac{d^2N_{\perp}}{d\Omega d\lambda}. \tag{3}$$

Here the following designations are used: $\alpha = 1/137$ — fine structure constant, λ — ChR wave length, $\omega = 2\pi c/\lambda$, c — velocity of light in vacuum, L — thickness of the radiator, $Z = \sqrt{\varepsilon - \sin^2\theta}$, $\beta = \sqrt{1-\gamma^{-2}}$, γ — Lorentz factor, $\beta_y = \beta\sin\psi$, $\beta_z = \beta\cos\psi$, $n_x = \sin\theta\sin\varphi$, $n_y = \sin\theta\cos\varphi$, $n_z = \cos\theta$, where θ, φ — polar and azimuthal angles characterizing the wave vector of ChR photon in the vacuum.

It should be noted that the formulas (1)–(3) describe the radiation into „the front" hemisphere. For the flat geometry shown in Fig. 1, at the azimuthal angle $\varphi = 0$ near the polar angle $\theta \approx \psi$ the „forward" transition radiation propagates.

If the angle of inclination of the radiator ψ meets the condition of

$$\psi < \theta^0_{\text{ch}} = \arccos\big(1/\beta n(\lambda)\big), \quad n(\lambda) = \sqrt{\varepsilon(\lambda)}, \tag{4}$$

then the expressions (1)–(3) describe the ChR in the vacuum for the azimuthal angle $\varphi = \pi$ near the polar angle θ_{ch} [9,10]:

$$\theta_{\text{ch}} = \arcsin\big\{n(\lambda)\sin\big[\arccos\big(1/\beta n(\lambda)\big) - \psi\big]\big\}. \tag{5}$$

If instead of (4) the opposite condition

$$\psi > \theta^2_{\text{ch}}, \tag{6}$$

is met, a part of the ChR cone is emitted into the vacuum near the azimuthal angle $\varphi = 0$ [10].

The paper [11] developed an alternative approach for calculation of the characteristics of radiation arising as the charge passes through the radiator of the finite dimensions in all three directions — the so called model of polarization currents (polarization current model, PC). Based on this method, the paper [12,13] produced the analytical formulas, similar to (1)–(3), making it possible to calculate the number of ChR photons for any geometry of the radiator. As in the model by V.E. Pafomov [8], the components of the ChR field in the vacuum were calculated using Fresnel refraction coefficients for a flat infinite interface [10,11]. In particular, for a plate with transverse dimensions $a, b \gg \lambda$ the following expressions were produced:

$$\frac{d^2N_{\|}}{d\Omega d\lambda} = 4\alpha \frac{n_z^2}{\left[(1-\beta_y n_y)^2 - \beta_z^2 n_z^2\right]^2} \left|\frac{\varepsilon - 1}{\varepsilon}\right|^2 \frac{L^2}{\lambda^3}$$

$$\times \left|\mathrm{sinc}\left(\pi \frac{L}{\lambda} \frac{1-\beta_z L - \beta_y n_y}{\beta_z}\right)\right|^2 \left|\frac{\varepsilon}{Z + \varepsilon\cos\theta}\right|^2$$

$$\times \left|(\beta_z^2 + \beta_y n_y + \beta_z Z - 1)\sqrt{n_x^2 + n_y^2} - \beta_y\beta_z Z \frac{n_y}{\sqrt{n_x^2+n_y^2}}\right|^2, \quad (7)$$

$$\frac{d^2N_{\perp}}{d\Omega d\lambda} = 4\alpha \frac{n_z^2}{\left[(1-\beta_y n_y)^2 - \beta_z^2 n_z^2\right]^2} \left|\frac{\varepsilon - 1}{\varepsilon}\right|^2 \frac{L^2}{\lambda^3}$$

$$\times \left|\mathrm{sinc}\left(\pi \frac{L}{\lambda} \frac{1-\beta_z L - \beta_y n_y}{\beta_z}\right)\right|^2$$

$$\times \left|\beta_y^2\beta_z^2 \frac{n_x^2}{n_x^2+n_y^2}\,\varepsilon \frac{\sqrt{\varepsilon}}{Z+\cos\theta}\right|^2, \quad (8)$$

$$\frac{d^2N}{d\Omega d\lambda} = \frac{d^2N_{\|}}{d\Omega d\lambda} + \frac{d^2N_{\perp}}{d\Omega d\lambda}. \quad (9)$$

Here $\mathrm{sinc}(x) = \sin x/x$, the other designations are the same, as in (1)−(3). The produced formulas are simpler and clearer. ChR is emitted in the vacuum for geometry defined by zero value of the argument sinc:

$$1-\beta_z Z-\beta_y n_y = 1-\beta\cos\psi\sqrt{\varepsilon-\sin^2\theta}-\beta\sin\psi\sin\theta\cos\varphi = 0. \quad (10)$$

For the geometry, where the wave vector of ChR photon is located in the plane (y, z), from (10) the ratio is produced for the angle θ_{vac} in the vacuum (for the azimuthal angle $\varphi = \pi$):

$$\theta_{\mathrm{vac}} = \psi + \arcsin\{n(\lambda)\sin[\arccos(1/\beta n(\lambda) - \psi)]\}. \quad (11)$$

Note that the formulas (7)−(9) together with ChR describe the transition radiation. At first glance it seems that the quadratic dependence on the thickness of the radiator in these expressions has no relation whatsoever to the transition radiation. We will show that it is not so. For simplicity let us consider orientation $\psi = 0$, for which $d^2N_{\perp}/d\Omega d\lambda = 0$.

For angles θ of the corresponding „forward" BTR ($\theta \sim \psi$) from the metal target, for which the condition $|\varepsilon(\lambda)| \gg \sin^2\theta$ is fair, the function argument $\sin \mathrm{c}^2(x)$ may be directed to the infinity, which makes it possible to make the following substitution:

$$\sin \mathrm{c}^2(x) = \frac{\sin^2(x)}{x^2} \to \frac{1/2}{x^2}, \quad (12)$$

when we obtain

$$\frac{L^2}{\lambda^3}\left|\sin \mathrm{c}\left(\pi \frac{L}{\lambda} \frac{1-\beta\sqrt{\varepsilon-\sin^2\theta}}{\beta}\right)\right|^2$$

$$\to \frac{L^2}{\lambda^3} \frac{\beta^2\lambda^2}{2\pi^2 L^2\left(1-\beta\sqrt{\varepsilon-\sin^2\theta}\right)^2}$$

$$= \frac{\beta^2}{2\pi^2\lambda\left(1-\beta\sqrt{\varepsilon-\sin^2\theta}\right)^2}. \quad (13)$$

This results in

$$\frac{d^2N_{\|}}{d\Omega d\lambda} = \frac{2\alpha}{\pi^2}\frac{1}{\lambda}\frac{\beta^2\cos^2\theta\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}\left|\frac{\varepsilon-1}{\varepsilon}\right|^2$$

$$\times \left|\frac{\gamma^{-2}-\beta\sqrt{\varepsilon-\sin^2\theta}}{1-\beta\sqrt{\varepsilon-\sin^2\theta}}\right|^2 \left|\frac{\varepsilon}{\sqrt{\varepsilon-\sin^2\theta}+\varepsilon\cos\theta}\right|^2. \quad (14)$$

Within the limit $|\varepsilon| \to \infty$ the obtained expression reduces to the formula

$$\frac{d^2N_{\|}}{d\Omega d\lambda} = \frac{2\alpha}{\pi^2}\frac{\beta^2\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}\frac{1}{\lambda}, \quad (15)$$

which provides the doubled value of the available Ginzburg−Frank formula, since formally the radiation from the two interface surfaces is considered.

Calculation results

Let us consider the characteristics of ChR for the conditions corresponding to the experiment [6]:

$\gamma = 50$; $n(\lambda) = \mathrm{const} = 1.05$; $L = 1$ mm; $\lambda = 600$ nm; $\psi = 0$.

Fig. 2 shows the dependence of ChR photon output on the variable $n_y = \sin\theta\cos\varphi$ (in the right quadrant $\varphi = 0$, in the left one — $\varphi = \pi$).

The ChR maximum corresponds to the angle $\theta_m = \arcsin[n(\lambda)\sin\theta^0_{\mathrm{ch}}] = 18.6°$, besides, the radiation output in the area of the angles $\theta \sim \gamma^{-1}$ ($n_y \sim 0$, corresponding to the „forward" transition radiation) below the ChR output approximately by 3 orders.

Fig. 3 shows the „width" of the ChR cone $\Delta\theta \approx 0.0015$, which may be compared to the Tamm estimate [14] $\Delta\theta = 2\lambda/\pi L\sin\theta_{\mathrm{ch}} \approx 0.0012$. In the plane $\{n_x, n_y\}$ perpendicular to the electron bunch (in the detector plane) „the trace" from the ChR cone will be „seen" as the circumference with the radius $R\sin 18.6°$ (R — the distance from the radiator to the screen).

The photon spectrum $dN/d\lambda$ is calculated by integration of the expression (9) by the solid angle upon transition to variables n_x, n_y,

$$d\Omega = dn_x dn_y \Big/ \sqrt{1-n_x^2-n_y^2},$$

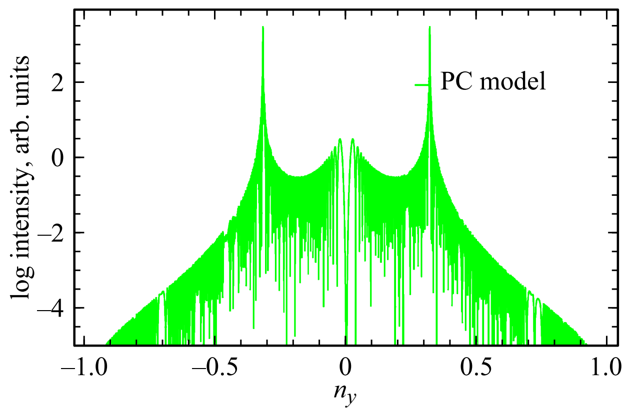

Figure 2. Intensity of radiation calculated according to the formula (9), from an aerogel radiator with the refraction index $n = 1.05$ in the front hemisphere (in logarithmic scale) depending on the angular variable $n_y = \sin\theta\cos\varphi$ (in the right quadrant $\varphi = 0$, in the left one — $\varphi = \pi$). The angle of inclination of the radiator $\psi = 0$ (normal incidence). The other parameters: $\gamma = 50$, radiator thickness $L = 1$ mm, wave length $\lambda = 600$ nm.

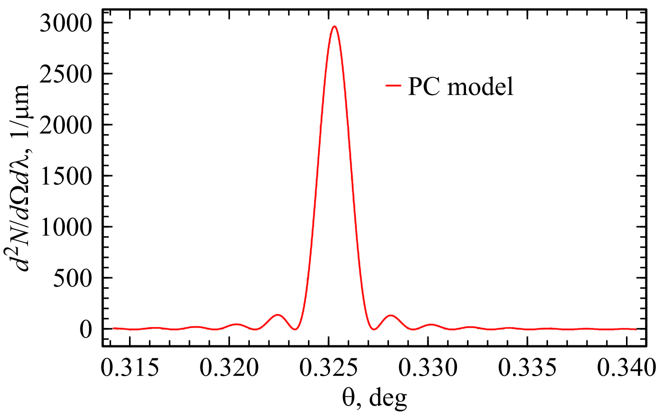

Figure 3. „Width of" the ChR cone for the same parameters as in Fig. 2.

$$\frac{dN}{d\lambda} = \int_{-1}^{1} dn_y \int_{-\sqrt{1-n_y^2}}^{\sqrt{1-n_y^2}} \frac{dn_x}{\sqrt{1-n_x^2-n_y^2}} \frac{d^2N}{d\Omega d\lambda}. \quad (16)$$

The result of integration for the different wave lengths is represented in Fig. 4 (points).

The same figure shows the adjustable dependence of the spectrum in the form of $dN/d\lambda = a/\lambda^2$, where $a = 4.28$ — the adjustable parameter. The ChR spectrum in the vacuum maintains the dependence on the wave length as in the medium (a/λ^2 with neglection of the radiation absorption).

The authors of the paper [6] estimated the output of ChR photons for the spectral range $400 < \lambda < 850$ nm from the radiator with thickness of 1 mm at $n = 1.05$ using the available formula [15]

$$\Delta N = 2\pi\alpha L(1 - 1/\beta^2 n^2)(1/\lambda_{\min} - 1/\lambda_{\max}) = 5.64\,\text{ph/e}^-. \quad (17)$$

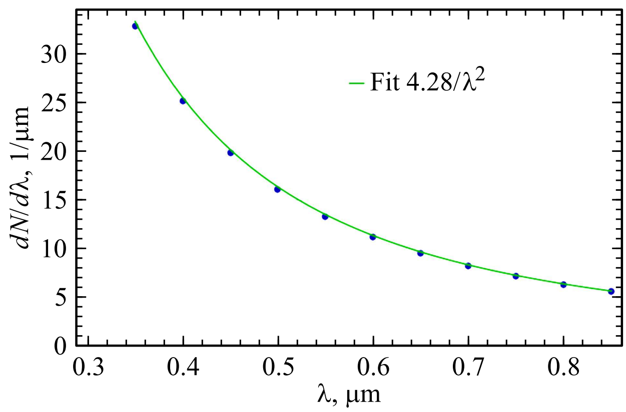

Figure 4. The photon spectrum $dN/d\lambda$ of ChR for the same parameters, as in Fig. 2, after the numerical integration by the full solid angle (points).

Note that the adjustable parameter $a = 4.28$ (Fig. 4) matches the theoretical value from the formula (17):

$$2\pi\alpha L(1 - 1/\beta^2 n^2) = 4.26 \approx a.$$

As a result of integration of the photon spectrum (16) in the same spectral range we obtain the value

$$\Delta N = 5.66\,\text{ph/e}^-,$$

which is close to the theoretical estimate.

The output of ChR photons increases as the optical density of the radiator increases. Thus, for example, for the radiator from corundum ($n = 1.76$) the number of ChR photons in the medium increases approximately by one order of magnitude compared to the considered case, however, to emit the radiation into vacuum, it is necessary to incline the radiator [16,17]. In such geometry only a part of the ChR cone is emitted from the radiator, but, as it will be shown below, the number of photons remains sufficient for diagnostics.

We will consider the characteristics of ChR for the radiator from corundum with thickness of 1 mm as previously in the system of coordinates, where the axis z' matches the direction of the electron bunch.

The complete solution to the equation (10) is recorded as

$$\sin\theta = \frac{-\sin\psi\cos\varphi + \cos\psi\sqrt{n^2\beta^2(1-\sin^2\theta\sin^2\varphi)-1}}{\beta(1-\sin^2\psi\sin\varphi)}. \quad (18)$$

Angles θ, φ in the produced solution are determined in the system, where the axis z is directed normally to the radiator surface (Fig. 1). The more convenient is the system, where the axis z' is directed along the electron momentum, the transition to which is carried out by the rotation of the initial one by the angle ψ. In the new system we will designate the angular variables via $\theta_{\text{vac}}, \varphi_{\text{vac}}$;

$$\cos\theta_{\text{vac}} = \cos\theta\cos\psi + \sin\theta\sin\psi\cos\varphi,$$

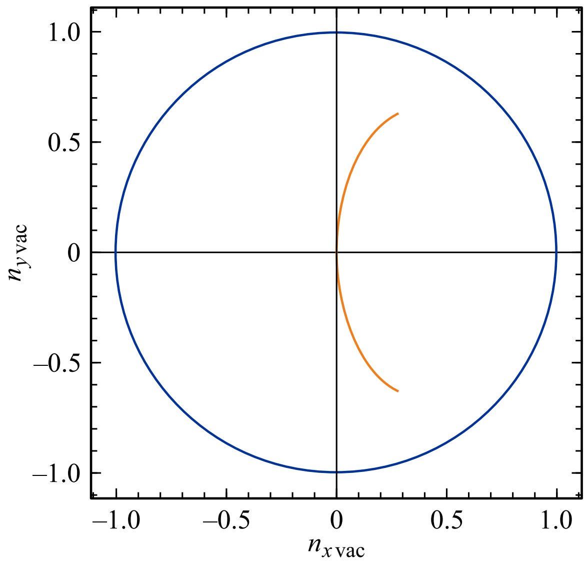

Figure 5. The part of the ChR cone emitted into the vacuum from the corundum radiator with $n = 1.76$, inclined at the angle $\psi = 24.25^\circ$ (curve in the center of the circle). The other parameters are the same as before.

$$\tan\varphi_{\mathrm{vac}} = \sin\theta\sin\psi/(\cos\theta\sin\psi - \sin\theta\cos\varphi\cos\psi). \tag{19}$$

As it follows from (18), (19) the polar and azimuthal angles (19) are determined by the azimuthal angle φ in the initial system of coordinates and depend on the inclination angle ψ. From (18) we obtain the ratio for determination of the inclination angle ψ for the specified angle of observation θ_{vac} at $\varphi = \pi$

$$\tan\psi = (\tan\theta_{\mathrm{ch}} - \beta\sin\theta_{\mathrm{vac}})/(1 - \beta\cos\theta_{\mathrm{vac}}). \tag{20}$$

Thus, for example, for corundum it is $\theta_{\mathrm{ch}} = 55.41^\circ$, and, accordingly, $\psi = 24.25^\circ$ for $\gamma = 50$. Only for the limited interval of angles φ the system (19) has the solution that determines the range of angles $\{\theta_{\mathrm{vac}}, \varphi_{\mathrm{vac}}\}$ and, accordingly, a part of the ChR cone, which propagates in the vacuum.

Fig. 5 shows a part of this cone emitted near the angle $\theta_{\mathrm{vac}} = 90^\circ$ from the corundum ($n = 1.76$) with thickness of 1 mm, inclined at the angle $\psi = 24.25^\circ$. „The trace" of the ChR cone is shown on the plane perpendicular to the wave vector (Fig. 1), where the angular variables look like

$$n_{x\mathrm{vac}} = \sin\theta_{\mathrm{vac}}\sin\varphi_{\mathrm{vac}}, \quad n_{y\mathrm{vac}} = \sin\theta_{\mathrm{vac}}\cos\varphi_{\mathrm{vac}}.$$

The maximum azimuthal angle, under which the ChR is emitted in this case ($\lambda = 600$ nm)

$$\varphi_{\mathrm{vac}}^{\max} \approx \pm 38^\circ.$$

The angular width of the ChR cone in this geometry calculated for both models is shown in Fig. 6. The good match of results should be noted (displacement of the maximum does not exceed 0.005°).

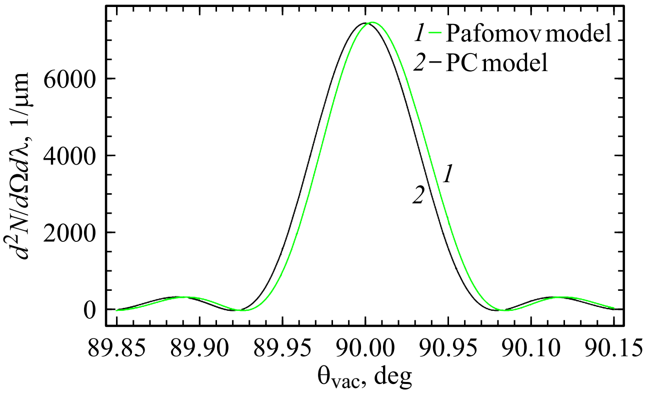

Figure 6. The width of the Cherenkov cone for geometry with $\psi = 24.25^\circ$, calculated according to the model of polarization currents (*2*) and the Pafomov's model (*1*).

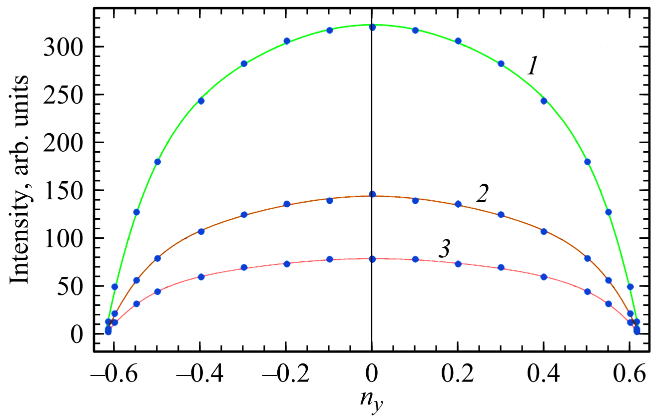

Figure 7. Dependence of ChR intensity $\frac{dN}{dn_y d\lambda}$ on variable n_y after integration by variable n_x for different wavelengths ($\lambda = 400$ (*1*), $\lambda = 600$ (*2*), $\lambda = 800$ nm (*3*)).

Fig. 7 shows the dependence of the ChR output on variable n_y after integration by n_x. In the used approximation ($n(\lambda) = 1.76 = \mathrm{const}$) the dependence of the ChR characteristics on the wavelength reduces to the standard dependence a/λ^2, which is illustrated in Fig. 8, where the calculation results $dN/d\lambda$ are shown for the considered geometry.

In the spectral range $400 < \lambda < 850$ nm the output of ChR photons from the corundum target is calculated using the same procedure:

$$\Delta N = 5.42\ \mathrm{ph/e^-},$$

i.e. slightly less than in the full cone from the aerogel radiator with $n = 1.05$.

Fig. 9 shows „traces" of the ChR cones on the screen located at the distance of $R = 100$ mm from the radiator. The origin of coordinates corresponds to the angle of radiation $\theta = 90^\circ$. The right „trace" illustrates the distribution of ChR from the radiator inclined at the angle $\psi = 21^\circ$, the central one — $\psi = 24.25^\circ$, the left one — $\psi = 28^\circ$. It should be

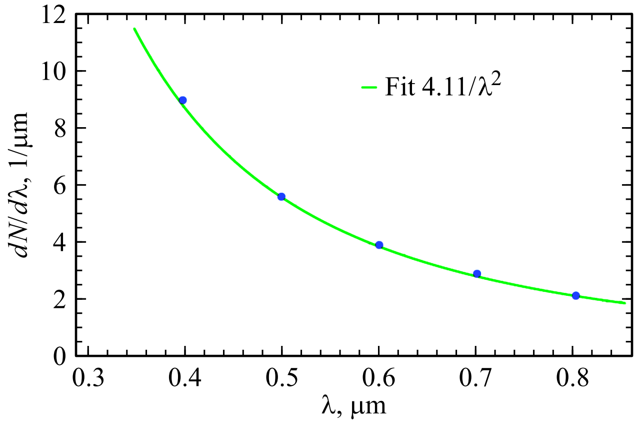

Figure 8. The photon spectrum $dN/d\lambda$ for the part of the cone of radiation ejected into vacuum after numerical integration by the solid angle for the same parameters as in Fig. 5.

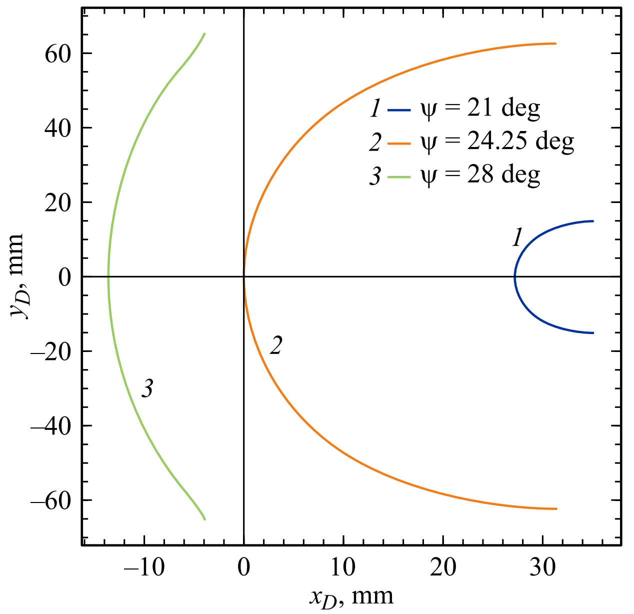

Figure 9. Distribution of „traces“ of ChR cones on the surface of the detector placed at the angle $\theta_{\text{vac}} = 90^\circ$. Angle of inclination of the radiator: $\psi = 21^\circ$ (*1*), $\psi = 24.25^\circ$ (2), $\psi = 28^\circ$ (*3*).

noted that for the angles of inclination $\psi < 21^\circ$ ChR is not emitted at the angle $\theta_{\text{vac}} = 90^\circ$.

Conclusion

To conclude, note that the model of polarization currents, which does not take into account the processes of photon rereflection in the radiator provides the identical results with the Pafomov model for thicknesses $L \gg \lambda$. For ultrarelativistic electrons, the output of ChR at the specified angle θ_{vac} is determined by the characteristics of the radiator (L, n) and its angle of inclination. The wider the angle of inclination and, accordingly, the wider is the angle of output θ_{vac}, the higher is the integral intensity of ChR. For registration of the maximum intensity of radiation, it is necessary to use the focusing lens, which under the conditions of the detection equipment location under wide angles θ_{vac} is easy.

Use of the radiators with the frequency dispersion will result in widening of the ChR cone (up to several degrees [17]) in the registered range of wavelengths, however, the selection of the optical scheme may eliminate the potential losses in the output of the ChR photons making it onto the detector.

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Conflict of interest

The authors declare that they have no conflict of interest.

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