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The use in medicine of speckle-like structures with scaling of spatial spectra

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A group of interdisciplinary issues related to the use of concepts of modern fractal optics in ophthalmology and art therapy is considered. To extend the range of test optical images presented to patients during treatment and to increase the efficiency of light stimulation, it is proposed to include in the list of used images speckle-like light structures with multifractal geometry and geometry of polynomial attractors. To substantiate this proposal, an analysis of the scaling characteristics of their spatial spectra was performed; the analysis showed that they are similar to the relevant characteristics of fractal images that have already proven their efficiency in medical practice.

Keywords: fractals, speckles, art therapy, ophthalmology.

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Fractal methods and approaches have entered the practice of modern medicine (see e.g. [1,2]). In particular, fractal speckle technology has enabled developing new diagnostic methods [3–5] and allowed increasing the efficiency of treatment procedures in ophthalmology [6–9] and art therapy [10–13]. Art therapy (treatment involving the phenomenon of beauty) is based on presenting the patient with images of various kinds (often fractal) which satisfy certain aesthetic characteristics. The aesthetic impact of fractal images on humans [14], which has been noted by many researchers, can be largely explained by the scaling (scale invariance) of the imagesśpatial spectra. The matter is that visual signals carrying information on the images are processed in the cerebral cortex based on their spatial spectrum structures [15]. Since, as shown in a number of studies [16,17], fractal images (speckle structures among them) possess self-similar spatial spectra; in contemplating them there is no need to perceive (process) the spectra in a wide frequency range, fixing their low frequency part is sufficient. This speeds up and facilitates the process of visual perception of objects being looked at and, hence, creates a feeling of comfort and aesthetic pleasure. Strengthening of interneuron links in the cerebral cortex, which takes place in this case, helps curing a number of eye diseases (e.g. glaucoma). Thus, fractal light stimulation may be regarded as a universal method for improving human health.

However, it is necessary to explore the possibility of transferring the above-formulated concept to other, differently structured light fields. This is because many objects whose images are used in art therapy are of natural origin and can by no means always be described by perfect fractals. Therefore, one of the tasks considered in this work is to establish the type of spatial spectra transformation during transition from fractal light distributions to multifractal ones which are characteristic of many natural objects. Another task being solved is to identify the scaling in the spectral characteristics of light structures that implement the nonlinear dynamics approaches.

Distribution of the multifractal light wave field was defined by the modernized two-dimensional Weierstrass function having the following form [18]:

$$W_{k,m} = \sigma\left[\sum_{v=0}^{V}\sum_{n=0}^{N}\left[\left[b^{(D_0+d\sin[2\pi rs(k+m)]-2)n}\right.\right.\right.$$
$$\times\sin\left[2\pi s b^n\left[\left(k-\frac{K+1}{2}\right)\sin(\alpha v)\right.\right.$$
$$\left.\left.\left.\left.\left.+\left(m-\frac{K+1}{2}\right)\cos(\alpha v)\right]+\psi_n+\psi_v\right]\right]\right]\right]+A. \qquad (1)$$

Here $W_{k,m}$ is the field amplitude, k, m are the discrete transverse coordinates ($0 \leqslant k,\ m \leqslant K$), σ is the normalization factor, N is the number of harmonics, V is the number of azimuthal components, n is the harmonic number, v is the index of the wave azimuthal component, α is the elementary azimuthal angle of rotation, b — is the scaling parameter, s is the scale parameter, ψ_n, ψ_v are the phases of field components, A is the additional component with a uniform distribution of the field amplitude.

Figure 1 demonstrates the general view of the speckle-like light wave and the field amplitude distribution in the transverse direction, which are defined by (1). The calculations were performed for the following set of parameters: $N = 5$, $V = 47$, $\sigma = 0.15$, $b = 2$, $s = 0.03$, $d = 0$, $r = 2$, $K = 255$, $A = 3$, $\alpha = \pi/48$, $D_0 = 1.25$, $n = 0{-}N$, $v = 0{-}V$, $m = 0{-}K$, $k = 0{-}K$, $\psi_v = \mathrm{rnd}(v)2\pi/(v+1)$, $\psi_n = \mathrm{rnd}(n)4\pi/(n+1)$. Since $d = 0$, the structure is monofractal at these parameters. Its spatial spectrum calculated using the fast Fourier transform is presented in Fig. 2, *a*. The figure shows that, despite the disordered

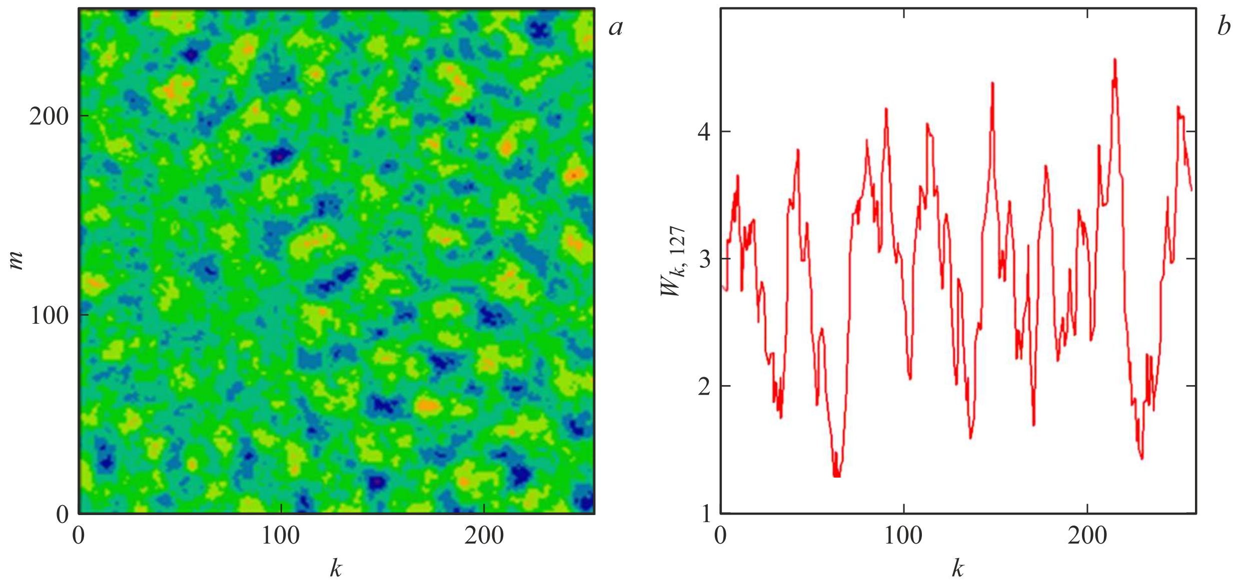

Figure 1. *a* — speckle-like emission (general view), *b* — field distribution cross-section.

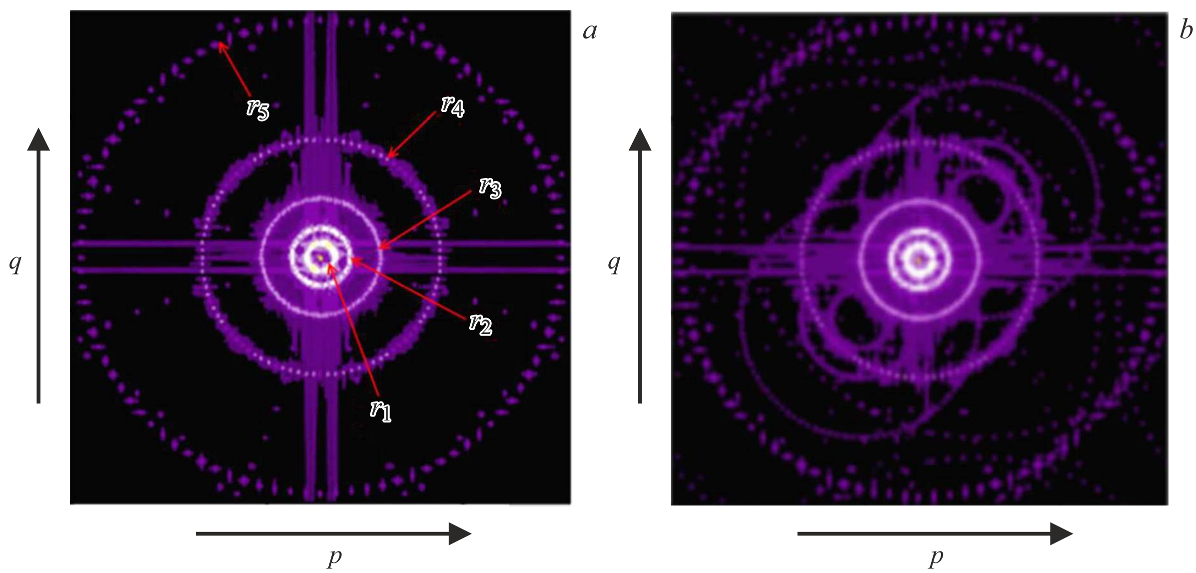

Figure 2. Spectra of the monofractal (a) and multifractal (b) structures. r_1, r_2, r_3, r_4, r_5 are the radii of the spectral maxima circumferences, p, q are the spatial frequencies.

distribution of the field amplitude, the spectrum is characterized by a high degree of symmetry. The spectral maxima form a system of concentric circles of decreasing intensity whose radius ratio r_i matches the scaling coefficient $b = 2$ $(r_2 : r_1 = r_3 : r_2 = r_4 : r_3 = r_5 : r_4 = 2)$. A similar spectrum is inherent to fractal images with a favorable perception background.

Now consider what will be the result of transition to multifractal representation of the light structure. This transition may be performed by changing d. Assume for definiteness that, unlike the previous case, $d = 0.2$. Estimates show that such an increase in d does not lead to any significant variations in the field statistics but causes noticeable structural disturbances in the spatial spectrum. This is clearly shown in Fig. 2, *b* where the features of spatial frequencies distribution are presented graphically. Variations in the spectrum are caused by emergence in it of additional ring elements located close to the monofractal maxima.

Quantitative estimates of the spectrum variations may be obtained by determining correlation coefficient C between spectral distributions of the monofractal and multifractal. If the spectra are presented in the logarithmic scale, coefficient C turns to be $C = 0.88$. Such a rather high correlation coefficient indicates a significant stability of the initial spectrum structural morphology. After the transition to the multifractal, the amplitude distribution standard deviation

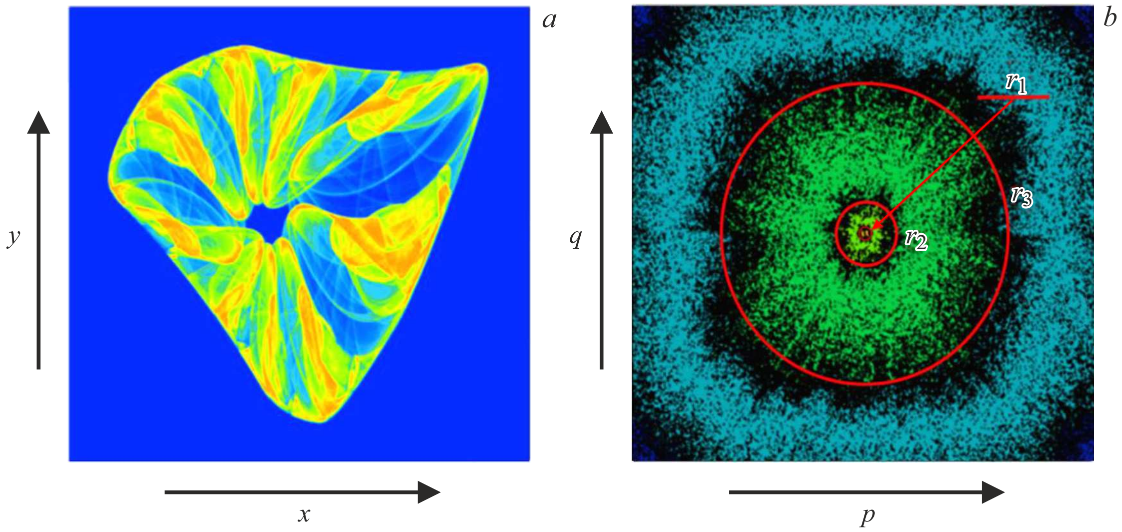

Figure 3. The attractor structure (a) and spectrum (b).

did not vary significantly and remained at the level of $\xi = 0.29$.

In the case when variations in the fractal dimension covered the entire possible range ($D_0 = 1.5$, $d = 0.49$), the correlation coefficient decreased to $C = 0.7$ while the standard deviation increased to $\xi = 0.35$. However, in the case of such a seemingly insignificant deterioration in the correlation it is necessary to take into account the fact that the arising spurious spectral maxima could reach 0.8 of the neighboring monofractal maxima. This situation cannot be regarded as favorable for implementing many medical applications assotiated with processing optical signals in the cerebral cortex. Most likely, the effect of additional spectral maxima may be reduced by selecting parameters D_0 and d so that those maxima will not be higher than 15% of the monofractal maxima.

In literature [10] there are evidences of the fact that certain aesthetic features are inherent to some graphic illustrations of processes developing on the basis of deterministic chaos. Let us use the results of [19] to analyze the characteristics of attractors formed based on the following polynomials:

$$x_{n+1} = a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n y_n + a_4 y_n + a_5 y_n^2,$$

$$y_{n+1} = a_6 + a_7 x_n + a_8 x_n^2 + a_9 x_n y_n + a_{10} y_n + a_{11} y_n^2. \quad (2)$$

Depending on coefficients a_i, the iterative procedure allows constructing attractors of different configurations. In some cases, the attractorsṕhase trajectories form a system of structural fragments that renders the attractor a speckle-like appearance. Let us consider the properties of one of such attractors constructed by setting the following sequence of coefficients: $a_0 = 0$, $a_1 = -1$, $a_2 = 0.5$, $a_3 = -1.1$, $a_4 = -0.4$, $a_5 = 0.3$, $a_6 = 0.2$, $a_7 = 0.3$, $a_8 = -0.5$, $a_9 = 0.7$, $a_{10} = -1.1$, $a_{11} = 0.1$.

Fig. 3 presents the attractor pattern after 3000 iterations of construction and corresponding spatial spectrum. Despite some attractor pattern fragments having high intensity differ significantly in shape and position from classical speckles, the obtained structure spectrum has some common features with the spectrum of fractal speckles. Arrangement of the spectral maxima is axisymmetric, and circles characterizing it have radii whose ratio is close to a constant of 3.2 (scaling parameter). This indicates scaling of the attractorś spatial spectrum.

Thus, we can conclude that images of fragmentarily structured polynomial attractors are characterized by scaling of spatial spectra. This explains why they have aesthetic characteristics allowing them to be used in light therapy.

Conflict of interests

The authors declare that they have no conflict of interests.

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