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# Spin dynamics control in a double quantum dot under the conditions of the electric dipole spin resonance via the tunable spin-orbit coupling

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The effect of the Rashba and Dresselhaus contributions ratio into the spin-orbit coupling is considered on the spin trajectory on the Bloch sphere induced by the periodic electric field in a GaAs semiconductor double quantum dot under the conditions of the electrical dipole spin resonance. It is shown that the variations of the Rashba parameter which can be achieved by the gate voltage lead to the changes for the spin rotation plane in wide limits. The predicted effect can be used as an additional control parameter for the spin dynamics including the applications for the design of spin qubits.

**Keywords:** spin, Bloch sphere, double quantum dot, spin-orbit coupling, electric dipole spin resonance.

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## 1. Introduction

Structures with quantum dots based on $A^{III}B^{V}$ semiconductors attract attention due to various possibilities of joint control of charge and spin degrees of freedom. Due to the presence of a strong spin-orbit interaction (SOI) in these structures, the spin can be controlled using an alternating electric field in the conditions of electric dipole spin resonance (EDSR) [1,2] when the frequency of the field $\omega$ coincides with the Zeeman splitting $\Delta_Z$ (in the system of units with $\hbar = 1$). Structures with a double quantum dot (QD) are of a particular interest, where the effects of Landau–Zener–Stëckelberg–Majorana interference (LZSM) [3,4] are observed in case of tunneling that consist in resonant amplification or attenuation of tunneling under certain conditions with respect to the frequency and amplitude of the field, as well as the shift of the minimum potential (detuning) of one QD relative to the other and the Zeeman splitting of the levels [5,6].

It was shown in our previous studies that the effects of LZSM interference are nontrivially manifested in spin dynamics during tunneling under EDSR conditions. In particular, points appear in the system parameter space in which the conditions of EDSR in a single QD are satisfied together with the conditions of resonant tunneling into an adjacent QD with both spin conservation and spin flip [7]. It was found that in the presence of even weak tunneling into the second QD in the resonance conditions, the spin dynamics in the first QD accelerates, and spin rotation can be controlled on the Bloch sphere not only on the main harmonic at $\omega = \Delta_Z$, but also on spin resonance subharmonics at $k\omega = \Delta_Z$, where $k = 2, 3, \dots$ [8]. The evolution on subharmonics proceeds at lower frequencies $\omega_k = \Delta_Z/k$, which can contribute to its realization in strong magnetic fields, when the base harmonic $k = 1$ is poorly achievable over frequency due to hardware limitations. EDSR subharmonics were observed in experiments, including experiments with double quantum dots in an InAs-based nanowire on the dependence of the current passing through the structure in the parameter plane $(f, B_z)$, where $f$ is the frequency of the electric field, and $B_z$ is the amplitude of the constant magnetic field [9]. The generation of the second EDSR subharmonics was observed in experiments with QD based on Si/SiGe [10]. Theoretical predictions about the occurrence of EDSR subharmonics have been made in a number of other papers [11,12].

Since the value of the amplitude of Rashba's contribution to the SOI can be changed using the electric field of the gates within a fairly wide range, up to 100% of the initial value [13], the ratio of Rashba and Dresselhaus contributions to the SOI can be another control parameter of spin dynamics. We considered mainly Dresselhaus contribution to SOI in our paper [8]. We consider a different combination of Rashba and Dresselhaus contributions in this paper. Our goal is to select such modes for which the widest possible class of spin rotation operations is realized with spin resonance and its subharmonics. Spin rotations in various planes are considered the transition between which can be made by changing the ratio of Rashba and Dresselhaus contributions. The possibilities of such a control parameter open up another way in controlling spin evolution in semiconductor quantum dots, which can be useful for information storage and processing tasks on spintronics devices.

We consider in our paper a coherent, non-dissipative dynamics at relatively short times of the order of $100-200$ periods of the electric field at frequencies of $\sim 2$ GHz, i.e. at times of about $50-100$ ns. These times are

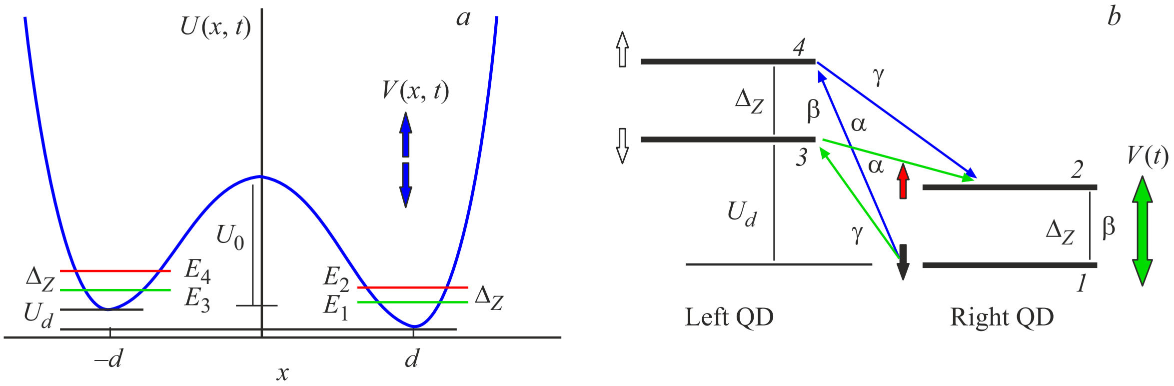


**Figure 1.** *a* — scheme of the lower four levels $E_1$–$E_4$ in a double quantum dot with potential $U(x, t)$ (blue curve) for the Hamiltonian (1). *b* — transitions between levels *1, 2* in the right QD and *3, 4* in the left QD with matrix elements $\alpha$, $\beta$, $\gamma$ for the Hamiltonian matrix notation (3). (A color version of the figure is provided in the online version of the paper).

comparable to the spin relaxation time for the mechanism of hyperfine interaction with nuclear spins [14,15]. The spin relaxation time can be quite short and amount to $\sim 10$–$20$ ns for GaAs-based structures [14], however, the presence of a magnetic field in Faraday geometry, when $B_z \parallel S_z(t=0)$, leads to an increase of the proportion of spins $S_z(t \to \infty)/S_z(0)$, not experiencing relaxation, i. e., the magnetic field performs a stabilizing function and slows down spin relaxation. The ratio of the Larmor frequency $\omega_B$ and the spin relaxation rate of $\delta$ gives an estimate of $\Omega_B/\delta \sim 20$ for a typical magnetic field of $B_z \sim 0.1$ T in our study, which results in a significant (up to 0.9 and higher) fraction of spins over large times, evolving without noticeable relaxation [14]. Moreover, the spin relaxation in double QD at larger times can be power-law rather than exponential when taking into account the mechanisms of spin blockade and exchange interaction, i. e., it can have a slower character [15]. The effects of dissipation in systems with LZSM interference have also been studied in a number of papers [4,16,17], where their impact over short times was mainly reduced to the spreading of the fine structure of interference patterns, but did not result in the disappearance of resonances. The estimates we mentioned suggest the possibility of observing coherent spin rotations at the discussed time intervals, when the effects of spin relaxation and dissipation can be ignored in the first approximation.

## 2. Model

We consider a structure with a double quantum dot created by gate fields in a two-dimensional electron or hole gas based on GaAs, as it was performed in experiments [5,6]. The tunneling process between neighboring points proceeds efficiently in a one-dimensional manner in such a system, and the potential energy has a profile shown in Figure 1. A typical Hamiltonian of the system has the form [7,8]

$$H = H_{2QD} + H_Z + H_{SO} + V(x, t). \qquad (1)$$

$H_{2QD}$ in (1) is the Hamiltonian of an effectively one-dimensional double quantum dot with a distance between the minima of the potential $2d$, $H_Z$ is a Zeeman term generating splitting of levels $\Delta_Z$, $H_{SO}$ is a contribution from SOI, which we take into account based on the linear approximation based on quasi-pulse $k_x$:

$$H_{SO} = \left(\alpha_R^{(0)}\sigma_y + \beta_D^{(0)}\sigma_x\right)k_x, \qquad (2)$$

where $\alpha_R^{(0)}$ and $\beta_D^{(0)}$ are the amplitudes of Rashba and Dresselhaus contributions to the SOI, and $\sigma_{x,y}$ are Pauli matrices. The term $V(x, t)$ in (1) describes the potential of a quasi-stationary electric field, which can include both the static bias potential $U_d$ of the bottom of one of the quantum dots (detuning) and the periodic potential of the electric field $V_d = f(x)\sin(\omega t)$ with amplitude $V_d$ and frequency $\omega$. The function $f(x)$ describes an addition to the symmetric potential of a double well corresponding to an electric field that shifts the levels in the right QD [7,8]. Figure 1 shows the scheme of the lower four levels $E_1$–$E_4$ together with the potential of a double quantum dot for the Hamiltonian (1), where $\Delta_Z$ is a Zeeman splitting of levels with a spin projection down (green lines) and up (red lines) relative to the direction of the magnetic field, $U_d$ shows the bias (detuning) of the bottom of the potential of the right QD relative to the left QD, and $V(x, t)$ corresponds to a non-stationary addition to the potential from a periodic electric field. The presence of the SOI in the Hamiltonian (1) determines the coupling of levels with different spin projections in an electric field and ensures the occurrence of EDSR under the following condition $k\omega = \Delta_Z$.

Tunneling processes are inextricably linked with the evolution of spin in the presence of a magnetic field and SOI, which makes it possible to control spin dynamics in case of interaction with a coordinate degree of freedom. At the same time, the system extends beyond the two-level approximation, since the dynamics involves at least a pair of spin-split levels in each of the two quantum dots. The

Hamiltonian of the system (1) within the framework of the four-level approximation can be written in matrix form in the basis of the functions $\psi_i = |\varphi_{L,R}| \uparrow\downarrow\rangle$, $i = 1-4$ localized in the right and left QD and having a spin projection down or up in the direction of the magnetic field. The numbering of the basis functions from $\psi_1$ to $\psi_4$ for the examples we are considering corresponds to the configuration of the levels shown in Figure 1, *a*, according to which the states $\psi_1$, $\psi_2$ form the Zeeman doublet in the right QD, to which a periodic electric field with potential $V(t) = U_d + V_d \sin(\omega t)$ is applied, and the states $\psi_3$, $\psi_4$ form the same doublet in the left QD. The Hamiltonian matrix in this case has the form [8]

$$H = \left\| \begin{matrix} -\frac{\Delta_Z}{2} + V(t) & i\beta_D + \beta_R & \gamma & i\alpha_D + \alpha_R \\ & \frac{\Delta_Z}{2} + V(t) & \alpha_D + \alpha_R & \gamma \\ & & -\frac{\Delta_Z}{2} & i\beta_D + \beta_R \\ h.c. & & & \frac{\Delta_Z}{2} \end{matrix} \right\|. \quad (3)$$

The parameter $\gamma$ in (3) is a matrix element of the tunnel coupling between adjacent potential minima shown in Figure 1, *a* in case of spin-preserving tunneling, *h. c.* denotes the Hermitian conjugation, $\gamma = 2-3\,\mu$eV for the considered structure [5,6]. The parameters $\alpha_{D,R}$ are matrix elements for tunneling with a spin flip due to the contribution of Dresselhaus and Rashba to the SOI, respectively. The parameters $\beta_{D,R}$ describe transitions inside the Zeeman doublet in the left or right QD with a spin flip, also due to the Dresselhaus or Rashba contribution to the SOI, i. e., the mechanism of EDSR without tunneling. The diagram of transitions between levels *1* and *2* in the right QD and levels *3* and *4* in the left QD together with the corresponding matrix elements $\alpha$, $\beta$, $\gamma$ is shown in Figure 1, *b*, where the black arrow in the right QD indicates the initial state with spin down at the level of $E_1$, and the red arrow shows the final state as a result of the EDSR process with an upward projection of spin at the level $E_2$. A double green arrow in Figure 1, *b* shows a periodic electric field with a potential of $V(t)$, applied mainly to the right QD.

Various cases of the ratio $s = |\alpha_R|/|\alpha_D|$ of Rashba and Dresselhaus contributions to the SOI will be the main variable parameter in the study of spin evolution in this paper, which can be implemented in experiments through the tunable amplitude of Rashba's contribution using the gate field [13]. The ratio $|\beta_R|/|\beta_D|$ in (3) changes proportionally to the same parameter $s = |\alpha_R|/|\alpha_D|$.

The wave function in representation (3) can be written as a four-component column vector $\mathbf{C}(t) = (C_1(t), C_2(t), C_3(t), C_4(t))$, for which the nonstationary Schrödinger equation $i\hbar \cdot \partial\mathbf{C}/\partial t = H\mathbf{C}$ with a matrix (3) can be written and solved, having the form of a system of ordinary differential equations for functions $C_n(t)$. This system is complemented by the initial condition $\mathbf{C}(0) = (1, 0, 0, 0)$ corresponding to the position of an electron or hole at the lower level $E_1$ in the right QD in Figure 1. After finding the functions $C_n(t)$ the dynamics of the projections of spin $S^R_{x,y,z}(t)$ in the right QD that is of interest to us can be described using $C_n(t)$ as follows:

$$\begin{cases} S^R_x(t) = \overline{C}_2(t)C_1(t) + \overline{C}_1(t)C_2(t) \\ S^R_y(t) = i\left(-\overline{C}_2(t)C_1(t) + \overline{C}_1(t)C_2(t)\right) \\ S^R_z(t) = |C_2(t)|^2 - |C_1(t)|^2 \end{cases}. \quad (4)$$

The trajectory of the end of the vector $\mathbf{S}(t) = (S^R_x(t), S^R_y(t), S^R_z(t))$ with components from (4) can be shown on the Bloch sphere of unit radius, which allows visualizing the dynamics of spin [7,8], in this case in the right QD where the evolution we are interested in takes place.

## 3. Evolution simulation results

We solved numerically the nonstationary Schrödinger equation in the matrix representation (3) using the same techniques as in our previous study [7,8]. Figure 2 shows the results for stroboscopic dynamics (depicted through an integer number of periods of the electric field $T = 2\pi/\omega$) for the vector of spin $\mathbf{S}(t) = (S_xR(t), S_yR(t), S_zR(t))$ with components from (4) shown on the Bloch sphere in the right QD. The trajectories *1–5* show the dynamics for different ratios $s = |\alpha_R|/|\alpha_D| = |\beta_R|/|\beta_D|$ between the Rashba and Dresselhaus parameters in the Hamiltonian (3). There is only the Rashba's contribution on the trajectory *1* which corresponds to the limiting case $s = \infty$, there is only the Dresselhaus contribution on the trajectory *2*, i. e. $s = 0$, the amplitude contributions on the trajectory *3* is the same, $s = 1$, the amplitude of the Rashba's contribution is twice as large on the trajectory *4*, $s = 2$, the amplitude of the Rashba's contribution on the trajectory *5* is three times larger, $s = 3$. The frequency of the periodic field in Figure 2 corresponds to the fundamental harmonic of the spin resonance, i. e. $\omega = \Delta_Z$. Other parameters in the Hamiltonian matrix (3) for evolution in Figure 2 are as follows: $\Delta_Z = 10.34\,\mu$eV, which corresponds to the magnetic field $B_z = 0.108$ T for $g$-factor $g = 1.35$ and the linear frequency of the electric field $f = 2.5$ GHz [5,6]. The tunnel splitting of the levels for the height barrier $U_0 = 4$ meV between the left and right QD in Figure 1, *a* is $\gamma = 2.2\,\mu$eV, the spin-orbital matrix element of the Dresselhaus SOI for tunneling with the spin flip $\alpha_D = 0.45\,\mu$eV, and for transitions in the same QD with a spin flip $\beta_D = 0.1\,\mu$eV. The magnitude of the bias (detuning) $U_d = -25\,\mu$eV, the amplitude of the periodic field $V_d = 75\,\mu$eV. Figure 3 shows results for the second subharmonics $k\omega = \Delta_Z$ which are similar to the results shown on Figure 2, where $k = 2$, i. e., for half the frequency of the electric field. A set of five trajectories corresponds to the same values $s$ for the amplitudes of the Rashba and Dresselhaus contributions as for the fundamental harmonic in Figure 2.

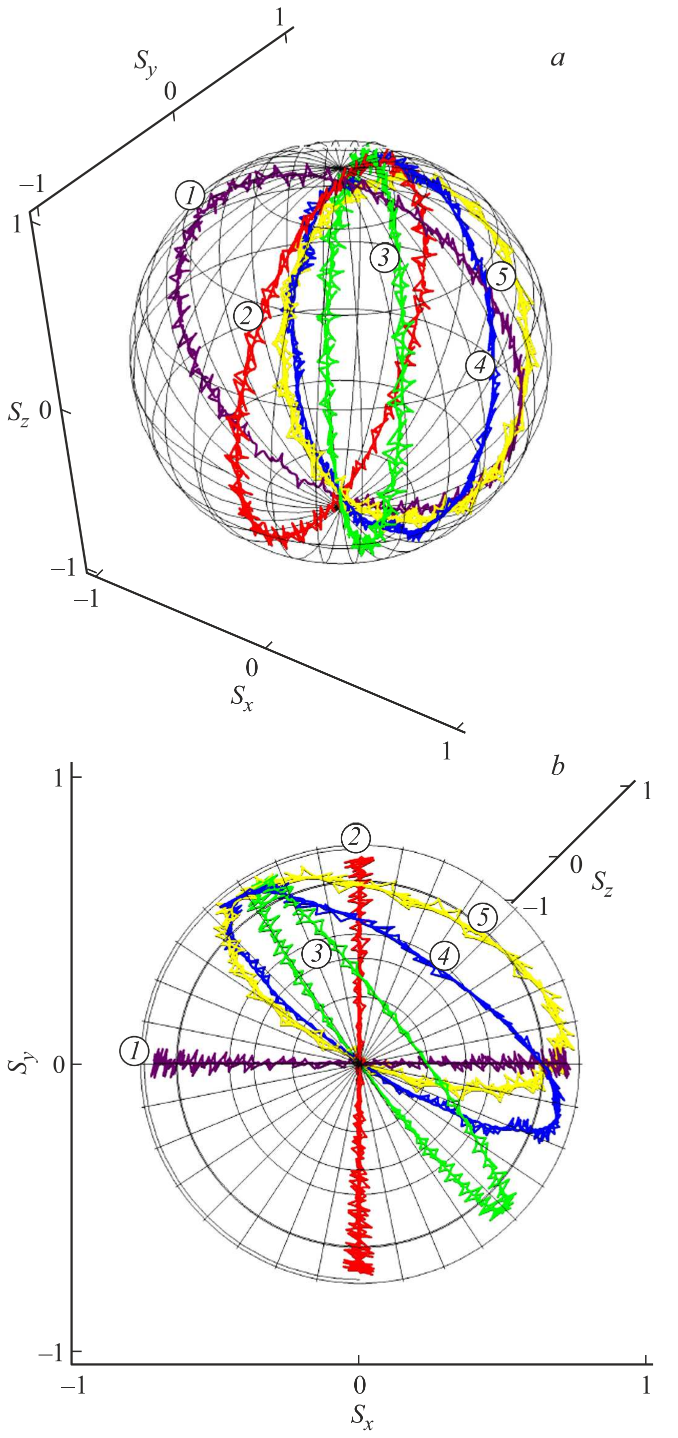


**Figure 2.** The spin dynamics on the lateral (*a*) and vertical (*b*) projections of the Bloch sphere in the right quantum dot, for the fundamental harmonic of the EDSR $\omega = \Delta_Z$. The trajectories *1–5* are plotted with different ratios $s = |\alpha_R|/|\alpha_D| = |\beta_R|/|\beta_D|$ between Rashba and Dresselhaus parameters in (3): *1* — $s = \infty$, only Rashba's contribution is present; *2* —$s = 0$, there is only a Dresselhouse contribution; *3* — $s = 1$; *4* —$s = 2$; *5* — $s = 3$. (A color version of the figure is provided in the online version of the paper).

## 4. Results and discussion

The results shown in Figures 2 and 3 suggest that a change in the ratio of the amplitudes of the Rashba and Dresselhaus contributions, which can be realized by the gate field by changing the value of the Rashba parameter, results in a rotation of the spin rotation plane in the range from 0 to $\pi/2$. This can be seen from the comparison of the two curves *1* and *2* with a limiting parameter *s* on the side (panels *a* in Figures 2 and 3) and vertical (panels *b* on

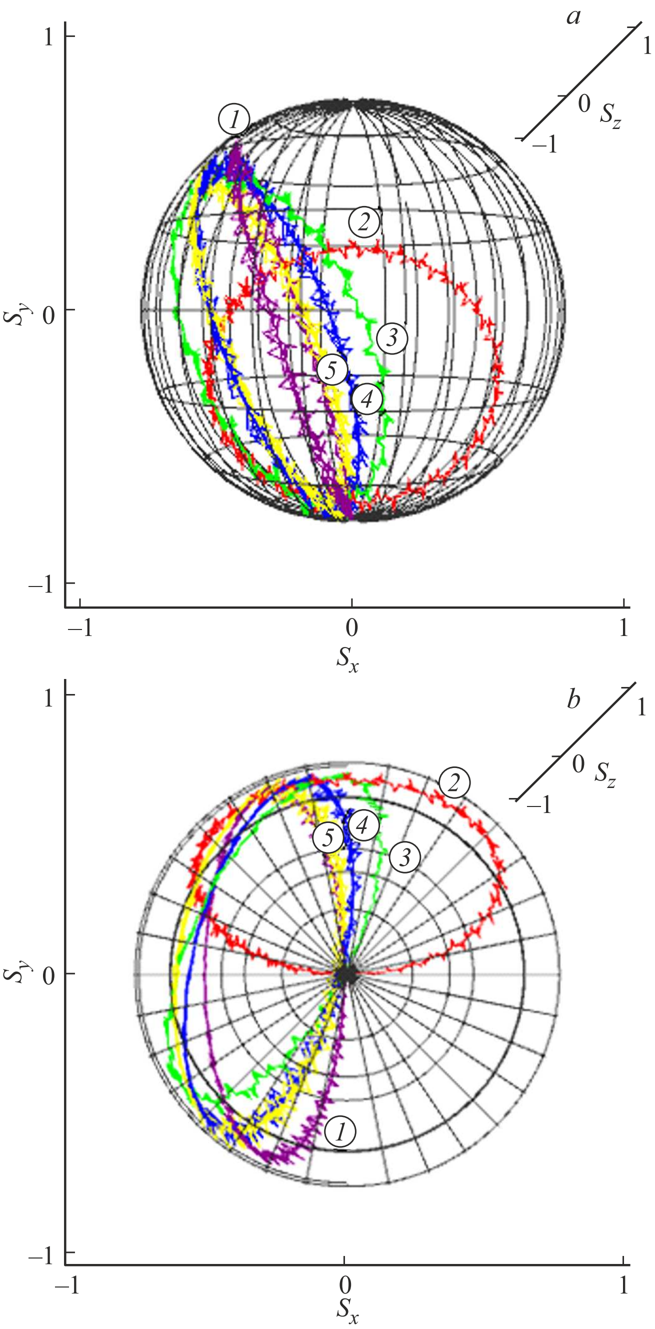


**Figure 3.** The same as in Figure 2, for the second subharmonic $k\omega = \Delta_Z$ with $k = 2$ (The colored version of the figure is available on-line).

Figures 2 and 3) projections of the Bloch sphere, while the specified rotation angle interval is achieved both on the main harmonic of resonance (Figure 2) and on the subharmonic $k = 2$ (Figure 3). This rotation of the spin rotation plane, together with the rotation of the spin from the south to the north pole during the resonance process, are the operations required to implement the concept of a spin qubit. It is possible to conclude that the tunable Rashba SOI results in the possibility of rotation for the spin rotation plane at EDSR, which will allow various spin operations to be performed on both the main harmonic and the subharmonics of EDSR.

## 5. Conclusion

The tunable spin-orbit interaction in semiconductor quantum dots can influence the position of the plane of spin rotations in an electric dipole spin resonance within a wide range through a change of the Rashba's contribution. Rotations of the spin rotation plane can be useful for implementing the concept of a spin qubit in semiconductor quantum dots with strong spin-orbit interaction.

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### Conflict of interest

The authors declare that they have no conflict of interest.

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