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# Spectral combs and periodic superradiant pulses in lasers with an asymmetric cavity

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Based on the Maxwell–Bloch equations, a numerical simulation of the regimes of asymmetric quasiperiodic generation of superradiance pulses in class D lasers with a homogeneous broadening of spectral line of a two-level active medium placed in a low-Q Fabry-Perot cavity with different reflection factors of mirrors is carried out. It is shown that in a wide range of laser parameters, the spectrum of its radiation is a comb consisting of a large number of equidistant lines and is discrete rather than continuous, despite the presence of dissipative (radiative) instability of the polarization waves with negative energy responsible for the induced collective Dicke superradiance.

**Keywords:** superradiant laser, asymmetric Fabry-Perot cavity, periodic pulses, spectral comb, discrete spectrum, continuous spectrum.

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## 1. Introduction

There are various possibilities for generating a periodic sequence of pulses with a spectrum in the form of a comb of equidistant discrete lines in the existing numerous lasers of classes A, B, C, where the photon lifetime $T_E$ in the cavity exceeds the phase relaxation time $T_2$ of the active medium polarization (i. e., the density of optical dipole moments of active centers) [1–13]. Such pulse sequences and spectral combs are obtained under a continuous-wave (CW) pumping, for example, due to synchronization of equidistant modes in a laser with a high-quality cavity and a wide spectral gain line of the active medium. However, an equidistant spectrum comb and periodic pulse generation can also be obtained in a laser with a low-Q cavity and a narrow, homogeneously broadened spectral line of the active medium using a nonlinear self-modulation of one laser mode in the absence of lasing of other cavity modes.

This statement is demonstrated in the paper by numerical solving the Maxwell-Bloch equations for a laser of the class D [1,14,15] with a low-Q asymmetric Fabry-Perot cavity, in which $T_E \ll T_2$ and in a wide range of parameters, a single-mode lasing of a periodic sequence of superradiance pulses under CW pumping is possible. Such an operation is owing to (i) the dissipative (radiative) instability of polarization waves that compose the laser polariton mode and have negative energy, and (ii) the dynamic nonlinearity of collective spontaneous Dicke superradiance that has an induced character in this case [16,17]. These factors, on the one hand, determine the instability of stationary (quasi-monochromatic) single-mode oscillations of a superradiant laser, and on the other hand, they allow the lasing of an irregular and even chaotic sequence of superradiance pulses with a quasi-continuous spectrum. The complex dynamics is possible under a strong CW pumping, many times greater than the laser threshold, in different dense (spatially and spectrally) ensembles of active centers, including semiconductor structures with impurities, excitons or electrons and holes in magnetized quantum wells (cf., for example, [10,18–31]).

The various lasing features, related mainly to the periodic operation we are interested in, are discussed in the following Secs. 4, 5, 6 for superradiant lasers with relatively moderate, strong and weak cavity asymmetry, respectively. The simplest laser model used and the basic requirements for its parameters, including the asymmetry of the cavity, as well as a qualitative description of the mechanism of periodic emission of identical coherent pulses under CW pumping are presented in the introductory Secs. 2 and 3. Main results and some open questions about the dynamics of superradiant lasers are given in Conclusion (section 6).

## 2. Superradiant laser with low-Q Fabry-Perot cavity

Despite the fact that the feasibility of lasing of a periodic sequence of superradiance pulses under CW incoherent pumping has already been discussed and demonstrated numerically in our works for rather complex combined cavities and active media with inhomogeneous broadening of the spectral line (see [13,14,32]), the specified periodic lasing has not been studied yet in the most popular model of a laser with a homogeneously broadened two-level active medium which is homogeneously distributed in an elementary Fabry-Perot cavity. This one-dimensional model is based on the semi-classical Maxwell–Bloch equations [14,15,33,34] for smoothly varying complex amplitudes of counterpropagating waves of the electromagnetic field and polarization of a medium consisting of 2-level centers with an optical dipole moment $d$ at the transition frequency $\omega_0$,

$$E = \mathrm{Re}\left[A_+(z,t)\exp(ik_0z - i\omega_0 t) + A_-(z,t)\exp(-ik_0z - i\omega_0 t)\right]/\sqrt{\varepsilon_0}, \quad (1)$$

$$P = \mathrm{Re}\left[P_+(z,t)\exp(ik_0z - i\omega_0 t) + P_-(z,t)\exp(-ik_0z) - i\omega_0 t)\right]\sqrt{\varepsilon_0}, \quad (2)$$

and for the inversion of populations of their energy levels, in which a half-wavelength grating $n_z(z,t)$ (with spatial period $\lambda_0/2 = \pi/k_0$ along the cavity axis $z$) separated from a smoothly inhomogeneous background $n(z,t)$ (both defined per one active center):

$$D(z,t) = n(z,t) + \mathrm{Im}\left[n_z(z,t)\exp(2ik_0z)\right]. \quad (3)$$

For certainty, the wavenumber $k_0 = \omega_0 c^{-1}\sqrt{\varepsilon_0}$ is assumed to coincide with the real part of the wavenumber of the working mode (and its frequency coincides with the transition frequency $\omega_0$) of the Fabry-Perot cavity with a length $B$ and mirrors having reflector factors (in field amplitude) of $R_2 < R_1$ and located at points $z = \pm B/2$, respectively ($c$ is the speed of light in vacuum, $\varepsilon_0$ is a dielectric permittivity of the active-medium matrix). Nonlinear dynamics of the considered physical quantities $n$, $n_z$, $p_\pm = P_\pm/(dN_0)$,

$$\alpha_\pm = a_\pm \frac{\nu_c}{\omega_0} = \frac{A_\pm}{2\pi d N_0} \equiv \frac{dA_\pm}{\hbar\nu_c\varepsilon_0} \quad (4)$$

obeys the well-known shortened Maxwell–Bloch equations presented in [15,34], as well as in [14], where the complex valued functions $\alpha_\pm$ are replaced by $a_\pm$ ($N_0$ is a concentration of active centers, $\hbar$ Planck's constant):

$$\left[\frac{\partial}{\partial\tau} \pm \frac{\partial}{\partial\xi}\right]\alpha_\pm = ip_\pm,$$

$$\left[\frac{\partial}{\partial\tau} + \Gamma_2\right]p_\pm = -in\alpha_\pm \mp \frac{n_z^{1,*}}{2}\alpha_\mp,$$

$$\left[\frac{\partial}{\partial\tau} + \Gamma_1\right](n - n_p) = \mathrm{Im}\,(\alpha_+^* p_+ + \alpha_-^* p_-),$$

$$\left[\frac{\partial}{\partial\tau} + \Gamma_1\right]n_z = \alpha_-^* p_+ - \alpha_+ p_-^*. \quad (5)$$

Here, the symbol $^*$ indicates a complex conjugation, which is present in the second equation only for wave amplitudes with lower signs. The pumping level, $0 < n_p \le 1$, and the rates of incoherent relaxation of population inversion and polarization of the active medium $\Gamma_{1,2} = 1/(T_{1,2}\nu_c)$ are also used. The decay rate of the field $\Gamma_E = 1/(T_E\nu_c)$ of a cavity mode is set by the boundary conditions $\alpha_+(-L/2) = R_1\alpha_-(-L/2)$ and $\alpha_-(L/2) = R_2\alpha_+(L/2)$ (and possible ohmic losses, which are omitted for simplicity in the first equation):

$$\Gamma_E = -\frac{\ln\sqrt{R_1R_2}}{L}. \quad (6)$$

The most important parameters are the cooperative length $B_c = c/\nu_c\sqrt{\varepsilon_0}$ and cooperative frequency

$$\nu_c = \sqrt{\frac{2\pi d^2\omega_0 N_0}{\varepsilon_0\hbar}}, \quad (7)$$

which are used for normalization of the spatial coordinate $\xi = z/B_c$, laser length $L = B/B_c$, time $\tau = \nu_c t$ and frequency detuning in the spectra $\Delta = (\omega - \omega_0)/\nu_c$. Then the following characteristic values of incoherent relaxation rates of polarization and population inversion $\Gamma_2 = 2\Gamma_1 = 0.02$, as well as the laser length $L = 2$ and four sets of mirror reflector factors were selected in most calculations: $(R_1, R_2) = (0.8, 0.5)$, $(0.9, 0.3)$, $(0.9, 0.1)$, $(0.5, 0.4)$, for which the field decay rate is $\Gamma_E = 0.23$, 0.33, 0.6, 0.4, respectively.

The equations (5) were solved for various laser parameters and levels of homogeneous CW pumping $n_p$ using the grid method and the modified Runge–Kutta method of the 4th order. Only the noise amplitudes of the polarization waves of the active medium were set finite, $|p_\pm| \sim 10^{-4}$, initial moment of time, and the electric field and the population-inversion grating were considered zero for simplicity: $\alpha_\pm = 0$, $n_z = 0$.

Non-stationary lasing takes place if the pumping level $n_p$ exceeds the so-called second laser threshold, which can be significantly higher than the first one, $n_0 = \Gamma_2\Gamma_E$, that corresponds to the appearance of a positive growth rate in the linearized equations (5) for the resonant polariton mode with the real frequency $\omega_0$

$$\Gamma = \sqrt{\bar{n} + \left(\frac{\Gamma_E - \Gamma_2}{2}\right)^2} - \frac{\Gamma_E + \Gamma_2}{2} \quad (8)$$

(normalized, as said, to the cooperative frequency (7)). For generality in this expression, in order to use the growth rate of the laser mode in further estimates in the presence of an inhomogeneous population inversion $n(\xi)$, its spatial average value $\bar{n}$ is given instead of the homogeneous pumping level $n_p$ considered here. According to numerous simulations [15,33–35], the nonstationarity of the resonant polariton mode is primarily originated from the excitation of neighboring modes (polariton or electromagnetic) if the Fabry-Perot cavity is sufficiently symmetrical, namely for the selected characteristic laser parameters, if the relative difference in the reflector factors of the mirrors is less than or of the order of 10%. We are interested in this paper in another mechanism of self-modulation of the resonant mode, associated with its own superradiant dynamics and in its pure form realized in more asymmetric cavities. It is described below and allows obtaining strictly periodic pulses of radiation at a not too high pumping level, for which other factors that violate the periodicity of lasing do not yet take place. They are attributable to the strong inhomogeneity of population inversion or the excitation of neighboring modes, which is inevitable under extreme pumping even in a very asymmetric cavity.

## 3. The mechanism of periodic lasing of superradiance pulses

To generate a pulse of mode superradiance in the considered laser of class D with $\Gamma_E > \Gamma_2$, it is actually required to achieve the inequality $\Gamma > \Gamma_2$, i.e., approximately $\bar{n} > 2\Gamma_2(\Gamma_E + \Gamma_2) \approx 2n_0$ according to (8). Its can be maintained for relatively short time, during a time $\Delta\tau$ less than or of the order of $\Gamma_1^{-1}$, but it should be sufficient for the pumping to have time to create a reserve of population inversion above the threshold value $n_0$ and ensure the development of dissipative, radiative instability of the polarization waves with negative energy [14,16,17]. In this case, the duration of the resulting pulse of the mode superradiance can be greater than or of the order of the photon lifetime in the cavity $T_E$, but it should be less than or of the order of the time of phase (incoherent) relaxation of polarization $T_2$.

The latter condition is known [15,16,36] to be necessary in the initial value problem of arising of collective spontaneous Dicke superradiance (get started with quantum or thermal field and polarization noises), and is essentially required for the considered periodic mode superradiance, which actually is an induced collective Dicke process. The only difference is that each subsequent pulse of this radiation does not originate spontaneously from noise, but is induced in a regular manner from the remnants of a coherent field (and polarization consistent with it) that have not yet leaved the low-Q cavity since the formation of the previous pulse. Therefore, self-modulation of the resonant polariton mode is attributable to the strong nonlinearity of the collective Dicke superradiance in these conditions, i.e., a sharp reset of the population inversion (locally, especially near mirrors, to negative values during the time interval of the order of $2/\max[\Gamma\nu_c]$ or several $T_E$) immediately after the generation and emission of the superradiance pulse.

The pulse repetition period in the generated sequence is mainly determined by the time of incoherent relaxation of population inversion $T_1$, which determines the characteristic time of the inversion increase in a partially deactivated medium under the action of CW pumping to an average level $\bar{n}$, significantly exceeding the second laser threshold, i.e. up to $\sim 2n_0$ and higher. The duration of the superradiance pulse is less than or of the order of time $T_2$ and cannot be less than the inverse cooperative frequency $\nu_c^{-1}$, and actually less than time $T_E$ at $L \sim 1$. In addition, the rate of incoherent relaxation of population inversion is always less than or of the order of the rate of phase relaxation of polarization in the two-level medium. Therefore, the described mechanism of self-modulation of the laser mode and, consequently, the formation of an equidistant comb of discrete spectral lines assumes the fulfillment of the inequalities $\Gamma_1 \lesssim \Gamma_2 \ll 1$. We perform calculations for $\Gamma_1 = 0.01$ at $\Gamma_2 = 0.02$ and $\Gamma_2 = 0.1$.

At the same time, the field decay rate is close to the value 0.3, i.e., $\Gamma_E \sim 0.3$, and is practically determined by the selected cavity length $L = 2$, since the reflector factors of the mirrors stand under the sign of the logarithm in the expression (6) and results in a factor of the order of 1 for adequate low-Q cavities. (It is difficult to achieve the threshold of lasing, especially the second laser threshold, for much shorter cavities, and the laser dynamics is irregular and corresponds to a quasi-continuous radiation spectrum for very long cavities due to the reabsorption of superradiance pulses.) As can be seen from the simulations provided below, the inverse duty cycle of the outcoming pulse sequence, and therefore the number of significant discrete lines in the spectrum turns out to be of the order of 10.

The range of pumping levels that ensure the implementation of this unique lasing and the nature of laser operation outside this range significantly depend on the asymmetry of the Fabry-Perot cavity. Three typical options are provided below.

## 4. Moderate asymmetry of the cavity

For a laser with mirror reflection factors of $R_1 = 0.8$ and $R_2 = 0.5$, a periodic sequence of superradiance pulses is generated at the pumping level of $n_p \sim 0.1$. According to Figure 1, with $n_p = 0.1$, the asymmetry factor of the laser radiation

$$r = \max\left[\frac{(1-R_2^2)|\alpha_+(\xi = L/2)|^2}{(1-R_1^2)|\alpha_-(\xi = -L/2)|^2}\right], \tag{9}$$

that is, the time-average ratio of the intensities of radiation from opposite mirrors in the steady lasing, is equal to $r = 3.5$ and is only 5% higher than this ratio for the cavity mode

$$r_0 = \frac{R_1(1-R_2^2)}{R_2(1-R_1^2)}. \tag{10}$$

The time-average level of population inversion $\langle\bar{n}\rangle_\tau = 0.034$ exceeds the threshold $n_0 = 0.0046$ almost 8 times and exceeds even stronger the average amplitude of the population-inversion grating $n_z$, which does not play a noticeable role, although it can reach a value of the order of 0.1 at particular time moments during superradiant lasing. The average (over the laser) population inversion $\bar{n}$ changes periodically, gradually increasing from a minimum value of 0.014 to a maximum of 0.05 and sharply decreasing again to a minimum. The decay time is slightly longer than the pulse duration $\tau_i \approx 10$, which is determined by the width of

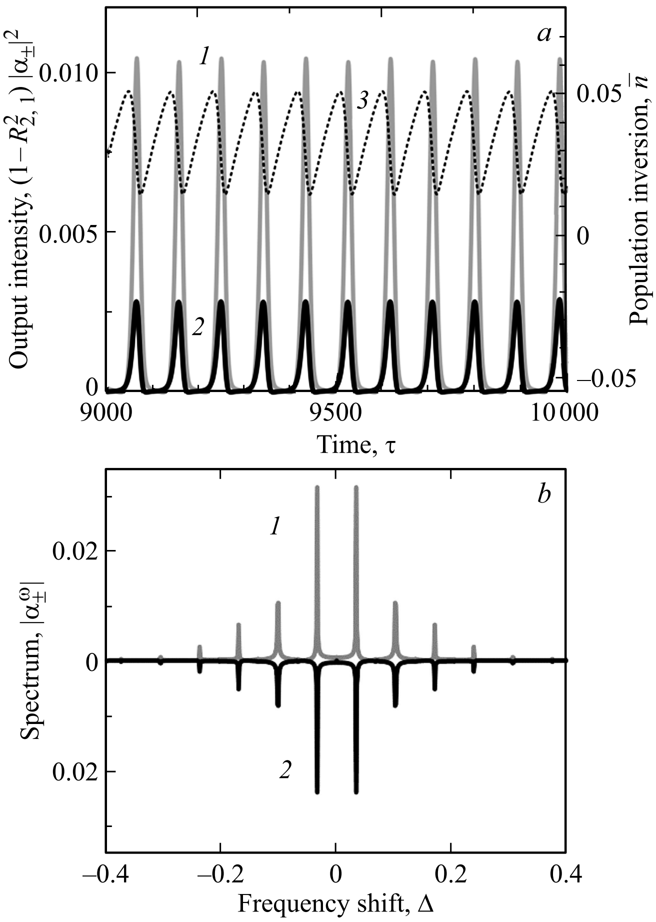


**Figure 1.** Periodic superradiance in a laser of length $L = 2$ with mirror reflection factors $R_1 = 0.8$ and $R_2 = 0.5$ at the pumping level $n_p = 0.1$ and polarization relaxation rate $\Gamma_2 = 0.02$. Oscillograms of the intensities of the outcoming radiation ($a$) and the amplitude spectra of the fields ($b$) at the opposite ends of the laser $\xi = L/2$ (light line *1*) and $\xi = -L/2$ (black *2*). The oscillogram of the average population inversion (dotted line *3*, corresponding to the ordinate axis on the right) is shown on the plot ($a$).

its intensity profile at half the maximum and approximately equal to twice the inverse growth rate of the polariton mode (8) $\Gamma_m \approx 0.12$ at the maximum population-inversion stage. The pulse repetition period is $\tau_s \approx 90$, i.e., it is a little less than the time $\Gamma_1^{-1} = 100$, and determines the step of the spectrum comb $2\pi/\tau_s \approx 0.07$.

A stationary single-frequency lasing of a polariton mode takes place at the pumping level $n_p \sim 0.01$ that does not significantly exceeds the threshold $n_0$, with an average population inversion slightly different from the threshold $n_0$ as well as with a very weak population-inversion grating and a radiation asymmetry factor $r = 3.62$, close to the „cold“ value $r_0 = 3.33$. A stationary lasing is also realized at a high pumping level $n_p \sim 0.5$, however, now the polariton mode is significantly changed by a strong asymmetric self-consistent population-inversion grating $n_z$, created by counterpropagating waves and having an amplitude of $\sim 0.1$. As a result, the grating and the radiation asymmetry factor determined by it radically depend on the pumping level, which, at the same time has almost no effect on the average (over the laser) population inversion. So, for $n_p = 0.5$ and 1, we get $r = 48.4$ and 97.7, whereas $\bar{n} = 0.04$ and 0.046, respectively.

## 5. Strong asymmetry of the cavity

In a more asymmetric cavity with $R_1 = 0.9$ and $R_2 = 0.3$, stationary lasing of the polariton mode is realized only at the pumping level $n_p \sim 0.01$, not much exceeding the threshold $n_0 = 0.0066$, and then again the average population inversion $\bar{n}$ differs slightly from the threshold $n_0$, the population-inversion grating $n_z$ is weak and the radiation asymmetry factor $r$ is close to the „cold“ value $r_0 = 14.3$.

Periodic lasing of superradiance pulses with an equidistant comb of the spectrum still occurs in a wide range of pumping levels at $n_p \sim 0.1$ and the qualitative conclusions of the previous section remain. Namely, $r \sim r_0$; the value of $\langle \tilde{n} \rangle_\tau$ exceeds the threshold $n_0$ several times; the population-inversion grating is insignificant, although it sometimes has an amplitude of $|n_z| \sim 0.1$; the average laser population inversion $\bar{n}$ periodically changes in magnitude in several times, sharply decreasing to almost zero during the emission of pulses; the time of this decrease is slightly longer than the duration of a pulse, determined by the doubled inverse growth rate of the polariton mode (8) at the moments when population inversion maxima are reached: $\tau_i \approx 2\Gamma_m^{-1} \sim 15$. As usual, the outgoing superradiance pulse leaves behind an area of an uninverted medium, where $n(\xi) < 0$, in the vicinity of the output mirror for a short time (fractions of time $T_1$).

Comparing these lasing features at $n_p = 0.1$ and 0.25, we have $r = 15.3$ and 12.1, $\langle \bar{n} \rangle_\tau = 0.03$ and 0.047, $\tau_s = 100$ and 120, $\tau_i = 20$ and 12, respectively. Thus, multiplication of the pumping level by 2.5 times increases significantly, but not very strongly the time-average population inversion (by 50%) and the period of the outcoming pulse sequence (by 20%), and therefore makes the spectral comb more frequent: from $2\pi/\tau_s = 0.06$ to 0.05. In addition, the width of the spectrum comb increases by $\sim 30\%$ (from $2\pi/\tau_i = 0.32$ to 0.5), since superradiance pulses become a third shorter with the indicated multiplication of the pumping level; at the same time, a pulse of almost half the intensity appears between two neighboring identical large pulses (large and small pulses are emitted in the opposite phase through the right and left mirrors of the cavity).

A very irregular (possibly chaotic) lasing of mode-superradiant pulses now takes place at a high level of pumping $n_p \sim 0.5$, instead of stationary generation. At $n_p = 1$, according to Figure 2a, random powerful short pulses are emitted from a poorly reflecting mirror (line 1), $n_p = 1$, and less powerful smoothed non-stationary radiation (line 2) comes out in the opposite direction, while the radiation asymmetry factor $r = 4.6$ is three times less

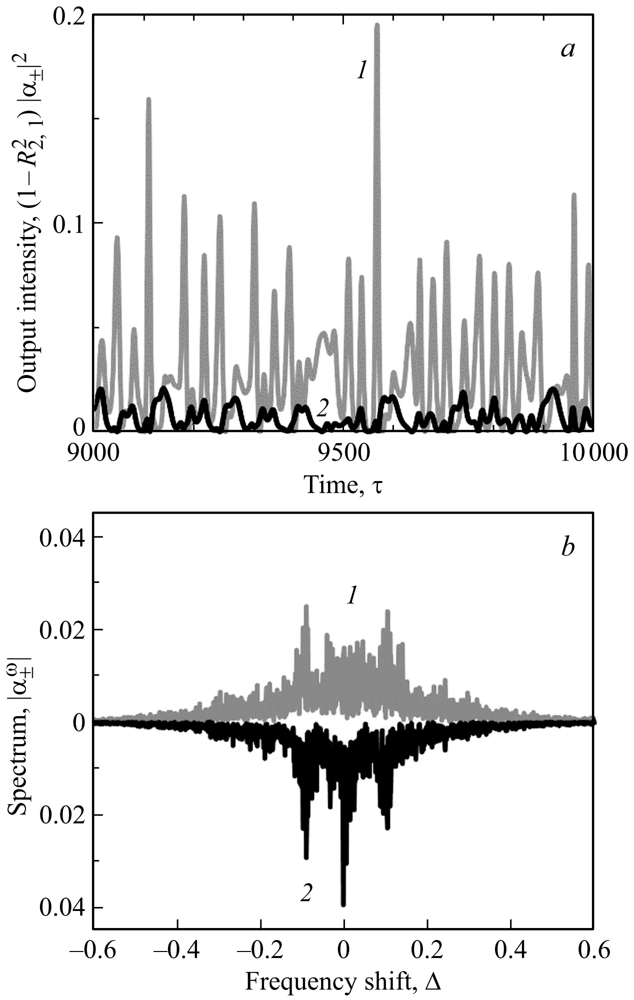


**Figure 2.** Irregular superradiance pulses in a laser of length $L = 2$ with mirror reflection factors $R_1 = 0.9$ and $R_2 = 0.3$ at the pumping level $n_p = 1$ and polarization relaxation rate $\Gamma_2 = 0.02$. Oscillograms of the intensities of the outcoming radiation (*a*) and the amplitude spectra of the fields (*b*) at the opposite ends of the laser $\xi = L/2$ (light line *1*) and $\xi = -L/2$ (black *2*).

than the „cold“ one $r_0$ and the time-average population inversion $\langle \bar{n} \rangle_\tau = 0.11$ is 17 times higher than the threshold $n_0$. Approximately the same dynamics is observed at $n_p = 0.5$ with $r = 5.8$ and $\langle \bar{n} \rangle_\tau = 0.094$, accompanied by strong local oscillations of the smoothly inhomogeneous component and the amplitude of the population-inversion grating within the ranges $-0.1{-}0.2$ and $0.1{-}0.4$ respectively (such oscillations are 1.5 times stronger at $n_p = 1$). However, in both cases, the spectrum is continuous with two well-defined smoothed peaks at frequencies around $\pm 0.1$. In the first case, it is 1.5 times wider and has a third central peak, which corresponds to a slowly changing component of radiation and is better represented at the output of a well-reflecting mirror than the opposite poorly reflecting one.

In a laser with an even greater, multiple difference of the reflection factors of the mirrors $R_1 = 0.9$ and $R_2 = 0.1$, where $\Gamma_E = 0.6$, i. e., the cavity has a very low-Q stationary lasing is realized only for the pumping level $n_p$ exceeding the threshold $n_0 = 0.012$ by no more than 5 times. Under these conditions, the average population inversion does not significantly exceed the threshold, $\bar{n} \leq 0.02$, but there is already a fairly significant population-inversion grating at $n_p = 0.06$ and even 0.05, increasing the radiation asymmetry factor from the „cold“ value $r_0 = 47$ to the values 197 and 157, respectively.

A strictly periodic lasing of superradiance pulses takes place in the rest of the pumping range, qualitatively demonstrating the same properties indicated above: a) well-formed short pulses for which the radiation asymmetry factor is suppresed, $r \lesssim r_0$; b) a spectral comb containing about 10 significant equidistant lines and having a width of the order of $\pi\Gamma_m$. The width of the comb monotonously triples from $\sim 0.4$ to 1.2, and its step $2\pi/\tau_s$ increases slightly less from $\sim 0.033$ to $\sim 0.09$ with an increase of the pumping level $n_p$ from 0.07 to 1.

The corresponding decrease of the pulse repetition period is accompanied by only a slight change of their shape in the pumping range of $0.07{-}0.1$, and is complemented in the range of $0.2{-}1$ by the appearance of significant intermediate pulses with an intensity of the order of 10–50% relative to the most powerful and short pulses, and the radiation profiles from opposite mirrors significantly differ in this case. The average population inversion $\langle \bar{n} \rangle_\tau$ increases by only 5% from 0.027 to 0.0283 in the first range, and the radiation asymmetry factor drops by about 30% from 68 to 49, remaining above the „cold“one: $r > r_0$. These values vary much more significantly in the second range — by 2.5 times from 0.039 to 0.098 and by 4 times from 28 to 7.3 ($r < r_0$), respectively.

## 6. Weak asymmetry of the cavity

In a more symmetrical cavity with $R_1 = 0.5$ and $R_2 = 0.4$, i. e., with a total of 20% difference of mirror reflection factors, when $r_0 = 1.4$, $\Gamma_E = 0.4$ and $n_0 = 0.008$, superradiance pulses are not generated and, according to numerical modeling, one stationary polariton mode is excited for all pumping levels above the threshold. The population-inversion grating associated with it is weak at $n_p \sim 0.01$, the radiation asymmetry factor $r$ is small and differs little from the „cold“value $r_0$, and the average population inversion $\bar{n}$ is close to the threshold $n_0$. The radiation asymmetry factor increases almost 10 times from $r = 2.8$ and 6.3 to 12.6 and 25.8 with an increase of pumping from $n_p = 0.1$ and 0.25 to 0.5 and 1, and the average population inversion changes only 1.5 times from $\bar{n} = 0.027$ and 0.037 to 0.044 and 0.05, respectively. A similar stationary lasing, already noted in Secs. 4 and 5, allows for an analytical description taking into account the key role of the population-inversion grating, which will be done in a separate paper.

For the present work, it is essential that a strictly periodic and quasi-periodic sequence of superradiance pulses will be generated, respectively if, for example, the relaxation

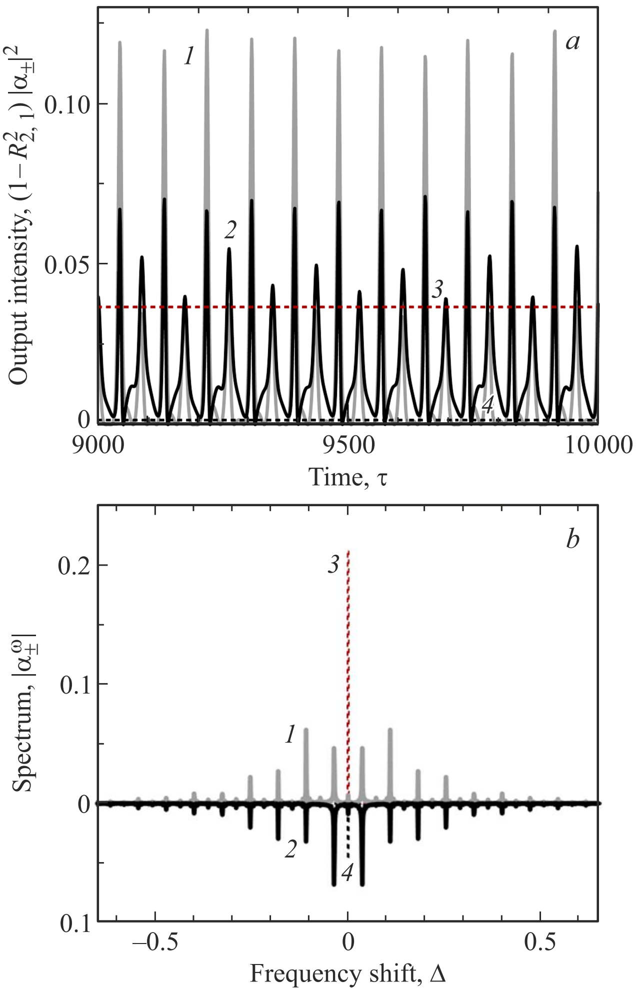


**Figure 3.** Quasi-periodic superradiance in a laser of length $L = 2$ with mirror reflection factors $R_1 = 0.5$ and $R_2 = 0.4$ at the pumping level $n_p = 1$ and the polarization relaxation rate $\Gamma_2 = 0.1$. Oscillograms of the intensities of the outcoming radiation (*a*) and the amplitude spectra of the fields (*b*) at the opposite ends of the laser $\xi = L/2$ (light line *1*) and $\xi = -L/2$ (black *2*). For comparison, the dashed lines show the oscillograms and spectra at the ends of the laser $\xi = L/2$ (light line *3*) and $\xi = -L/2$ (black *4*) for strongly asymmetric single-mode lasing with the same parameters, except with the relaxation rate, reduced by 5 times: $\Gamma_2 = 0.02$. (A color version of the figure is provided in the online version of the paper.)

rate of the polarization of the active medium is increased by 5 times at $n_p = 0.5$ and 1, i.e., taking $\Gamma_2 = 0.1$ (thereby changing the excess over the threshold $n_0 = \Gamma_2\Gamma_E$). In the first case, at $n_p = 0.5$, it has general properties already been specified in the Secs. 4, 5 and shown in Figure 1, and is characterized by the following numerical parameters: $\tau_i \approx 2\Gamma_m^{-1} = 9$, $2\pi/\tau_i = 0.7$, $\tau_s = 60$, $2\pi/\tau_s = 0.1$, $r = 1.6$, $\langle\bar{n}\rangle_\tau = 0.15 > n_0 = 0.04$.

In the second case, at $n_p = 1$, there is a large number of very weak spectral components that correspond to significantly weaker and more variable superradiance pulses following the same periodicity, but having a low-power variable pedestal, as can be seen from Figure 3, in addition to the discrete spectrum of the main equidistant comb, which corresponds to the periodic sequence of almost identical powerful superradiance pulses. In other words, a very weak almost continuous component appears in the spectrum at the pumping level $n_p$ close to 1. The quantitative parameters of the equidistant spectral comb we are interested in and the corresponding „averaged“ superradiance pulses, of course, change very noticeably with the considered twofold increase of pumping to the level $n_p = 1$: $\tau_i \approx 2\Gamma_m^{-1} = 6$, $2\pi/\tau_i = 1.04$, $\tau_s = 86$, $2\pi/\tau_s = 0.07$, $r = 1$, $\langle\bar{n}\rangle_\tau = 0.18 > n_0 = 0.04$.

The simulation of the dynamics of an asymmetric-cavity laser under CW pumping shows that the single-mode lasing of periodic superradiance pulses, which gives a discrete spectrum in the form of a comb, is possible if there is significant (in several times), but not excessive (by orders of magnitude) surpassing of the laser threshold $n_0 = \Gamma_2\Gamma_E$. It can be achieved by changing both the pumping level $n_p$ and the rates of phase relaxation of polarization $\Gamma_2$ and the decay of the field in the cavity (6) $\Gamma_E$, in particular, due to variations of its length $L$ (at $n_p > n_0$ and maintaining the conditions of a class D laser, including the condition for the rate of incoherent relaxation of population inversion $\Gamma_1 \lesssim \Gamma_2 \ll 1$). A single-mode stationary lasing takes place in the case of insufficient threshold exceeding. When the threshold $n_0$ is excessively exceeded, the discrete radiation spectrum is supplemented or replaced by a continuous radiation spectrum, which is attributable to the reabsorption of superradiance pulses, the desynchronization of inhomogeneous counterpropagating waves of the field and polarization, and the irregular population-inversion grating created by them inside an excessively long laser.

## 7. Conclusion

Thus, the spontaneous formation of a periodic sequence of very short coherent superradiance pulses of a resonant polariton mode is predicted and numerically studied for a wide range of parameters of an asymmetric laser of class D in the presence of CW incoherent pumping of an active two-level medium with a homogeneous broadening of the spectral line.

The analysis shows that the relaxation characteristics of the active medium and the cavity, including the rate of incoherent relaxation of population inversion $\Gamma_1$, the width of the spectral line of the laser transition $2\Gamma_2$ (both homogeneous and inhomogeneous) and the reflection factors of mirrors $R_{1,2}$ (as well as ohmic losses in the cavity) significantly affect the properties of the predicted strictly periodic lasing of superradiance pulses — their amplitude, duration, repetition rate, mirror asymmetry of emission and the corresponding parameters of the equidistant comb of the spectrum. This unique single-mode highly unsteady behaviour is actually an induced collective Dicke mode superradiance.

Further study of this intriguing operation and determination of the range of all class D laser parameters (its length, pumping level, etc.), allowing the implementation of the resulting equidistant spectrum comb, are of interest both for the fundamental physics of manyparticle systems with radiation interaction and, possibly, for practical applications of superradiant lasers with low-Q cavity. Attention needs to be paid to the studies of the features of such a pulsed superradiant lasing in case of CW pumping of more complex active media, which are placed into more complicated cavities, for example, in the presence of distributed feedback of counterpropagating waves, phase-shifting mirrors and frequency detuning of the cavity mode from the center of the spectral line of the medium.


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### Conflict of interest

The authors declare that they have no conflict of interest.

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