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Creation of a dynamic microcavity by collision of half-cycle light pulses in a resonant medium

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A simple theoretical approach is proposed, based on an approximate solution of the time dependent Schrödinger equation in the first order of perturbation theory, showing the possibility of creating a dynamic microcavity in the collision of half-cycle, extremely short light pulses in a multilevel medium. Based on the proposed approach, the possibility of controlling the parameters of a microcavity with an increase in the number of colliding pulses is shown.

Keywords: population difference gratings, dynamic cavities, extremely short light pulses, half-cycle pulses.

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Introduction

In recent years, tremendous progress has been seen in generation of ultrashort electromagnetic pulses with duration of a few electromagnetic field oscillations in the attosecond range [1,2]. This allows to study and control electron dynamics in a substance on intra-atomic time scales comparable with the orbital period of an electron in an atom [3–5]. Further reduction of the pulse duration leads to occurrence of pulses with inherently extremely short duration equal to a half of field period. Such pulses are called half-cycle or unipolar pulses [6]. They have a nonzero electric area defined in a given point of space $\mathbf{r}$ as an integral of field strength $\mathbf{E}$ over time t [7]

$$\mathbf{S}_E = \int \mathbf{E}(\mathbf{r}, t)dt. \quad (1)$$

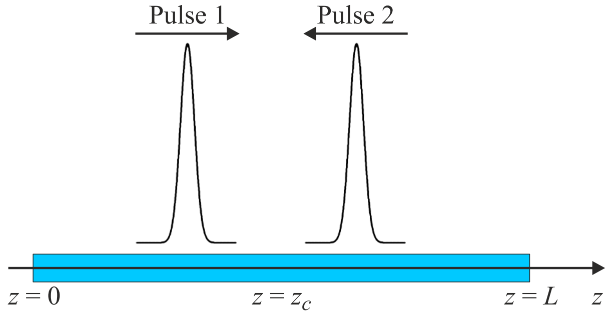
Half-cycle light pulses have good prospect of application for ultrafast control of atoms, molecules, quantum dots, charge acceleration, etc., see reviews [6–9] and the cited literature. When the pulse duration is shorter that the orbital period of the electron in an atom, then its action on a microobject is defined by the electric area, rather than by the pulse energy [10]. The half-cycle pulses are, thus, able to control quantum systems faster and more efficiently than traditional multicycle, bipolar pulses.

One of the possible applications is the generation of extremely short pulses and ultrafast control of population difference gratings when pulses do not overlap immediately in a medium or collide in the center of a medium [11–13], see also [14] and the cited literature. This effect occurs in the coherent interaction between pulses and a medium when the pulse duration and delays between pulses are shorter than the medium polarization relaxation time T_2. Occurrence of atomic population gratings in this case is attributed to the interaction between incident pulses and travelling polarization waves of a resonant medium induced by the previous pulse [11–14]. In case of a rare medium and when the pulse amplitude is small, it is suggested that the gratings occur due to the interference of the electric areas of incident pulses [15].

In [16], one interesting phenomenon has been found — possibility of inducing population gratings in a medium in the form of dynamic „microcavities" arising from collision of unipolar nonharmonic pulses in a two-level medium. In this case, the population difference in the pulse overlap area is almost constant, and on its periphery the population difference varies in space or has another value, other than that in the pulse overlap area. In this sense, a refractive index step of a medium occurs which suggests that a dynamic „microcavity" with controlled is formed in the medium.

In subsequent studies [17,18], the dynamics of such structures has been studied in more detail in a two-level medium. It has been shown that the effect of creation of dynamic microcavities also takes place in a three-level medium [19,20]. All these works have performed the analysis through the numerical solution of the Maxwell −Bloch system of equations. Meanwhile, the approximate analytical description of gratings was proposed only for the case when pulses do not overlap in a medium [11–14], and for overlapping unipolar pulses with unusual waveforms [20].

The objective of this study is to investigate a simple approach that theoretically predicts the occurrence of dynamic microcavities in collision of half-cycle light pulses in the center of a resonant medium. The approach is also used to describe the behavior of these microcavities and to evaluate the properties of the induced population difference gratings. This approach uses an approximate solution of the Schrödinger time equation through perturbation theory



A pair of half-cycle pulses moves over the medium from opposite directions and collides in the center of the medium in a point with coordinate $z = z_c$.

when the excitation pulse field amplitude is low, and is applicable to a rare multi-level medium. It is a logical generalization of a previous approach [11–14] used to calculate the dynamics of population gratings through a sequence of pulses that don't overlap in a medium.

Dynamics of microcavities

Let a pair of half-cycle pulses collides in a multi-level medium at some undetermined time in the center of the medium in a point with coordinate $z = z_c$ (Figure).

As in [11–15], for the purpose of the approximate analytical description of gratings, the medium is assumed to be rare, thus, neglecting the pulse waveform during propagation and the influence of particles on each other. It can be easily shown in this case that the problem of pulse sequence action on an extended medium is limited to a problem of excitation of a single atom or molecule by a variable-delay pulse sequence.

Dynamics of the quantum system in the pulse field is described by the Schrödinger time equation for the electron wave function [21]:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[\hat{H}_0 + V(t)\right]\psi. \tag{2}$$

Here, $\hbar$ is reduced Planck's constant, $\hat{H}_0$ is the system's Hamiltonian, V is the potential of interaction between an atom and external field that for the dipole approximation case is written as: $V(t) = -dE(t)$, d is the dipole moment of an atom. In a low-field approximation, the population of the k-th medium energy level is calculated to the first order in perturbation theory using the following expression [21]:

$$w_{1k} = \frac{d_{1k}^2}{\hbar^2}\left|\int E(t)e^{i\omega_{1k}t}dt\right|^2. \tag{3}$$

Here, d_{1k} is the transition dipole moment, ω_{1k} is the medium transition frequency. We take a temporal shape of incident half-cycle Gaussian pulses 1 and 2 transmitted with the delay Δ:

$$E(t) = E_{01}\exp\left[-t^2/\tau_1^2\right] + E_{02}\exp\left[-(t-\Delta)^2/\tau_2^2\right]. \tag{4}$$

Apparently, the electric areas of such pulses are $S_{E,1,2} = E_{01,2}\tau_{1,2}\sqrt{\pi}$. For simplicity of calculation of populations using expression (3), we assume below that the half-cycle pulse duration (4) is much lower than the resonant transition period of the medium $T_{1k} = 2\pi/\omega_{1k}$, $\tau_{1,2} \ll T_{1k}$ (sudden perturbation approximation [22–24]). In this case, the exponent under the integral in (3) is small compared with 1. When calculating the populations w_{1k}, the exponential factor $e^{i\omega_{1k}t}$ under the integral sign is approximately equal to 1 and is inessential.

According to the numerical experimental results [16–20], when a pair of unipolar pulses collides in the center of the medium, after the first collision in the vicinity of the pulse overlapping area, the population difference is constant in space. On either side, to the left an to the right of the overlapping area at $z \ll z_c$ and $z \gg z_c$, where the pulses don't overlap, a population difference grating may occur. Using expression (3), we derive the expressions for populations at $z \ll z_c$ and $z \gg z_c$ [14]:

$$w_k = \frac{d_{1k}^2 S_{E,1}^2}{\hbar^2} + \frac{d_{1k}^2 S_{E,2}^2}{\hbar^2} + 2\,\frac{d_{1k}^2}{\hbar^2}\,S_{E,1}S_{E,2}\cos\omega_{1k}\Delta. \tag{5}$$

Hence, w_k is defined by a sum of squares of the electric areas of pulses and depends periodically on the delay between pulses Δ, is defined by the interference of the pulse areas [15].

In the pulse overlapping area, a pair of pulses (4) acts as a single pulse whose electric area is equal to the sum of areas of the initial pulses $S_E = S_{E,1} + S_{E,2}$. And to calculate the population in the overlapping area using the expression, it necessary to set a zero delay between pulses $\Delta = 0$ in (5)

$$w_{1k} = \frac{d_{1k}^2}{\hbar^2}\,S_E^2. \tag{6}$$

Thus, in the vicinity of the point with $z = z_c$, the population in the pulse collision area is defined by a square of the total electric area of pulses. And outside the pulse overlapping area $z \ll z_c$ and $z \gg z_c$, the populations are described by expression (6). This expression shows that a periodic population grating may be created on the periphery of the pulse overlapping area. Dependence of the populations on the spatial coordinate is contained in the delay $\Delta \sim z/c$ that, in case of an extended medium, is proportional to the time of second pulse arrival at a point in the medium with coordinate z [13,14].

It is apparent that this approach makes it possible to predict the appearance of a microcavity with Bragg type mirrors in the medium — population in the pulse overlapping area is constant and defined by expression (5). On either side of the overlapping area, the medium population and, accordingly, the refractive index vary periodically in space according to expression (6). Results of this prediction agree with the numerical calculations using the Maxwell–Bloch system of equations that predict the appearance of such type of microcavity [16–20].

It is apparent that if these pulses collide again in he medium after some time, then the above-mentioned procedure may be also applicable to the calculation of the modified microcavity parameters. In case of the impact of 4 identical pulses, the filed strength will be written as

$$E(t) = E_0 \exp\left[-t^2/\tau^2\right] + E_0 \exp\left[-(t-\Delta)^2/\tau^2\right]$$
$$+ E_0 \exp\left[-(t-\Delta-\Delta_{23})^2/\tau_2\right]$$
$$+ E_0 \exp\left[-(t-\Delta-\Delta_{23}-\Delta_{34})^2/\tau^2\right] \qquad (7)$$

where Δ_{23} and Δ_{34} is the delay between the second and third, fourth pulses, respectively. $S_{E0} = E_0\tau\sqrt{\pi}$ is the electric area of pulses. For the bound state populations outside the pulse overlapping area at $z \ll z_c$ and $z \gg z_c$, we have [13]:

$$w_{1k} = 2\frac{d_{1k}^2 S_{E0}^2}{\hbar^2}\Big|1 + e^{i\omega_{1k}\Delta}$$
$$+ e^{i\omega_{1k}\tau} e^{i\omega_{1k}\Delta_{23}} + e^{i\omega_{1k}\Delta} e^{i\omega_{1k}\Delta_{23}} e^{i\omega_{1k}\Delta_{34}}\Big|^2. \qquad (8)$$

This expression is used to calculate the parameters of a dynamic microcavity modified as a result of the second collision of pulses. The similar applies to the calculation of the change in parameters of such cavity after the third collision, etc.

Conclusion

The study proposes a simple analytical approach based on an approximate solution of the Schrödinger time equation to the first order in perturbation theory the analysis of which is used to predict the possibility of creation and ultrafast control of dynamic microcavities induced during collision of half-cycle light pulses in the resonant medium. This approach is applicable when the incident pulse amplitude is small and the medium is rare. Results of the analysis using this approach predict the occurrence of a dynamic microcavity after pulse collision — population in the overlapping area is almost constant, and a harmonic population difference grating (Bragg type mirror) occurs on either side of the area. These results agree qualitatively with the results of numerical solution of the Maxwell-Bloch system of equations for the extended medium at small excitation field amplitudes [16–20]. Note also that the proposed approach may be used to calculate the dynamics of such cavities through the use of a sequence of a greater number of colliding pulses. The investigated microcavities may be interesting for short-term light storage systems, ultrafast optical switches and other ultrafast optical applications.

Conflict of interest

The authors declare that they have no conflict of interest.

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