12

The problem of correct measurements of the width and diverge angle of a laser beam

© A.M. Raitsin, S.M. Frolovichev

Moscow Technical University of Communications and Informatics, Moscow, Russia e-mail: arcadiyram@rambler.ru; sergeym.frolovichev@gmail.com

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> Using examples of known spatial distributions of intensity in the cross section of a laser beam in the plane of the emitter, formulas for the measurement error of the second moments in the far zone by a matrix radiation detector associated with determining the beam width and divergence angle using the existing standard ISO 11146:2021 "Lasers and laser-related equipment. Test methods for laser beam widths, divergence angles and beam propagation ratios, Part $1-2^{\circ}$.

> Gaussian, exponential and uniform spatial intensity distributions in the emitter plane are considered. It is shown that the use of the mentioned standard leads to incorrect measurements due to the divergence of the measured value. In this case, the conditions ensuring the convergence of results are practically impossible to fulfill. Recommendations for the measurement process are proposed that eliminate the noted drawback. Key words: metrology of laser radiation, measurement methods, moments of intensity distribution, matrix radiation receiver, laser beam width, divergence angle.

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Introduction

Standard method for measuring laser beam width and divergence angle is currently based on determining the initial moments of spatial intensity distribution in the beam cross-section [1].

Development of matrix radiation receiver (MRR) components provided the opportunity for almost real-time measuring the above-mentioned parameters. However, when measuring the MRR beam width and divergence angle using the initial moments of intensity distribution, there is a methodological measurement accuracy issue in that there are generally no finite values of the second moments of the radiated field.

This issue was first focused on by Yu.A. Ananyev in [2]. Discussion of the measured properties has identified that they gave reasonable finite value only when stringent conditions imposed on the form of field distribution were met. These stringent conditions have not been rigorously formulated, but it has been established that, when there are discontinuities in the dependence of spatial intensity distribution on transverse coordinates, the second moments of angular distribution associated with the parameter M^2 , width and divergence angle become unrestrictedly large.

In [3], such conditions are formulated and it is proved theoretically that the second moments diverge also for all continuous spatial intensity distributions, if the field amplitude modulus or intensity at the boundary of the emitter aperture is nonzero, which occurs in actual practice. However, despite this circumstance, International Standard ISO 11146-2005 has been introduced since the early 2000s. This standard specified methods for measuring laser beam width and divergence angle and contained the above-mentioned measurement accuracy issue. In 2008, this standard was adopted in the Russian Federation as GOST R ISO 11146-2008 and represented the complete authentic text of ISO 11146-2005. In 2021, a new version of the international standard was published [1], where the measurement accuracy issue was not resolved.

This fact has encouraged the authors to return to the issue identified by Yu.A. Ananyev as early as in 1990 and to show the errors arising in practice through the case study method [1].

This study is the visual illustration of the above-mentioned incorrect measurements and its objective is to investigate the second moment measurement error induced by nonzero values of the amplitude modulus or intensity at the boundary of the emitter aperture as well as by the limited MRR measurement range. A number of known spatial distributions of amplitude in the emitter plane was chosen for illustration: Gaussian, exponential and uniform.

The choice of these distributions is determined by the fact that the Gaussian spatial amplitude distribution with quickly decreasing intensity in laser beam cross-section that is of interest when investigating laser systems corresponds to the radiating fundamental mode of the resonator. Uniform distribution is also of interest for practical applications; it features constant amplitude in the emitter plane that is nonzero at the aperture boundaries. The exponential amplitude distribution is intermediate between the Gaussian and uniform distributions. This model is visible and convenient for calculations and leads to far-field intensity distribution with weakly decreasing "wings" resembling the Cauchy distribution.

Divergence of the measured quantity impairs the uniformity of laser beam parameter measurements. Metrological aspects of this issue have been discussed in detail in [4,5], where it has been shown that the issue may be resolved by considering the lower level of the MRR dynamic range.

The study also proposes the recommendations for the measurement process to rectify the above-mentioned short-coming.

In [6], an attempt has been first made to characterize laser radiation using the spatial movement language and a parabolic behavior of the dependences of the second moments of intensity distribution on distance z. The parameter M^2 governing the laser beam "quality" is examined on the same basis [7].

Reasoning leading to such dependence of the second moments is based on the quasioptics equation. This equation implies the known connection between the complex field amplitude in the radiating field beam cross-section U(x, y, z) with a z coordinate and the complex radiating field amplitude in the source plane $u(x_1, y_1, 0)$ [8]:

$$U(x, y, z) = A(z) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x_1, y_1, \mathbf{0})$$

$$\times \exp\left(i \frac{\pi}{\lambda z} \left((x - x_1)^2 + (y - y_1)^2\right)\right) dx_1 dy_1, \quad (1)$$

where

$$A(z) = \exp(i2\pi z/\lambda)/(i\lambda z),$$

 λ is the radiation wavelength.

We write the radiating field characteristics in terms of the first and second moments of intensity distribution I(x, y, z) normalized to the zeroth moment:

$$\overline{m}_{10}(z) = m_{10}(z)/m_{00}, \quad \overline{m}_{01}(z) = m_{01}(z)/m_{00},$$
$$\overline{m}_{20}(z) = m_{20}/m_{00}, \quad (2)$$

$$\overline{m}_{02}(z) = m_{02}(z)/m_{00}, \quad \overline{m}_{11}(z) = m_{11}(z)/m_{00},$$

$$m_{10}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xI(x, y, z)dxdy, \qquad (3)$$

$$m_{01}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y I(x, y, z) dx dy, \qquad (4)$$

$$m_{20}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 I(x, y, z) dx dy, \qquad (5)$$

$$m_{02}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} y^2 I(x, y, z) dx dy, \qquad (6)$$

$$m_{11}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy I(x, y, z) dx dy, \qquad (7)$$

$$m_{00}(z) = m_{00}(0) = m_{00} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y, z) dx dy, \quad (8)$$
$$I(x, y, z) = |U(x, y, z)|^{2}.$$

For the Gaussian elliptical intensity distribution, the dependence of the beam width at the intensity level $1/\exp(2)$ on the distance z is defined using the following equations [1] in terms of the normalized moments:

$$\begin{aligned} d_{x}(z) &= \\ 2\sqrt{2 \Big[\sigma_{20}^{2}(z) + \sigma_{02}^{2}(z) + \gamma \sqrt{\left(\sigma_{20}^{2}(z) - \sigma_{02}^{2}(z) \right)^{2} + 4\sigma_{11}^{2}(z) \Big]}, \\ d_{y}(z) &= \\ 2\sqrt{2 \Big[\sigma_{20}^{2}(z) + \sigma_{02}^{2}(z) - \gamma \sqrt{\left(\sigma_{20}^{2}(z) - \sigma_{02}^{2}(z) \right)^{2} + 4\sigma_{11}^{2}(z) \Big]}, \\ g_{20}(z) &= \overline{m}_{20}(z) - \left(\overline{m}_{10}(z) \right)^{2}, \\ \sigma_{20}^{2}(z) &= \overline{m}_{02}(z) - \left(\overline{m}_{01}(z) \right)^{2}, \\ \sigma_{11}(z) &= \overline{m}_{11}(z) - \overline{m}_{10}(z) \overline{m}_{01}(z), \\ \gamma &= \frac{|\sigma_{20}^{2}(z) - \sigma_{02}^{2}(z)|}{\sigma_{20}^{2}(z) - \sigma_{02}^{2}(z)} = \begin{cases} 1, & \text{if } \sigma_{20}^{2}(z) \geq \sigma_{02}^{2}(z), \\ -1, & \text{if } \sigma_{20}^{2}(z) < \sigma_{02}^{2}(z). \end{cases} \end{aligned}$$

Equation (1) in [8] has been derived under the assumption on transverse dimensions of the source. Therefore, the replacement of the finite limits of integration in (1) with the infinite limits throughout the emitter positioning plane is an assumption to be defined.

For this, in [8] it is assumed that the modulus of field amplitude or radiation source intensity outside the emitter aperture is equal to zero, but is not necessarily equal to zero at the aperture boundary.

In [3] it is shown that for all continuous intensity distributions the condition under which the second moments of radiating field exist is reduced to the intensity (amplitude) I(x, y, 0) equal to zero at the aperture boundaries in the emitter plane, which is not generally fulfilled in a real measurement process. Expressions (5) and (6) represent diverging integrals and width characteristics (9) of the radiating field lose their meaning.

In actual practice, the second moments of the measured field are calculated:

— by a limited region of space Ω defined by the MRR aperture size in the measurement plane;

— using MRR with a limited dynamic measurement range relative to the intensity distribution $0 < r \le I(x, y, z)/I_{\text{max}} \le 1$, where r is the lower limit of detection of the MRR measurement range.

In this case, the integration in (3)-(8) is conducted in finite limits and the second moments always exist.

However, the result of such measurements is applicable to the characteristics of the measured field, rather that of the radiating field that has no such characteristic, which defines the methodological measurement accuracy issue described in [4,5].

Second moment of the measured laser beam field with the Gaussian spatial amplitude distribution in the emitter plane

We will deal with the far-field distribution of laser beam intensity in the Fraunhofer approximation [8]

$$I(x, y, z) = \frac{1}{\lambda^2 z^2} \left| \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} u(x_1, y_1, 0) \right| \\ \times \exp\left(-i \frac{2\pi}{\lambda z} (xx_1 + yy_1)\right) dx_1 dy_1 \right|^2, \quad (10)$$

integration in (10) is performed within the square emitter aperture with the linear dimensions $T \times T$,

$$u(x_1, y_1, 0) = R^{4(x_1^2 + y_1^2)/T_0^2}$$
(11)

— the Gaussian spatial distribution of amplitude in the emitter plane equal to $R(0 < R \le 1)$ with $x_1 = \pm T_0/2$; $y_1 = 0$ or $x_1 = 0$; $y_1 = \pm T_0/2$.

If $T = T_0$, then the amplitude at the emitter aperture boundaries is also equal to *R*.

Then, we consider the normalized second moment (2) of the measured field intensity distribution (10) within the limited square aperture of MRR with the linear dimensions $L \times L$ in the measurement plane. The normalized second moment is calculated using equations (5) and (8) where finite limits of integration are replaced with infinite limits from -L to L.

After substitution of (11) into (10) and transformation for the second moment of the Gaussian beam (2) we obtain

$$\overline{m}_{20}^G(z) = \frac{m_2^G(z)}{m_0^G(z)},$$
(12)

$$m_2^G(z) = \int_{-L}^{L} x^2 \left(\int_{-T/2}^{T/2} R^{4x_1^2/T_0^2} \cos\left(\frac{2\pi x}{\lambda z} x_1\right) dx_1 \right)^2 dx,$$
(13)

$$m_0^G(z) = \int_{-L}^{L} \left(\int_{-T/2}^{T/2} R^{4x_1^2/T_0^2} \cos\left(\frac{2\pi x}{\lambda z} x_1\right) dx_1 \right)^2 dx. \quad (14)$$

Integral (13) with finite values of T and $L \to \infty$ generally diverges and the second moment (12) does not exist, while integral (14) always has a finite value.



Figure 1. Dependence of the second moment on ρ with T = 0.01 m for R = 0.2 (1), 0.01 (2), 0.15 (3), 0.1 (4).

We consider diverging integral (13) and introduce $u = \frac{x}{\lambda z}$ for convenience. After calculating (13) with $T_0 = T$ with respect to x_1 , we obtain

$$m_{2}^{G}(z) = \frac{2\lambda^{3}z^{3}b^{2}}{\pi} \int_{0}^{p} u^{2} \exp(-2b^{2}u^{2})$$
$$\times \left(\operatorname{erf}(a+ibu) + \operatorname{erf}(a-ibu) \right)^{2} du, \qquad (15)$$

where

$$a = \sqrt{-\ln R}, \quad b = \frac{\pi I}{2\sqrt{-\ln R}},$$
$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_{0}^{u} \exp(-x^{2}) dx, \quad \rho = \frac{L}{\lambda z}.$$

Expression (13) after normalization to $\lambda^3 z^3$ may be written as

$$m_2^{*G}(\rho) = \frac{m_2^G(z)}{\lambda^3 z^3} = \frac{2b^2}{\pi} \int_0^{\rho} u^2 \exp(-2b^2 u^2) \times \left(\operatorname{erf}(a+ibu) + \operatorname{erf}(a-ibu) \right)^2 du.$$
(16)

Figure 1 shows the dependence $m_2^{*G}(\rho)$ calculated using equation (16) for various *R*. It can be seen that as ρ grows the above-mentioned quantity increases and more considerable change occurs at larger *R*.

If the emitter aperture is infinite $(T \to \infty)$ and

$$\lim_{\substack{x_1 \to \infty \\ y_1 \to \infty}} R^{4(x_1^2 + y_1^2)/T_0^2} = 0,$$

then integral (16) converges at $\rho \rightarrow \infty$ and the following equation is valid

$$m_2^{*G} = \lim_{\rho \to \infty} m_2^{*G}(\rho) = \frac{\sqrt{-2\ln R}}{4\pi^{3/2}T_0}.$$
 (17)

Optics and Spectroscopy, 2024, Vol. 132, No. 5

For the actually existing emitter aperture with the finite dimensions $T = T_0$, at whose boundaries $R \to 0$, divergence of integral (16) with the growth of ρ is negligible. Thus, for small R = 0.01 (Figure 1, curve 2) with $T = T_0 = 0.01$ m, $m_2^{sG}(1500) \approx 13.646$ that differs from limit (17) $m_2^{sG} \approx 13.626$ by $\approx 0.15\%$.

We determine the main parameters affecting the divergence of integral (16) by considering the asymptotic expansion of its subintegral function at lager u.

We write (16) as a sum of two summands

$$m_2^{*G}(\rho) = m_2^{*G}(\rho^*) + \frac{b^2}{2\pi} \int_{\rho^*}^{\rho} u^2 \exp(-2b^2 u^2) \\ \times \left(\operatorname{erf}(a + ibu) + \operatorname{erf}(a - ibu) \right)^2 du.$$
(18)

where ρ^* may be assumed as a parameter defining some value in the MRR measurement range beginning from which the asymptotic expansion of the subintegral function of the second summand is valid.

The first summand defined in the finite limits of integration is a finitely defined integral, the second summand — is the diverging integral at $\rho \to \infty$.

If

$$u \gg a/b = \frac{-2\ln R}{\pi T},$$

the square of sum included in the second summand (18) may be reduced to the asymptotic form [9]

$$\left(\operatorname{erf}(a+ibu) + \operatorname{erf}(a-ibu)\right)^{2} = \frac{4R^{2} \exp(2b^{2}u^{2})}{\pi b^{2}u^{2}}$$
$$\times \left(\sin^{2}(2abu) - \frac{a\sin(4abu)}{bu} + o(1/u)\right).$$

Then

$$\frac{b^2}{2\pi} \int_{\rho^*}^{\rho} u^2 \exp(-2b^2 u^2) \left(\operatorname{erf}(a+ibu) + \operatorname{erf}(a-ibu) \right)^2 du$$
$$= \frac{2R^2}{\pi^2} \left(\int_{\rho^*}^{\rho} \left(\sin^2 \pi T u + \frac{2\ln R}{\pi T u} \sin 2\pi T u + o(1/u) \right) du \right),$$
(19)

whence it follows that the divergence of integral (18) at $\rho \rightarrow \infty$ is defined by two first summands of the subintegral function. With the first summand making the main contribution to the divergence. As a result, we get

$$\int_{\rho^*}^{\rho} u^2 \exp(2b^2 u^2) \left(\operatorname{erf}(a+ibu) + \operatorname{erf}(a-ibu) \right)^2 du$$
$$\approx \frac{2R^2}{\pi^2} \int_{\rho^*}^{\rho} (\sin^2 \pi T u) du = \frac{R^2 \Delta \rho}{\pi^2}$$
$$\times \left(1 - \frac{\sin \pi T \Delta \rho}{\pi T \Delta \rho} \cos \left(\pi T \rho^* \left(2 + \frac{\Delta \rho}{\rho^*} \right) \right) \right),$$

32 Optics and Spectroscopy, 2024, Vol. 132, No. 5

where $\Delta \rho = \rho - \rho^*$.

Taking into account (19), the following approximate expression is valid for (18)

$$m_2^{*G}(\rho) \approx m_2^{*G}(\rho^*) + \frac{R^2 \Delta \rho}{\pi^2} \times \left(1 - \frac{\sin \pi T \Delta \rho}{\pi T \Delta \rho} \cos\left(\pi T \rho^* \left(2 + \frac{\Delta \rho}{\rho^*}\right)\right)\right). \quad (20)$$

Since various lower levels of the measurement range may be defined for MRR depending on ρ , then from (20) it follows that the measured moments of the same distribution will differ, which impairs the uniformity of beam diameter and divergence angle measurements. It is apparent that the value of the above-mentioned level will be limited only by the lower limit level r of the MRR measurement range. As the measurement range is extended (i.e. as ρ and, respectively, $\Delta \rho$ increase), the second moment will grow. The quantity

$$\delta^G(R) \approx \frac{m_2^{*G}(\rho) - m_2^{*G}(\rho^*)}{m_2^{*G}(\rho^*)},$$
(21)

where

$$\begin{split} m_2^{*G}(\rho) &- m_2^{*G}(\rho^*) \approx \frac{R^2 \Delta \rho}{\pi^2} \\ &\times \left(1 - \frac{\sin \pi T \Delta \rho}{\pi T \Delta \rho} \cos \left(\pi T \rho^* \left(2 + \frac{\Delta \rho}{\rho^*} \right) \right) \right), \end{split}$$

where may be considered as an approximate relative measurement error of the second moment induced by the divergence of integral (16).

It is clearly seen from (21) that the error contains a linear component $\frac{R^2 \Delta \rho}{\pi^2}$ increasing with the growth of $\Delta \rho$ and *R*. With small *R*, the error is low, which is in line with the results shown in Figure 1 (curve 2).

To derive the main conclusions, we consider two lower levels of the MRR measurement range equal to 0.001 and 0.0001 for various R and define the relative measurement error $\delta^G(R)$ at T = 0.01 m using equation (21), where $m_2^{*G}(\rho^*)$ and $m_2^{*G}(\rho)$ are calculated using exact equation (16), ρ^* corresponds to the relative spatial intensity distribution at 0.001, and ρ — at 0.0001.

The lower level of the relative measurement range is limited by the performance capabilities of the MRR to be used. Thus, for ORCA-Flash4.0 V3 Digital CMOS C13440-20CU camera the lower limit is $r = 2.7 \cdot 10^{-5}$ and the lower level r^* of the measurement range shall not be below this value.

Thus, we consider the second moment measurement error of the same distribution measured by two MRR with different lower measurement range levels depending on R.

The calculation results are shown in Figure 2, curve *I*). It is seen that, when T = 0.01 m, $\delta^G(R)$ grows considerably



Figure 2. Dependence of the second moment measurement error on *R* at T = 0.01 m for (1) the Gaussian $\delta^G(R)$ and (2) exponential $\delta^{\exp}(R)$ spatial intensity distributions in the emitter plane.

as *R* increases beginning from some percent points at $R \approx 0.1$ to $\approx 28\%$ at R = 0.2.

It is fair to say that measurements of the same Gaussian parameter of the MRR beam with different lower measurement range levels are very different and incomparable.

Second moment of the measured laser beam field with the exponential spatial amplitude distribution in the emitter plane

We consider the far-field spatial distribution of the laser beam intensity (10) with exponential spatial distribution of amplitude on the square aperture $T \times T$ in the emitter plane

$$u(x_1, y_1, 0) = R^{2(|x_1| + |y_1|)/T_0},$$
(22)

 $R (0 < R \le 1)$ — amplitude at $x_1 = \pm T_0/2$; $y_1 = 0$ or $x_1 = 0$; $y_1 = \pm T_0/2$.

If $T_0 = T$, then the amplitude at the emitter aperture boundaries is equal to *R*.

Calculation of the second moment using equation (2) including (5) and (8) for distribution (22) gives the expressions equivalent to (12)-(14)

$$\overline{m}_{20}^{\exp}(z) = \frac{m_2^{\exp}(z)}{m_0^{\exp}(z)},$$



Figure 3. Dependence $m_2^{\text{*exp}}(\rho)$ at T = 0.01 m for R = 0.5 (1), 0.3 (2), 0.2 (3), 0.1 (4).

where the integral

$$m_2^{\exp}(z) = \int_{-L}^{L} x^2 \left(\int_{-T/2}^{T/2} R^{2|x_1|/T_0} \cos\left(\frac{2\pi x}{\lambda z} x_1\right) dx_1 \right)^2 dx$$

diverges,

$$m_0^{\exp}(z) = \int_{-L}^{L} \left(\int_{-T/2}^{T/2} R^{2|x_1|/T_0} \cos\left(\frac{2\pi x}{\lambda z} x_1\right) dx_1 \right)^2 dx$$

- converges at $L \to \infty$.

When $T_0 = T$, the integral that diverges at $\rho \to \infty$ is written as follows after normalization to $\lambda^3 z^3$

$$m_{2}^{*\exp}(\rho) = \frac{m_{2}^{\exp}(z)}{\lambda^{3}z^{3}} = \frac{2T^{2}}{\ln^{4}R}$$

$$\times \int_{0}^{\rho} u^{2} \left(\frac{u\pi RT\sin(\pi Tu) + \ln R\left(R\cos(\pi Tu) - 1\right)}{1 + \pi^{2}T^{2}u^{2}/\ln^{2}R}\right)^{2} du,$$
(23)

where $\rho = \frac{L}{\lambda_z}$.

The parenthesized subintegral function expression in (23) is characterized by slowly decreasing "wings" at $u \to \infty$. Therefore, divergence of integral (23) is more significant compared with (18).

Figure 3 shows the dependence of the second moment on ρ for various *R* calculated using equation (23). As ρ and *R* increase, the above-mentioned quantity grows infinitely as (18).



Figure 4. Dependence $m_2^{*\text{uniform}}(\rho)$ at T = 0.01 m for uniform intensity distribution in the emitter plane (R = 1).

As above, we consider two lower levels of the MRR measurement range equal to 0.001 and 0.0001 for various R and define the second moment measurement error $\delta^{\exp}(R)$ at T = 0.01 m using an equation equivalent to (21), where $m_2^{\exp}(\rho^*)$ and $m_2^{\exp}(\rho)$ are calculated using equation (23), ρ^* corresponds to the relative spatial intensity distribution at 0.001, and ρ — at 0.0001.

The calculation results are shown in Figure 2, curve 2). It is seen that at the finite value of *T*, the measurement error grows considerably as *R* increases beginning from $\approx 12\%$ at $R \approx 0.01$ to $\approx 54\%$ at $R \approx 0.2$.

The measurements, as for the Gaussian bean, are incomparable and characterized by even higher error.

Second moment of the measured laser beam field with the uniform spatial amplitude distribution in the emitter plane

Expression equivalent to (16) with uniform amplitude distribution in the emitter plane may be derived from (16) by the passage to the limit at $R \rightarrow 1$.

It can be easily shown that in this case

$$m_2^{*\text{uniform}}(\rho) = \frac{\rho}{\pi^2} \left(1 - \frac{\sin(2\pi T\rho)}{2\pi T\rho} \right). \tag{24}$$

Figure 4 shows almost linear dependence of the growth of $m_2^{*\text{uniform}}(\rho)$ plotted using (24). In this case, the measurement error of $m_2^{*\text{uniform}}(\rho)$ measured by different MRRs may exceed 100%.

Conclusions

The results of the distribution case study clearly prove that the method for measuring laser beam width and divergence angle, as described in [1], based on using the initial moments of spatial intensity distribution gives invalid results.

The general reason for such results, as specified in the introduction, is the divergence of the second moments of the radiating field, if the values of amplitude R (intensity) at the emitter aperture boundaries are nonzero, as is actually the case. The study shows that the divergence is negligible at small R, but R cannot be controlled during the measurement process. Moreover, for the uniform spatial intensity distribution, always R = 1.

It follows from the curves shown in Figure 2 that the divergence of the second moments induces considerable measurement error, therefore parameters of the same laser beam measured by different MRRs are incomparable.

Thus, it is wrong to use the second moment as a universal characteristic for measuring beam parameters, which poses the measurement accuracy issue. From metrology standpoint, a non-existent quantity such as the diverging second moment of a radiating field cannot be used as a standard measure. If in some particular cases the evaluation of the second moment gives an acceptable result, in particular at small R (curve 2 in Figure 1), the second moment cannot be reliably reproduced due to the lack of control of R and a variety of the types of spatial intensity distribution.

According to the authors, if the described characteristic for measuring beam parameters is not given up, the only possible solution of the issue is to constrain artificially the relative distribution range of the radiating field intensity by some consistent values of the lower level r^* ($r^* \ge r$), and these values shall be specified during calibration.

It is reasonable to set and specify r^* according to the desired lower level of the dynamic intensity measurement range of MRRs used in various measuring systems. In this case, the measured beam widths and divergence angles become dependent on r^* , but for different MRRs with the same r^* the measurement results will be comparable. However, the mere fact of such dependence is a shortcoming of this measurement method as has been also focused on in [2], and it has been emphasized that a promising approach to such measurements was to use the aberration factor [10] characterizing the parameters of an emitter. Together with the aberration factor, it is suggested also to consider a generalized laser beam width [11,12] in the measurement plane, which will be the subject of further research and will define the method for measuring beam width and divergence angle without the above-mentioned shortcomings.

Conflict of interest

The authors declare that they have no conflict of interest.

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