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Integral area of charges for a given distribution of electric area of pulses

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A general solution to the problem of determining the electric charge density distribution in a vacuum that provides a given spatial distribution of the electric pulse area is presented and discussed. An example of charge distribution for obtaining a spherically symmetric distribution of the electric area is given.

Keywords: electric pulse area, unipolar electromagnetic pulses, field of moving charges in a vacuum.

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Generation of increasingly shorter pulses necessary to control transient processes is one of the most fast-moving modern laser physics and nonlinear optics areas as it appears from the Nobel Prize 2023 [1]. The main method here is the coherent addition of many optical harmonics, i.e. radiation spectrum expansion. Transition into the high-frequency spectrum range also facilitates pulse duration reduction. Thus, a pulse duration of 43 as was successfully achieved in the X-ray range [2].

The so obtained pulses contain numerous field oscillations (cycles) during which the orientation of the electric strength $\mathbf{E}$ varies significantly. The electric area for them is accordingly close to zero

$$\mathbf{S}_E(\mathbf{r}) = \int \mathbf{E}(\mathbf{r}, t)dt, \tag{1}$$

where $\mathbf{r}$ is the radius vector and t is the time. Note that this value is named „time integral of field" in book [3] and then in [4] and many other further papers. At the same time, another option for pulse reduction is possible reducing the number of cycles up to the limit — one half-cycle (unidirectional pulse). The electric area for such pulses is already nonzero. This is very important, because it means that a mechanical pulse proportional to the electric area may be transmitted by the shortest possible pulses to electric charges interacting with radiation.

Various methods for generating pulses with nonzero electric area, that will be called unidirectional pulses, are discussed in [5]. Solution of the problem of determining the electric area of a pulse generated at the pre-defined charge motion in vacuum is described in [6], see also [7,8]. This paper focuses on the inverse problem — determination of distribution of electric-charge density that ensures the required spatial distribution of the electric area of pulses.

We shall specify in advance that not any distributions of electric area are allowable. Actually, they shall first satisfy the general relation expressing a vortex-free nature of the electric are field [7,9]

$$\operatorname{rot} \mathbf{S}_E = 0. \tag{2}$$

Then, it is reasonable that we will be interested in charge distributions localized in a finite spatial domain. The analysis shows that for them the far-field electric area at distances R that are much longer than the localization domain sizes shall decrease not slower than R^{-3} [7,10].

At the specified restrictions, solution of the formulated problem is given by [6,7]

$$\operatorname{div} \mathbf{S}_E = 4\pi Q, \tag{3}$$

where the integral charge density is introduced

$$Q(\mathbf{r}) = \int_{-\infty}^{\infty} \rho(\mathbf{r}, t)dt \tag{4}$$

($\rho(\mathbf{r}, t)$ is the charge density). Note that from (3) and from charge conservation it follows that the system shall be generally charge-neutral

$$q_0 = \int \rho(\mathbf{r}, t)d\mathbf{r} = 0. \tag{5}$$

Therefore, the system shall contain both positive and negative charges (ions).

One of the types of electric area distributions for which condition (2) is met automatically is the radially symmetric distribution with a single nonzero radial component in the spherical coordinate system:

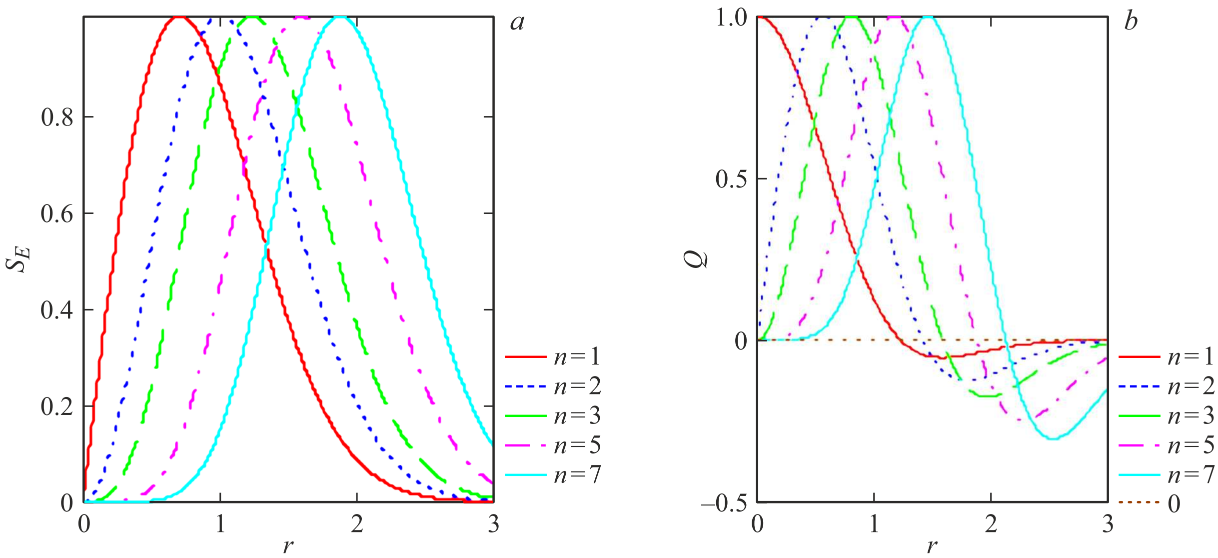
$$\mathbf{S}_E = \big(S_{E,r}(r), 0, 0\big).$$

Whereby the integral charge density is spherically symmetric:

$$Q(r) = \frac{1}{4\pi r^2}\frac{d}{dr}(r^2 S_{E,r}). \tag{6}$$

For example:

$$S_{E,r} = S_0 r^n \exp(-\alpha r^2), \quad n \geq 1. \tag{7}$$



Normalized profiles of the radial component of electric area of pulse written as (7) (*a*) and of the corresponding electric charge density (*b*); radius r in terms of $\alpha^{-1/2}$.

The radial component of electric area is sign-constant and positive provided that $S_0 > 0$. The maximum value is reached at $\alpha r_{\max}^2 = n/2$, with

$$\max S_{E,r} = S_0[n/(2\alpha)]^{n/2} \exp[-n(r/r_{\max})^2/2]. \quad (8)$$

Normalized-to-maximum distributions of this component are shown in Figure 1, *a* at several values of n. According to (6), such distribution of electric area is formed with the integral charge density distribution

$$Q(r) = \frac{S_0}{4\pi}[(n+2) - 2\alpha r^2]r^{n-1}\exp(-\alpha r^2). \quad (9)$$

Positive ($r < r_0$) and negative ($r > r_0$) charges are separated by a sphere with radius

$$r_0 = \sqrt{(n+2)/(2\alpha)},$$

that grows with n. The profiles have their valley and peak, respectively, at

$$\alpha r_{\pm}^2 = \frac{1}{4}\left(2n + 3 \pm \sqrt{8n+17}\right). \quad (10)$$

The normalized-to-maximum distributions of the radial component of electric area corresponding to the integral charge density profiles in Figure *a* are also shown in Figure *b*. The figure shows that the positive charge at $n = 1$ is concentrated in the central region, and as $n$grows — the positive charge is concentrated in the spherical layer of increasingly larger radius.

The approach described herein does not fix the electric current density structure and does not allow us to determine the duration and shape of the generated pulses. To find complete electric and magnetic strength behavior, complete system of Maxwell equations shall be solved [11].

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Conflict of interest

The author declares that he has no conflict of interest.

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