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Analytical model of a laser Gaussian statistically averaged beam with random homogeneous and isotropic phase distortions of the field

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In the Fresnel approximation, an analytical model of a Gaussian laser beam with random phase distortions of the field has been developed. It is accepted in the work that random phase distortions of the field are distributed according to the normal law, statistically homogeneous and isotropic. The propagating beam is represented by the sum of two components: diffraction-limited and partially coherent (scattered by phase inhomogeneities). In turn, the partially coherent component is represented by the sum of statistically independent subbeams, each of which has a zero average statistical field. The distribution of subbeams by radiation power is related to the dispersion of phase distortions. The study of the spatial structure of subbeams was carried out using the methods of the theory of spatial moments. Analytical relations have been obtained and studied that uniformly approximate the distribution function of the average statistical radiation flux depending on the size of the receiver and the distance to the observation plane without restrictions on the amplitude and scale of random phase distortions of the field. The research results can be used in the development and optimization of laser transceiver optical systems, in methodologies for measuring the parameters and quality of laser beams.

Keywords: Gaussian laser beam, random phase distortions of the field, radiation flux, partially coherent component, axial intensity, beam width.

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1. Introduction

Spatial energy characteristics of a laser beam depend considerably on phase distortions (PD) of the field [1]. PD sources are active medium nonuniformities within a laser resonator [2,3], manufacturing errors and roughness [4] of beam shaping optics mirrors, laser beam propagation medium nonuniformities [5], etc. Due to this fact, theoretical and practical interest in the development of receiving/transmitting laser systems is associated with the investigations of intralaser output radiation deformations [2,3,6,7], phase structure features of the field [6,7], spatial energy structure of the propagating laser beam [6–12]. Analytical research methods for investigating laser optical systems considering deterministic aberration and diffraction deformations (PD) of a laser beam are summarized in [6,10,11]. Note that due to the lack of information and problems of describing the field PD sources, influence of the PD sources is often considered using statistical models. In this case, statistical approach to the field PD description allows an in-depth analysis to be performed to investigate the main characteristics of the formed radiation considering all features of the receiving/transmitting optical systems [7–9].

However, analytical studies of beams with random PDs are generally restricted [8,9] to a limiting case of PD with small measurement amplitude. The objective of the study is to develop an analytical model and analytical investigations of an average spatial energy structure of a propagating laser beam without restrictions on the amplitude and scope of variation of random field PDs. Modeling was performed using a laser beam representation in the form of a sum of two statistically independent components — diffraction-limited and partially coherent (scattered by phase nonuniformities) components with zero average value. In diffraction approximation using the spatial moment theory methods [11,12] (for propagating laser radiation intensity distribution), analytical relations were derived and investigated for uniform approximation of the average radiation flux distribution function depending on the receiver sizes and distance to the observation plane.

2. Basic equations

Field distributions $U_0(\boldsymbol{\rho})$ of the radiation intensity $I_0(\boldsymbol{\rho})$ and power flux $W_0(\rho)$ for the Gaussian beam in the absence of PD are written as:

$$U_0(\boldsymbol{\rho}) = \sqrt{\frac{4}{\pi}}\exp(-2\rho^2),$$

$$I_0(\boldsymbol{\rho}) = \left(\frac{4}{\pi}\right)\exp(-4\rho^2),$$

$$W_0(\rho) = 1 - \exp(-4\rho^2). \quad (1)$$

Here, the vector $\boldsymbol{\rho} = \mathbf{r}/a$, $\mathbf{r}$ is the radius vector of the point in the output aperture plane, a is the beam radius.

Then the propagating beam characteristics are examined in a cylindrical coordinate system. The Z axis of this system coincides with the optical axis and $\boldsymbol{\rho} = (x, y)$ is the radial vector, $\rho = |\boldsymbol{\rho}|$.

Considering the field PDs concentrated in the output aperture plane of the optical system, the kernel of the distribution integral in the Fresnel approximation [1] is written as:

$$H(\boldsymbol{\rho}_1, \boldsymbol{\rho}) = (-iN_z) \times \exp\left\{i\pi\left[\left(\frac{2z}{\lambda}\right) + N_z(\boldsymbol{\rho}_1 - \boldsymbol{\rho})^2 - N_f\boldsymbol{\rho}_1^2\right]\right\}\exp(i\varphi(\boldsymbol{\rho}_1)). \quad (2)$$

Here, $N_z = a^2/(\lambda z)$ and $N_f = a^2/(\lambda f)$ are the Fresnel numbers, λ is the radiation wavelength, z is the distance to the observation plane, f is the focal distance; $\varphi(\boldsymbol{\rho}_1)$ — is the field PD function. In expression (2), $\boldsymbol{\rho}_1$ in the output aperture plane, $\boldsymbol{\rho}$ is in the observation plane.

The field phase distortion function $\varphi(\boldsymbol{\rho}) = 2\pi L(\boldsymbol{\rho})/\lambda$, $L(\boldsymbol{\rho})$ is the corresponding geometrical wave surface [1]. Statistical characteristics of $\varphi(\boldsymbol{\rho})$ will be taken as follows: average PD is equal to zero, $\overline{\varphi(\boldsymbol{\rho})} = 0$; symbol $\overline{(\ldots)}$ hereinafter denotes the statistical averaging operation; dispersion $\sigma^2 = \overline{\varphi^2(\boldsymbol{\rho})}$ is homogeneous; correlation factor is isotropic and equal to $K(\boldsymbol{\rho}) = \exp(-\rho^2/c^2)$, $c = C_\varphi/a$ is the normalized correlation radius, C_φ is the correlation radius. The field correlation function is written as

$$\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \overline{\exp\{i[\varphi(\boldsymbol{\rho}_1) - \varphi(\boldsymbol{\rho}_2)]\}} = \exp\{-\sigma^2[1 - K(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)]\}, \quad (3)$$

where $\boldsymbol{\rho}_{1,2}$ are the points in the output aperture plane $z = 0$.

Find the intensity distribution function $\overline{I(\boldsymbol{\rho})}$ in the z plane. For this, substitute correlation function (3) as Taylor's series in $\sigma^2 K(\rho)$ into average intensity expression [1]. Considering (1) and (3) for $\overline{I(\boldsymbol{\rho})}$, the following can be derived

$$\overline{I(\boldsymbol{\rho})} = \exp(-\sigma^2)\sum_{n=0}^{\infty}\frac{\sigma^{2n}I(\rho/\eta_n)}{n!\eta_n^2}, \qquad \eta_n = \sqrt{1 + \frac{n}{c_z^2}}, \quad (4)$$

where

$$I(\rho) = \left(\frac{4}{\pi\rho_z^2}\right)\exp\left[-4\left(\frac{\rho}{\rho_z}\right)^2\right] \quad (5)$$

— is the intensity distribution in the z plane in the absence of PD;

$$\rho_z = s_z\left(\frac{2}{\pi N_z}\right), \quad s_z = \sqrt{1 + \left[\frac{\pi}{2}(N_z - N_f)\right]^2}, \quad (6)$$

ρ_z — is the beam radius without PD; $c_z = cs_z$ is the effective correlation radius in the z plane; c_z is minimum in focus, $c_f = c$. Relations (3)–(5) imply that the full power of the Gaussian beam is equal to 1 and does not depend on the distance z to the observation plane.

The axial intensity $\overline{I(0)}$ and Strehl number [1] St for the beam formed in the z plane are equal

$$\overline{I(0)} = St\left(\frac{4}{\pi\rho_z^2}\right), \quad St = {}_1F_1(1; 1 + c_z^2; -\sigma^2), \quad (7)$$

where ${}_1F_1(\ldots)$ is the degenerate hypergeometric function [13]. With $\sigma^2 \ll 1$, $St \approx \exp[-\sigma^2/(1 + c_z^2)]$.

From equation (4), we derive the expression for the average radiation flux through a round receiving plate with the radius ρ:

$$\overline{W(\rho)} = \exp(-\sigma^2)\sum_{n=0}^{\infty}(\sigma^{2n}/n!)W(\rho/\eta_n). \quad (8)$$

Here, $W(\rho) = 1 - \exp[-(4\rho/\rho_z)^2]$ is the radiation flux without field PDs. With $\sigma^2 << 1$,

$$\overline{W(\rho)} \approx W(\rho) - \sigma^2\left[W(\rho) - W\left(\rho\Big/\sqrt{1 + 1/c_z^2}\right)\right].$$

Equations (4), (8) that represent the average radiation intensity and flux distributions as functional series are the basic relations for further analysis.

3. Spatial moments and beam width

Consider the characteristics of the Gaussian beams with random field PDs using the spatial moment theory methods [11,12] for the function of intensity distribution in beam cross-section. This theory defines the basic spatial energy characteristics of the propagating laser beam: beam width and waist position, and energy divergence angle.

Let's introduce the n-th relative spatial moment into the study:

$$m_n = \frac{\int \overline{I(\boldsymbol{\rho})}(x^n + y^n)d^2\boldsymbol{\rho}}{\int I(\rho)(x^n + y^n)d^2\boldsymbol{\rho}}, \quad n = 2, 4, \ldots \quad (9)$$

For the moments m_2 and m_4 considering (4), (5), we derive the following algebraic expressions

$$m_2 = 1 + \frac{\sigma^2}{c_z^2}, \qquad m_4 = m_2^2 + \frac{\sigma^2}{c_z^2}. \quad (10)$$

Assuming (10) as a system of equations, we find the following for calculation of random field PDs

$$\sigma^2 = (m_2 - 1)^2/(m_4 - m_2^2), \qquad c = (1/s_z)\sqrt{(m_2 - 1)/(m_4 - m_2^2)}.$$

Thus, the fourth spatial moment together with the second standard moment (Q factor M^2) are used, in particular, to find the dispersion and correlation radius of the random field PDs according to the experimental results.

Equations (9), (11) are examined and considered below for solution of the analytical approximation problem of functional series (4), (8) for the average radiation intensity and flux distribution functions.

3.1. Beam width in the statistical model

In the spatial moment theory, the beam diameter $2A_Z$ (width) on the z plane and distance $z = z_b$ to the waist plane are the main beam characteristics. Depending on PD, A_Z is proportional to the propagation coefficient M_z^2 (M_z^2 is the Q factor). The beam radius in this case is equal to

$$A_z = M_z^2 a_z, \qquad a_z = \rho_z a, \tag{11}$$

where a_z is the beam radius without PD. In the z plane, within the region with the radius A_Z, more than $\sim 90\%$ of the full beam power propagates, $\overline{W(A_z/a)} > 0.9$.

Within the statistical approach considering (5), (9) and (11), the relative beam radius is equal to

$$\frac{A_z}{a} = \left(\frac{2}{\pi N_{ez}}\right)\sqrt{1 + \left[\frac{\pi}{2}\left(N_{ez} - N_{ef}\right)\right]^2}. \tag{12}$$

Here, $N_{ez} = N_z/M_f^2$, $N_{ef} = N_f/M_f^2$ are the effective Fresnel numbers, $M_f^2 = \sqrt{1 + \sigma^2/c^2}$. Thus, in the spatial moment theory, the average beam with random PDs is equivalent to the beam without PDs with the Fresnel number reduced in M_f^2.

The beam radius is minimum in the waist plane $z = z_b$. From (12), we find

$$\min A_z = \frac{a}{\sqrt{1 + (\pi N_{ef}/2)^2}}, \quad z_b = \frac{f}{1 + [2/(\pi N_{ef})]^2}. \tag{13}$$

From equations (13) and general ideas of the properties of laser beams with field PDs, it is clear that, as beam PDs grow, the waist moves closer to the output aperture plane and the waist diameter and distance between the focus and waist,

$$f - z_b = f \Big/ \left[1 + (\pi N_{ef}/2)^2\right],$$

increase as σ^2/c^2 grows. At the distance $z = 2z_b$, the beam width is equal to the initial beam width (determined in the output aperture plane).

The following shall be noted here. Expression (12) for the Q factor M_f^2 includes σ^2/c^2. In physical meaning, it is proportional to the average squared wavefront angles [14]:

$$\sigma^2/c^2 = \overline{\langle[\mathrm{grad}\varphi(\boldsymbol{\rho})]^2\rangle} \Big/ 4 = \sigma^2\left[-dK(\rho)/d\rho^2|_{\rho=0}\right],$$

where the angle brackets $\langle\ldots\rangle$ mean averaging over the output aperture area. Whereby $\sigma^2/c^4 \sim (m_4 - m_2^2)$ is inversely proportional to the average squared radius of field curvature in the output aperture plane. Therefore, the spatial moment m_4 used in the given model considers more consistently not only the beam width, but also the possible deviations from the normal beam curvature radius.

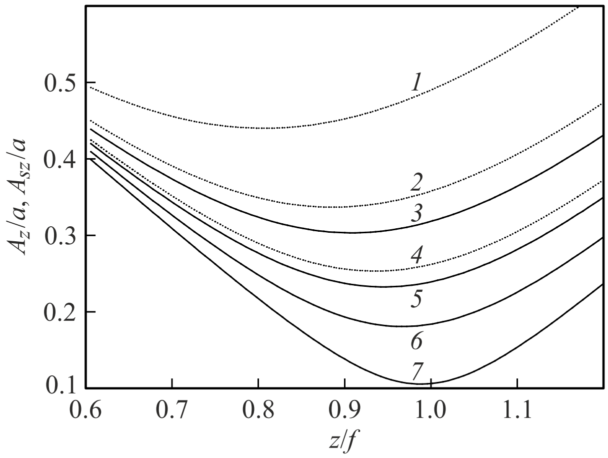

Figure 1. Beam width A_z/a (curves *3, 5* and *6*) and width of the partially coherent component A_{sz}/a (curves *1, 2* and *4*) depending on z/f at $N_f = 6$, $\sigma^2 = 0.5$. Curves *1* and *3* — at $c = 0.25$; curves *2* and *5* — at $c = 0.35$; *4* and *6* at $c = 0.5$. Curve *7* — for the diffraction-limited beam.

3.2. Average beam as a sum of coherent and partially coherent beams

Taking into account PDs, the field in the output aperture plane will be written as

$$U_0(\rho)\exp(i\varphi(\boldsymbol{\rho})) = U_0(\rho)\exp(-\sigma^2/2) + U_s(\boldsymbol{\rho}),$$

$$U_s(\boldsymbol{\rho}) = U_0(\rho)[\exp(i\varphi(\boldsymbol{\rho})) - \exp(-\sigma^2/2)].$$

Here, $U_s(\boldsymbol{\rho})$ is the random field with zero average value. Thus, the average intensity distribution in the z plane considering (4) is described by the following relation:

$$\overline{I(\boldsymbol{\rho})} = \exp(-\sigma^2)I(\rho) + [1 - \exp(-\sigma^2)]\,\overline{I_s(\boldsymbol{\rho})}. \tag{14}$$

In expression (14), $I(\boldsymbol{\rho})$ is distribution (5) without PDs,

$$\overline{I_s(\boldsymbol{\rho})} = \sum_{n=1}^{\infty}\frac{\sigma^{2n}I(\rho/\eta_n)}{n!\eta_n^2[\exp(\sigma^2) - 1]} \tag{15}$$

is the intensity distribution in partially coherent beam component. In (14), $\exp(-\sigma^2)$ and $[1 - \exp(-\sigma^2)]$ are the coherent and scattered component powers.

The power of partially coherent component grows as the dispersion grows and is equal to the coherent component power at $\sigma^2 \approx 0.7$. When $\sigma^2 > 0.7$, the spatial energy structure of the beam begins to be defined by the scattered beam component.

According to (14), (15), we find for the M_z^2-factor

$$M_z^2 = \sqrt{\exp(-\sigma^2) + [1 - \exp(-\sigma^2)]M_{sz}^4},$$

whereby M_{sz}^2 defines the scattered beam component width,

$$M_{sz}^2 = \sqrt{1 + \frac{M_z^4 - 1}{1 - \exp(-\sigma^2)}}. \tag{16}$$

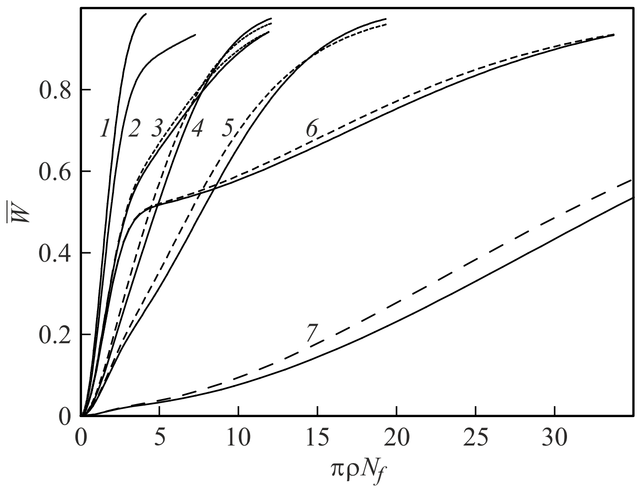

Figure 2. Radiation flux $\overline{W(\rho)}$ in the optical system focus in the two-component approximation. Curve *1* — for the diffraction-limited beam; *2* — at $\sigma^2 = 0.2$, $c = 0.3$; *3* — $\sigma^2 = 0.7$, $c = 0.3$; *4* — $\sigma^2 = 0.7$, $c = 0.1$; *5* — $\sigma^2 = 2$, $c = 0.5$; *6* — $\sigma^2 = 2$, $c = 0.3$; *7* — $\sigma^2 = 4$, $c = 0.1$. Solid line is the approximated calculation, dashed line near the solid one is the accurate calculation.

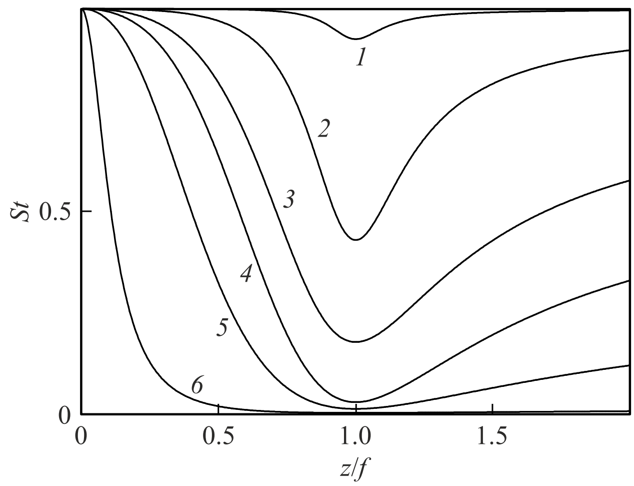

Figure 3. Dependence of the Strehl number *St* on the relative distance z/f at several dispersions σ^2 and correlations radii c, $N_f = 6$. Curves *1–6* correspond to $(\sigma^2, c) = (0.4, 2)$, (1.4, 0.7), (2, 0.3), (5, 0.3), (5, 0.15), (6, 0.03).

Thus, in the spatial moment theory, the scattered beam component width is calculated by equation (13) considering the substitution $N_{ez} = N_z/M_{sf}^2$, $N_{ef} = N_f/M_{sf}^2$ at

$$M_{sf}^2 = \sqrt{1 + (\sigma^2/c^2)/[1 - \exp(-\sigma^2)]}.$$

The minimum beam widths for the diffraction-limited and quasi-coherent components are achieved at different distance z_{db} and z_{sb}:

$$z_{db} = \frac{f}{1 + [2/(\pi N_f)]^2},$$

$$z_{sb} = \frac{f}{1 + [2M_{sf}^2/(\pi N_f)]^2}. \quad (17)$$

Therefore, the beam with random PDs may be treated as a sum of coherent and partially coherent components with different wavefront curvature radii.

Figure 1 shows the typical calculation results for the beam width A_z/a and scattered beam component A_{sz}/a depending on z/f.

Waist positions for the diffraction-limited component z_{db}, scattered component z_{sb} and whole beam z_b satisfy the inequality $z_{sb} < z_b < z_{db} < f$. As PDs grow, the z_b and z_{sb} planes approach the output aperture plane.

The shown result makes it possible to consider in more detail the ray paths in the receiving-transmitting laser optical systems and to optimize the system design parameters more accurately.

4. Analytical relations for $\overline{I(\rho)}$, $\overline{W(\rho)}$ and computational experiment results

We will derive approximated analytical relations to approximate functional series (4), (8) for average intensity and radiation flux at the specified field PD dispersion σ^2 and correlation radius c (or at the known spatial moments m_2 and m_4).

4.1. Two-component beam model

In the two-component model, the partially coherent beam component is approximated by the Gaussian beam whose width is larger that of the diffraction-limited component. Taking into account the results of [15] and expression (16), for the relative scattered beam component width we have

$$\overline{I(\boldsymbol{\rho})} \approx DI(\rho) + \frac{1-D}{\mu^2} I\left(\frac{\rho}{\mu}\right),$$

$$\overline{W(\rho)} \approx DW(\rho) + (1-D)W\left(\frac{\rho}{\mu}\right), \quad \mu = M_{sz}^2, \quad (18)$$

where $D = \exp(-\sigma^2)$. Approximation accuracy (18) of radiation flux (8) is illustrated in Figure 2.

Relative error of the analytical calculation of the radiation flux $\overline{W(\rho)}$ achieves 20%, absolute error is lower than 0.07–0.1.

In the statisctical laser beam model of interest, the amplitude of field formed in the specified space point is equal to the sum of the regular component and random component whose average value is equal to zero. If the random field fluctuation amplitude is quite high, then caustic products with extremely high and zero radiation intensity may be formed. Let's determine the parameters of random PDs at which paraxial intensity distribution

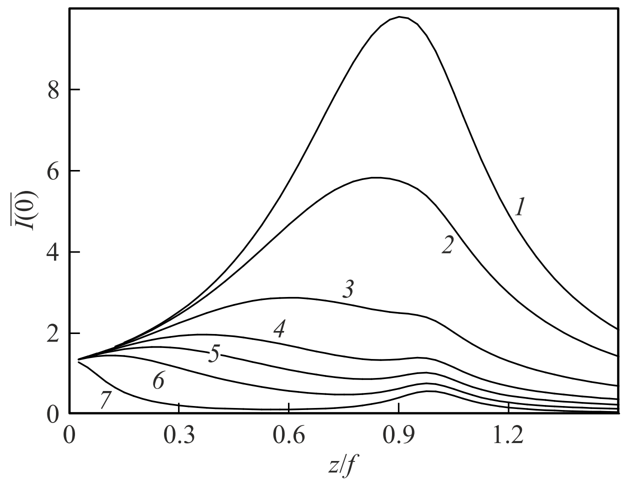

Figure 4. Dependence of the axial intensity $\overline{I(0)}$ on z/f for a beam with the Fresnel number $N_f = 6$ at $\sigma^2 = 5$. Curves *1–7* correspond to the correlation radii $c = 0.7$, 0.5, 0.3, 0.2, 0.15, 0.1, 0.03.

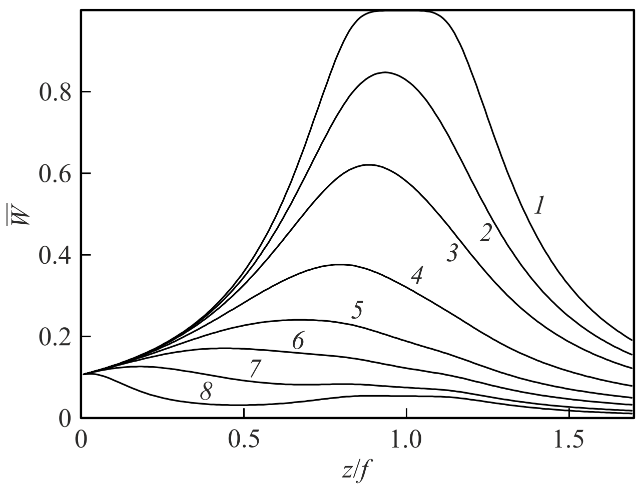

Figure 5. Dependence of the radiation flux on z/f at the receiver with the radius $\rho = 1/N_f$ for the beam with the Fresnel number $N_f = 6$. Curves *2–8* correspond to the correlation radii $c = 0.8$, 0.5, 0.3, 0.2, 0.14, 0.08, 0.03 at $\sigma^2 = 3$. Curve *1* — for the diffraction-limited beam.

does not contain points with zero values. Note that the given condition is sufficient for the absence of phase screw dislocations in the beam [6,7]. There are no dislocations within the given laser beam model, if the regular field component intensity is higher than the average random component intensity. Taking into account relation (14), the condition of interest is equivalent to

$$\exp(-\sigma^2)I(\rho) > [1 - \exp(-\sigma^2)]\overline{I_s(\boldsymbol{\rho})}. \tag{19}$$

It follows rom this inequality that there are no dislocations in the paraxial area limited by the diffraction radius $\rho = 0.7\rho_z$ (with $W(\rho) \approx 0.86$), when

$$\sigma^2 < \ln[1 + \mu^2 \exp(-2(1 - 1/\mu^2))].$$

If $\sigma^2 < 0.55$, inequality (19) is satisfied at any $\mu > 1$. Generally, μ depends on z. Therefore, at fixed PD parameters, satisfiability of (19) and local regions with zero intensity in the beam depend on the observation plane position. The given conclusions agree with the results of [7], where the wavefront screw dislocations were studied experimentally and using the computational mathematics methods.

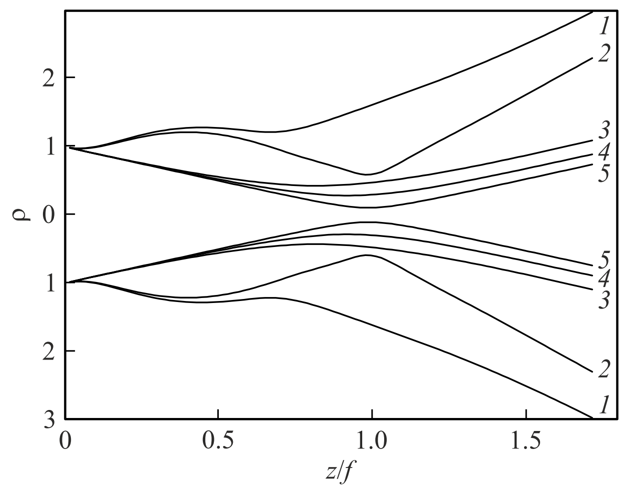

Figure 6. Beam radius depending on z/f. Curves *1* and *2* correspond to the beam with the following parameters: $\gamma_{\bar{W}} = 0.25$, $N_f = 14$, $c = 0.012$ at $\sigma^2 = 1.5$ and $\sigma^2 = 0.5$. Curves *3, 4* — $\gamma_{\bar{W}} = 0.63$, $N_f = 6$, $\sigma^2 = 1.5$ at $c = 0.3$ and $c = 0.5$. Curve *5* — for the diffraction-limited beam with $N_f = 6$ at $\gamma_{\bar{W}} = 0.63$. The beam radius ρ is normalized to $a_d = a\sqrt{\ln[1/(1-\gamma_{\bar{W}})]/2}$ that is equal to the beam radius in the output aperture plane.

4.2. Three-component beam model

Then, we will refine the beam model by considering the partially Gaussian beams:

$$\overline{I_s(\boldsymbol{\rho})} = \frac{1}{2}\left[\frac{1}{\mu_1^2} I\left(\frac{\rho}{\mu_1}\right) + \frac{1}{\mu_2^2} I\left(\frac{\rho}{\mu_2}\right)\right],$$

$$\overline{W_s(\rho)} = \frac{1}{2}\left[W\left(\frac{\rho}{\mu_1}\right) + W\left(\frac{\rho}{\mu_2}\right)\right]. \tag{20}$$

$\mu_{1,2} > 1$ define the divergence of the partially coherent field component. Let's find the correlation between $\mu_{1,2}$ and the beam parameters. According to definition (9) of the spatial moments considering representation (20), we obtain the set of equations

$$St = D + (1 - D)\left(\frac{\mu_1^2 + \mu_2^2}{2\mu_1^2\mu_2^2}\right), \tag{21}$$

$$m_2 = D + (1 - D)\left(\frac{\mu_1^2 + \mu_2^2}{2}\right),$$

$$m_4 = D + (1 - D)\left(\frac{\mu_1^4 + \mu_2^4}{2}\right). \tag{22}$$

Assuming (22) as a system of equations in $\mu_{1,2}$, we find

$$\mu_{1,2} = \sqrt{m_{s2}\left(1 \pm \sqrt{\frac{m_{s4}}{m_{s2}} - 1}\right)},$$

$$m_{sn} = \frac{m_n - D}{1 - D}, \quad n = 2, 4, \tag{23}$$

where m_{sn} are relative spatial moments for quasi-coherent field component. By means of algebraic transformations of expression (23), it is easy to derive

$$\mu_{1,2} = M_{sz}^2 \sqrt{1 \pm \left(1 - \frac{1}{M_{sz}^4}\right) k_s},$$

$$M_{sz}^2 = \sqrt{1 + \frac{\sigma^2/c_z^2}{1 - \exp(-\sigma^2)}},$$

$$k_s = \sqrt{\frac{1 - (1 + \sigma^2)\exp(-\sigma^2)}{\sigma^2}}. \tag{24}$$

Whereby the Strehl number considering equations (21), (24) is equal to

$$St \approx D + \frac{1 - D}{\mu^2}, \quad \mu = M_{sz}^2 \sqrt{1 - (1 - 1/M_{sz}^4)^2 k_s^2}. \tag{25}$$

$k_s = k_s(\sigma^2)$ is maximum at$\sigma^2 = 1.79$, $max[k_s(\sigma^2)] = 0.546$.

The above-mentioned equations solve the problem of analytical approximation of functional series (4), (8). When using equations (24), (25) for the Strehl number, the absolute calculation error does not exceed 0.01, the relative error is lower than 3.4%. So, the absolute error is lower than min(0.01; $0.034St$). The absolute error of the radiation error calculation is lower than 0.017, the relative error is lower than 3.7%.

4.3. Computational experiment results

Some results of calculations of the average intensity and power flux for the focused laser beam depending on the relative distance z/f and (σ^2, c) are shown in Figure 3–6. In the computational experiment, the σ^2 varied in the range of $[0; 10]$. Taking into account the rule 3σ [16], the amplitude of random wavefront deformations did not exceed 1.6λ.

Figure 3 illustrates the dependence of the Strehl number on the distance to the observation plane.

The Strehl number gradually increases as σ^2 decreases and/or c_z grows. At fixed σ^2 and c, St depending on z is minimum in focus, when $c_z = c_f = c$. When z decreases in the region upstream of focus, $c_z \to \infty$ and St approaches 1. As z grows in the range of $z > f$, the effective correlation radius c_z grows and the Strehl number approaches the value defined at

$$c_z = c_\infty = c\sqrt{1 + (\pi N_f/2)^2}.$$

Figure 4 shows the results of calculations of the axial intensity $\overline{I(0)}$.

$\overline{I(0)} = (4/\pi\rho_z^2)St$ depends on z in a more complex way than the integral beam width $2A_z$ (Figure 1) and the Strehl number (Figure 3). The presence of PD may result in appearance of the second local maximum in the intensity distribution along the optical axis. As PDs grow, the global beam intensity peak moves towards the output aperture. the above-mentioned effect was experimentally observed in [17].

Let's evaluate the conditions at which the dependence of $\overline{I(0)}$ on z has two peaks. According to equations (16), (18), it is clear that the second intensity peak (and waist) for the partially coherent component of the focused beam shall be in the region where z is much lower than the focal distance f. This condition is satisfied when the relative width μ of the beam component of interest is comparable with the Fresnel number N_f; more exactly $\mu^2 - 1 \sim N_f^2$, $N_f > 1$. In addition, the radiation intensity of the partially coherent component shall be comparable with the intensity of the diffraction-limited component, therefore $\sigma^2 > \ln(1 + \mu^2)$.

Figure 5 shows individual calculations of the radiation flux through the plate with the fifed radius ρ depending on the distance to the observation plane.

Curves in Figure 5 are identical to curves in Figure 4 for the axial intensity, but are smoother as could be expected.

Figure 6 illustrates the calculated beam radius ρ at the specified (fixed) relative radiation flux level $\gamma_W = \overline{W(\rho)} < 1$.

The local minimum beam radius moves towards the radiating aperture as σ^2 increase and/or c decreases.

At the end of Section 4, it should be noted that the derived and examined analytical relations approximate the average radiation flux distribution function depending on the receiver dimensions and distance to the observation plane without restrictions to the random field PD amplitude and scale. The absolute error of the derived analytical relation doesn't exceed 0.017, the relative error is lower than 3.7%. Additional investigations show that the given analytical model of the Gaussian beam with random PDs admits expansion to the version when the random field PD function is the sum of several statistically independent components.

5. Conclusion

In the Fresnel approximation, an analytical model has been developed and analytical studies have been performed to investigate the spatial energy characteristics of the average laser Gaussian beam with random normally distributed field PDs with the Gaussian correlation function without restrictions to the field PD amplitude and scale.

The field of the propagating laser Gaussian beam with random PDs is represented by a sum of two components: coherent (diffraction-limited) and partially coherent (scattered by phase nonuniformities), the latter has the zero average value. Analytical relations have been derived and examined to approximate evenly the average radiation flux distribution function depending on the receiver dimensions and distance to the observation plane without restrictions to the random field PD amplitude and scale.

It is shown that the field PDs may lead to appearance of the second peak in the average intensity and radiation flux distribution along the optical axis and to the shift of the global peak closer to the output aperture of the system. These effects are caused by the presence of the partially coherent component in the beam. The axial intensity of this beam component has its peak that is closer to the output aperture than the intensity peak for the diffraction-limited component. Analytical relations have been derived for calculation of the propagating beam width and beam waist position depending on the field PD dispersion and correlation radius.

Of interest is the possible development of the described Gaussian beam model to a more general case when the wavefront deformation has several sources and the phase deformation function of the field is equal to the sum of several randomly distributed components.

The results of the research may be used for the development and optimization of receiving-transmitting laser optical systems, laser beam parameter and quality measurement techniques.

References

[1] M. Born, E. Wolf. *Principles of Optics* (Pergamon, Oxford, 1969).

[2] V.V. Valuev, V.G. Naumov, N.E. Sarkarov, P.A. Svotin. Quantum Electron., **28** (1), 14 (1998). DOI: 10.1070/QE1998v028n01ABEH001129

[3] V.A. Gurashvili, A.M. Zotov, P.V. Korolenko, A.P. Napartovich, S.P. Pavlov, A.V. Rodin, N.E. Sarkarov. Quantum Electron., **31** (9), 821 (2001). DOI: 10.1070/QE2001v031n09ABEH002053

[4] A.S. Toporets. *Optika sherokhovatoy poverkhnosti* (Mashinostroenie, L., 1988) (in Russian).

[5] V.E. Zuev. *Rasprostranenie lazernogo izlucheniya v atmosfere* (Radio i svyaz', M., 1981) (in Russian)

[6] P.A. Nosov, I.I. Pakhomov, A.F. Shirankov. Vestnik MGTU im. N.E. Baumana. Ser. „Priborostroenie", **9** (9), 167 (2012) (in Russian). DOI: 10.18698/2308-6033-2012-9-363

[7] A.M. Zotov, A.V. Averchenko, P.V. Korolenko, N.N. Pavlov. Izvestiya RAN. Seriya fizicheskaya, **82** (1), 15 (2018) (in Russian). DOI: 10.7868/S0367676518010039

[8] S.A. Akhmanov. Yu.E. D'yakov, A.S. Tchirkin. *Vvedenie v statisticheskuyu radiofiziku i optiku* (Nauka, M., 1981) (in Russian).

[9] S.M. Rytov, Yu.A. Kravtsov, V.I. Tatarsky. *Vvedenie v statisticheskuyu radiofiziku. Tchast' 2. Sluchajnye polya* (Nauka, M., 1978) (in Russian).

[10] P.A. Nosov, V.Y. Pavlov, I.I. Pakhomov, A.F. Shirankov. J. Opt. Technol., **78** (9), 586 (2011) DOI: 10.1364/JOT.78.000586

[11] A.E. Siegman. IEEE J. Quant. Electron., 27 (5), 1146 (1991). DOI:10.1109/3.83370

[12] GOST R ISO 11146-1-2008 [Electronic resource]. URL: https://docs.cntd.ru/document/1200076797

[13] H. Bateman, A. Erdelyi. *Higher Transcendental Functions* (McGraw Hill, NY., 1953).

[14] V.I. Kislov, V.G. Taranenko. Radiotekhnika i elektronika, **31** (11), 2187 (1986).

[15] V.I. Kislov, E.N. Ofitserov. Phys. Wave Phenom., **27** (1), 24 (2019). DOI: 10.3103/S1541308X19010059

[16] G.A. Korn, T.M. Korn. *Mathematical handbook for scientists and engineers; definitions, theorems, and formulas for reference and review* (McGraw Hill, NY., 1968).

[17] N.P. Badalyan, V.V. Kiiko, V.I. Kislov, A.B. Kozlov. Quantum Electron., **38** (5), 477 (2008). DOI: 10.1070/QE2008v038n05ABEH013622

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