Dedicated to the memory of Alexander Alexandrovich Tomasov

On the Optimization of Energy Extraction from a Supercapacitor under an Impulse Load

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The problem of energy extraction from a supercapacitor (within the given duration τ) under an impulse load has been considered. It has been shown that for each τ there exists an optimal load value at which the maximum energy will be released. In a simple model of a single RC- element the problem can be solved analytically. For more complex models of supercapacitors (such as self-similar ladder RC- networks, tree-like RC- networks, etc.), numerical simulations have been conducted. The simulations have shown that the sharpness of the maximum decreases with increasing τ and with the degree of distributiveness of RC- network. A computer program has been developed to model impulse loads directly in the time domain for any equivalent supercapacitor circuit. This allows to consider nonlinear systems and avoid the need for complex conversion of impedance $Z(\omega)$ into the time domain. The developed approach (together with the simulation program) can be directly applied to solving practical problems related to the impulse operation mode of supercapacitors.

Keywords: supercapacitor modeling, Impedance Spectroscopy, IEC 62391, inverse relaxation, binary tree model, impulse load.

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Introduction

Modern supercapacitors (SC) have high power characteristics, their use can significantly improve the parameters of power supply units. Improvement of power characteristics is relevant for many electronic devices (for example, the GSM standard specifies the transmission time (with power in watts) of 4.6 ms with a full period of 0.575 ms, specialized DC−DC chips for were created for pulse GSM load [1]), high-power pulsed lasers, cars, and a number of other applications.

We proposed a method for supercapacitor diagnostics in Ref. [2]: the supercapacitor is shorted for a given time τ, the current is measured in the external circuit, after which effective $C(\tau)$ and $R(\tau)$ are obtained from the invariants of charge $\int I dt$ and energy $R\int I^2 dt$. Here τ is an equivalent of the frequency ω in impedance measurement, and the parametric graph $[C(\tau), R(\tau)]$ is a generalization of the usually considered Nyquist dependence $[\mathrm{Re}Z(\omega), \mathrm{Im}Z(\omega)]$. The experimental detection (also confirmed by simulation for a number of equivalent SC schemes) of a linear dependence $[C(\tau), R(\tau)]$ in the range of several orders of magnitude τ is an important result [2].

This measuring technique was extended in the present work to apply to the pulsed energy extraction mode typical for supercapacitors. The supercapacitor is discharged according to the method to the load R during a given τ and the energy released in the external circuit is considered as a function of the load R. The dependence of the behavior of such curves on the internal equivalent structure of supercapacitors is studied in the paper. An analytical answer was obtained for a simple RC-chain, numerical simulation was performed for self-similar ladder RC-models and tree-like RC-networks. The existence of an optimal external load value for each given τ is shown. The sharpness of the maximum depends on the τ and the distribution of the RC-network. A dual formulation of the problem can also be considered: find the time τ for a given external load for which this load is optimal. The developed computer program [3] allows solving both direct and dual problems for an arbitrary equivalent supercapacitor circuit, thereby directly applying the developed approach to technical applications.

A supercapacitor is a hierarchical porous structure, the equivalent circuit of which is usually represented as a RC-network. Fig. 1 shows two most frequently considered models. A real-world SC model usually contains a combination of several simple models. The models in Fig. 1 does not contain leaks (parallel-connected R): leaks are very small for most modern SC and may not be taken into account at times less than units of minutes and even hours. The methods of studying the properties of supercapacitors (both measuring the characteristics of a real supercapacitor and simulation) can be classified as consideration in the frequency space [4–6] and consideration in the time space [7–11].

Th impedance spectroscopy (the study of impedance as a function of frequency) is an effective technique for studying the properties of materials and reactions on electrodes [12]. The system is studied using a low-amplitude harmonic test signal. The amplitude and phase of the signal (voltage U and current I) are measured, which makes it possible to calculate the complex impedance $Z(\omega) = U(\omega)/I(\omega)$. The frequency range varies by many orders of magnitude, usually $10^{-3}-10^{6}$ Hz, which allows obtaining information about porous structures. Interpretation is the most difficult step in impedance measurement. The analysis of the impedance dependence consists in the initial assumption of an equivalent circuit, the nominal values of the elements of which are obtained by optimization to best match the $Z(\omega)$-model of the experimental curve. It is known [6,12] that different equivalent circuits may exhibit identical impedance behavior. The simulation in impedance spectroscopy assumes the linear nature of the dynamics of SC, then it is a fairly simple task, which is generally reduced to solving a system of linear Kirchhoff equations. The solution for $Z(\omega)$ is represented as a ratio of polynomials from ω with complex coefficients for a system with a finite number R and C . An analytical solution is possible for $Z(\omega)$ in some cases for a system with an infinite number of RC-elements, for example, the nTE element (4) considered below, corresponding to a self-similar ladder circuit (5) with $n_C = 1/n_R = n$.

The impedance measurement has such complexity in relation to supercapacitors that the result in the form of $Z(\omega)$ is inconvenient for analysis in applications. Yes, $Z(\omega)$ obtained in the linear mode as a response to a test harmonic signal of small amplitude can be converted into a response to an arbitrary waveform, however, its behavior is nonlinear in typical high-current SC modes and such a conversion introduces significant errors [13]. Therefore, measuring characteristics immediately in time space are of particular interest. The cyclic voltometry, charge/discharge mode with a given direct current, etc. can be named among the most common techniques [14,15]. The development of techniques in time space is the subject of a number of studies [7–11,16–18].

The measuring technique [2] has been expanded in this paper to apply to the study of supercapacitors in the mode of pulsed energy extraction. Unlike simulation in frequency space, which is reduced to solving a system of linear Kirchhoff equations, simulation directly in time space requires solving a system of differential equations of an order equal to the number of capacitors. The main advantage of the approach is to obtain results directly in the form required in practice. The nonlinear properties of supercapacitors can also be easily taken into account by introducing nonlinearities into the model. This paper considers the simulation of an arbitrary RC-network in time space in relation to the problem of pulsed energy extraction.

1. Setting the task of pulsed energy extraction

Load R is connected to a supercapacitor charged to the voltage U_0 („Charge" key) for a specified time τ in accordance with the circuit diagram shown in Fig. 1, *c*. At what load will the maximum energy be released in the external circuit

$$E = \int_0^{\tau} I^2(t) R dt. \tag{1}$$

Since the structure of the supercapacitor is a complex hierarchical RC-system, in general this problem cannot be solved analytically, a numerical consideration of complex RC-networks will be given below. Let us consider the simplest RC-system of a series-connected resistor and capacitor, which corresponds to Fig. 1, *a* with one link. Let us denote them as R_i, C. The answer can be obtained analytically For such a RC-system. Since the voltage on the supercapacitor decreases exponentially with an external load R

$$U_C(t) = U_0 \exp\left(-\frac{t}{(R+R_i)C}\right), \tag{2}$$

the energy released in the external circuit (1) when the „Load" key is closed for a given time τ is obtained by integrating the power; the analytical expression —

$$E = \frac{U_0^2}{2(R+R_i)} RC \left[1 - \exp\left(-\frac{2\tau}{(R+R_i)C}\right)\right]. \tag{3}$$

We immediately obtain known answers for extreme cases from here: if $\tau \to 0$, then the maximum energy is released at $R = R_i$ and will be equal to $\tau U_0^2 \frac{R}{(R+R_i)^2}$, and if $\tau \to \infty$, then the maximum energy is released at $R \gg R_i$ and will be equal to the total energy stored in the supercapacitor $CU_0^2/2$.

This simple model shows the general form of dependence: there is a specific optimal load for each given time of energy extraction τ, which ensures the release of maximum energy in the external circuit. A dual formulation is also possible:find the time τ for which this load is optimal for a given external load. Figure 2, *a* shows the dependence $E(R)$ (3) for the considered model system $R_i = 1\Omega, C = 1$F for a series of values τ. This dependence has a maximum corresponding to the optimal load R_{opt}. The maximum is observed at $R = R_i$ at small τ , the maximum shifts to the area of high loads R with an increase of R and becomes much less pronounced. Fig. 2,*b* shows the dependence of the optimal load value R_{opt} on time τ. This dependence is almost linear for this model.

Let us apply the considered approach to supercapacitors with a more complex equivalent circuit. It is a tree-like RC-structure in the general case with a random distribution

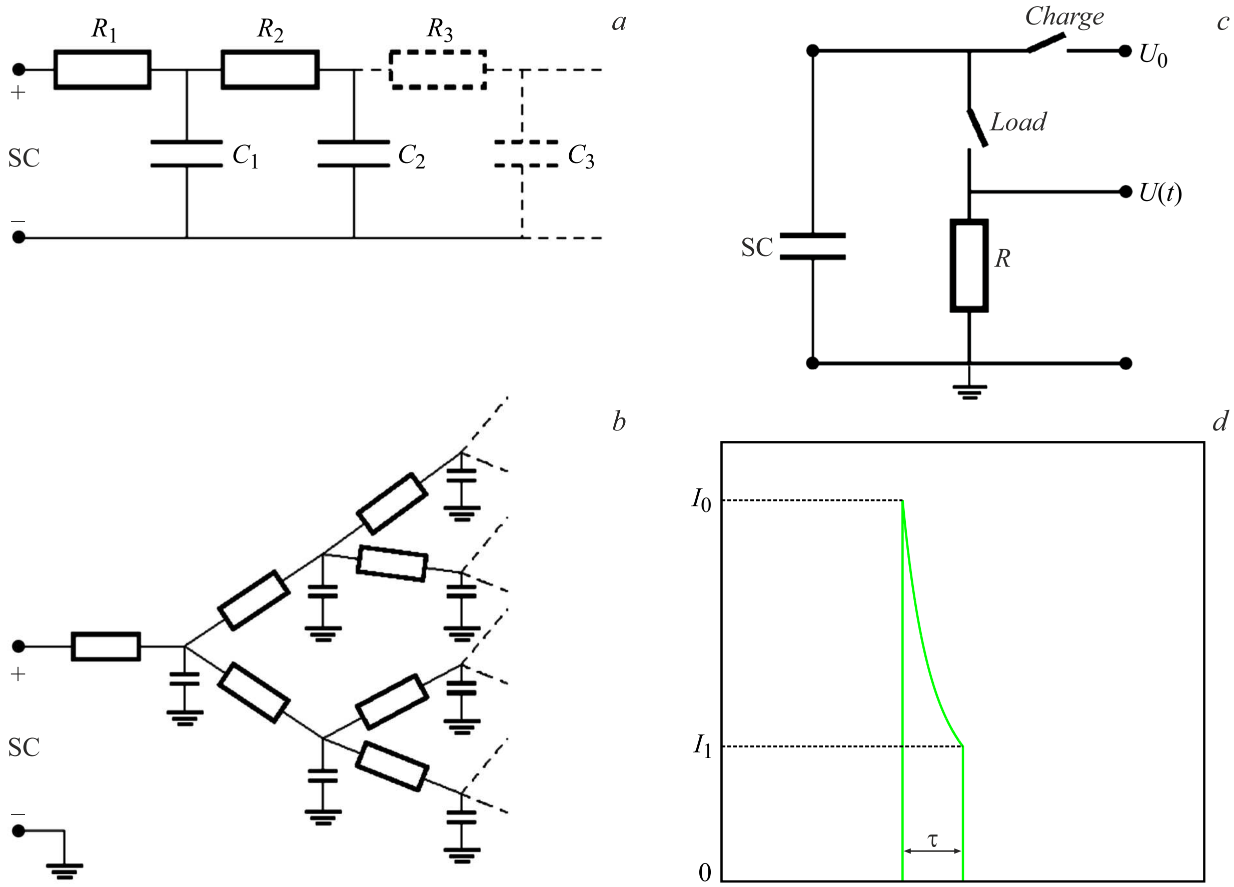

Figure 1. Equivalent supercapacitor circuits: *a* —horizontal ladder *RC*-circuit, *b* — example of a multilevel tree (binary tree); *c* — measurement circuit $U(t) = RI(t)$; $d - I(t)$.

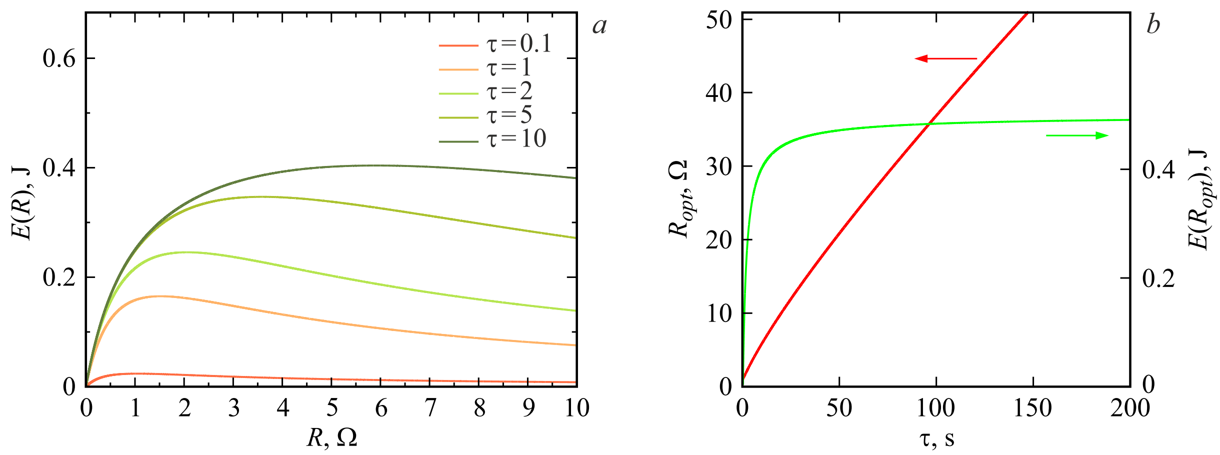

Figure 2. Dependence of the energy released in the external circuit for a simple supercapacitor model (serial connection $R_i = 1\,\Omega$, $C = 1\,\mathrm{F}$ with $U_0 = 1\,\mathrm{V}$, $E_{\max} = 0.5\,\mathrm{J}$) on the magnitude of the external load R. The model allows an analytical solution (3); *a* — dependence of the released energy on the load value R at a fixed time of energy extraction $\tau = 0.1$; 1; 2; 5; 10 s. An optimal R_{opt} is observed (corresponding to the maximum $E = E(R_{opt})$), which increases with τ. The maximum energy becomes less pronounced with increase of τ; *b* — the dependence of the optimal load value R_{opt} on τ. R_{opt} depends on τ almost linearly for model (3); the standard $R_{opt} = R_i$ is observed for small τ .

of R and C. We proposed a new efficient circuit element in [2] corresponding to an infinite *RC*-network with n descendants at each node — n-Tree Element (nTE). Its impedance can be expressed analytically in the case of identical R and C:

$$Z(\omega) = \frac{1}{2}\left[R + \frac{1-n}{j\omega C}\right] \pm \sqrt{\frac{1}{4}\left[R + \frac{1-n}{j\omega C}\right]^2 + \frac{Rn}{j\omega C}}. \tag{4}$$

The advantageous difference of the nTE element from other circuit assemblies used in impedance measurement (CPE and others) is its clear physical essence. The model corresponds to an infinite ladder scheme for $n = 1$ (one descendant for each node) (Fig. 1, *a*) and can be used to model a diffusion-limited Warburg element, for $n = 2$ — infinite a binary tree (Fig. 1, *b*) , for $n = 3$ — a tree with three descendants in each node. The self-similar structure of the equivalent supercapacitor circuit is an active subject of the study. In particular, a self-similar RC-network can model the Warburg element [19,20] with different values of power α. Consideration of three-dimensional RC-networks is of particular interest [21].

The study of RC-networks is usually carried out in the frequency space (parameter ω), and the goal is to obtain a complex impedance $Z(\omega)$. The study is carried out directly in time space in this paper (parameter τ). This approach does not require subsequent conversion from frequency space to time, however, studying a supercapacitor directly in time space requires more complex numerical simulation — solving a system of differential equations of an order equal to the number of capacitors. The above system (3) with a single RC-element corresponds to a self-similar system (5) of finite length with $n_R = 0$ or $n_R = \infty$.

Ladder model (Fig. 1, *a*) with self-similar ($R_{k+1}/R_k = n_R$, $C_{k+1}/C_k = n_C$) elements R and C:

$$R_k = n_R^k R, \tag{5 a}$$

$$C_k = n_C^k C \tag{5 b}$$

has a number of interesting properties. For instance, the Warburg element $Z(\omega) = 1/\omega^\alpha$ with various α can be modelled using such a chain. The analytically solvable nTE model (4) is a special case of a self-similar RC-network $n = n_C = 1/n_R$.

Let us consider a ladder system ($n_R = n_C = 1$, Fig. 1, *a*) of 31 (the number is selected for comparison with a binary tree) RC-elements with $R = 1\,\Omega$, $C = 1\,\mathrm{F}$ and a binary tree ($n_C = 1/n_R = 2$) of 4 levels, with a total of 31 RC-elements with $R = 1\,\Omega$, $C = 1\,\mathrm{F}$, and two self-similar systems (5 a), (5 b) with $n_C = 1$, $n_R = 1.2$ and $n_C = 1$, $n_R = 0.8$. Although a chain with identical or self-similar R and C seems to be too artificial an approximation, it can be expected that it can adequately describe porous materials with a predominant pore size made from plant precursors (for example, NORIT [22]), and fractal materials with a single similarity dimension [23].

The Ngspice circuit simulator program was used for the simulation. An updated version of the simulation code used in [2] is available in [3].

2. Simulation results

The transition from a system with a single RC to a system with distributed RC changes the nature of the curve of energy released during τ depending on the magnitude of the load. The severity of the maximum optimal load in distributed systems (Fig. 3) becomes even less noticeable compared to a singleRC- system in Fig. 2. The manifestation of the maximum now decreases not only with an increase of τ, but also with an increase of the degree of distribution RC. The smoothing of the maximum with the increase of the degree of distribution can be seen in systems of the self-similar type (5 a), (5 b): in case of transition from the system $n_C = 1/n_R = 2$ to $n_C = 1$, $n_R = 0.8$, to $n_C = 1$, $n_R = 1$ and further to $n_C = 1$, $n_R = 1.2$. This smoothing is attributable to the transition from one-exponential dependence (3) to distributed dependencies determined by the hierarchical RC. The graphs shown in Fig. 3 are similar to model (3). The effect of multi-exponential dependencies can be clearly demonstrated on dependencies $R_{opt}(\tau)$. We built the dependence of R_{opt} on τ in Fig. 4, for a system of 31 RC-elements with $n_C = n_R = 1$, the dependence $E(R)$ of which is shown in Fig. 3, *a* similar to the dependence previously built for the model (3) in Fig. 2, *b*. A stronger deviation from linear dependence is observed for distributed systems.

The question arises about the possibility of curves with several maxima. The presence of several maxima is undoubtedly possible if it is possible to include inductors in an equivalent circuit. In the absence of inductors, this behavior is practically not observed for a wide variety of models of RC-networks with all possible RC ratings and various values τ.

The task of ensuring a given load value introduces an additional technical complexity for experimental measurements while the hierarchical RC-networks can be simulated directly using the attached program [3]. The supercapacitor is shorted for a time τ in the methodology [2], and effective $R(\tau)$ and $C(\tau)$ are found from the invariants $\int I dt$ and $\int I^2 dt$; it is easier to perform such diagnostics experimentally, since it is not required to ensure a certain load value. The use of the obtained values $R(\tau)$ and $C(\tau)$ in the simplest oneRC-model with an analytical solution can be considered as an approximate method (3). Moreover, the values of the effective R and C can be approximately obtained from impedance measurements, as[1] $Z(\omega) = R_{\mathrm{eff}} + \frac{1}{j\omega C_{\mathrm{eff}}}$, assuming $\tau = 1/\omega$. The found R_{eff}, C_{eff} and the specified τ can also be used after that as values of parameter R, C and τ in analytical expression (3). For example, it is possible to estimate the effective values of $R_{\mathrm{eff}} = 0.04\,\Omega$ and $C_{\mathrm{eff}} = 3\,\mathrm{F}$ for a commercially available IC-505DCN2R7Q supercapacitor with a rated capacity of 5 F and a limit voltage of 2.7 V from Fig. 6 of the article [2] for $\tau = 0.1$ s. Similar values are obtained from

[1] It should be noted that the approach simulating a supercapacitor with a parallel connection of R and C as $Y(\omega) = 1/Z(\omega) = 1/R_{\mathrm{eff}} + j\omega C_{\mathrm{eff}}$ is better used for systems with significant charge leaks.

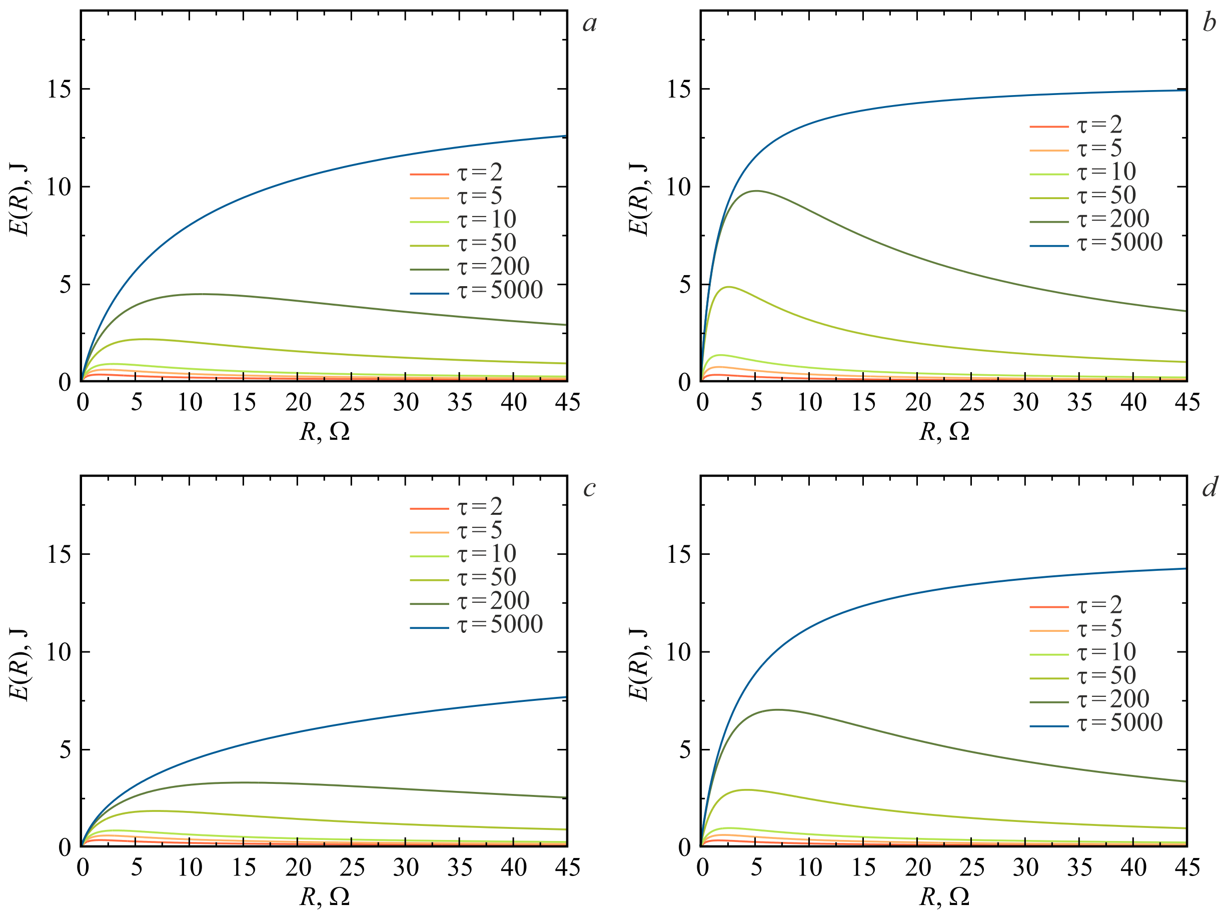

Figure 3. Dependence of the energy released in the external circuit for the supercapacitor model: *a* — in the form of *RC*-circuit (Fig. 1, *a*; all $R = 1\,\Omega$, $C = 1$ F, 31 *RC*-element, $n_C = n_R = 1$); *b* — binary tree (Fig. 1, *b*; all $R = 1\,\Omega$, $C = 1$ F, a tree with 4 levels $(2^{4+1} - 1 = 31$ *RC*-elements), $n_C = 1/n_R = 2$); *c* — self-similar system (5 a), (5 b) of 31 *RC*-element with $R_1 = 1\,\Omega$, $R_{k+1}/R_k = n_R = 1.2$, $C = 1$ F, $n_C = 1$); *d* — self-similar system (5 a), (5 b) of 31*RC*-element with $R_1 = 1\,\Omega$, $R_{k+1}/R_k = n_R = 0.8$, $C = 1$ F, $n_C = 1$). For all considered systems $U_0 = 1$ V, $E_{\max} = C_\Sigma U_0^2/2 = 15.5$ J. Graphs are given for energy extraction time $\tau = 2$; 5; 10; 50; 200; 5000 s.

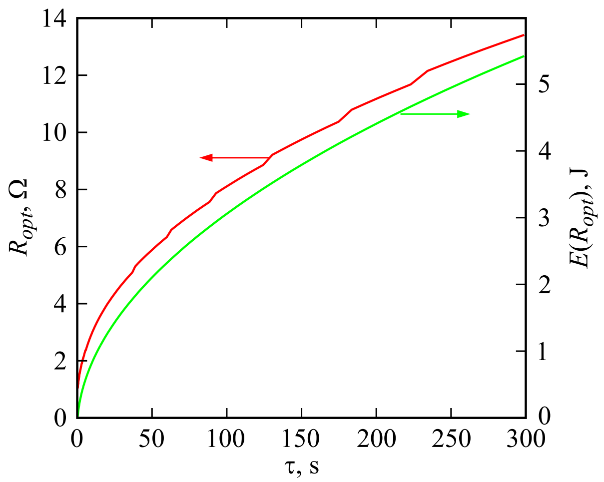

Figure 4. Dependence of the value of optimal load R_{opt} on time τ for a system of 31 *RC*-elements with $n_C = n_R = 1$, which roughly corresponds to the Warburg element with $\alpha = 0.5$. A much stronger deviation from linearity is observed compared to one*RC*-model (3) (Fig. 2, *b*).

impedance $[\mathrm{Re}Z(\omega), \mathrm{Im}Z(\omega)]$ measurements: $R_{\text{eff}} = 0.08\,\Omega$ and $C_{\text{eff}} = 3$ F, fig. 7 of the article [2] for $\omega = 1/\tau$ without using the reverse relaxation method [24]. Figure 5 shows a graph of the energy released in the external circuit of the IC-505DCN2R7Q supercapacitor as a function of the load at $\tau = 0.1$ s, $U_0 = 1$ V. In the simplest version, the proposed technique allows estimating the energy released in the external circuit only on the basis of effective R and C of paper [2].

3. Discussion of results

The results of the study show that there is an optimal load for each time of energy extraction in case of a pulsed discharge of a supercapacitor. The sharpness of the maximum depends on the time of energy extraction and on the degree of distribution of the *RC*-system. The simulation approach used in time space allows obtaining the result directly, without converting from frequency space, which would be necessary for simulation of the impedance

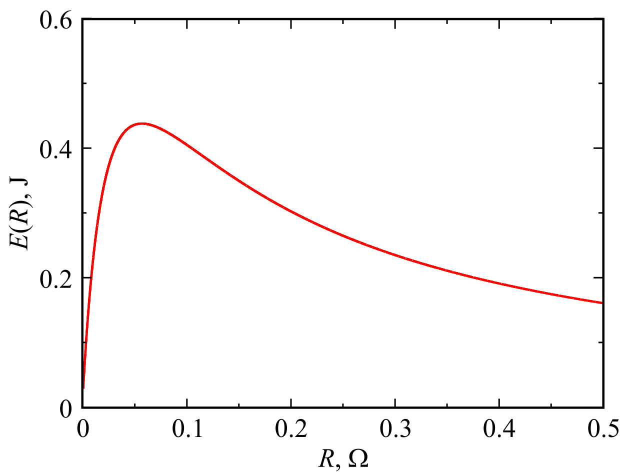

Figure 5. The dependence of the energy released in the external circuit for the IC-505DCN2R7Q supercapacitor with nominal capacity of 5 F and limit voltage of 2.7 V at $\tau = 0.1$ s, $U_0 = 1$ V. The result was obtained by preliminary estimation of effective$R_{\text{eff}} = 0.04\,\Omega$ and $C_{\text{eff}} = 3$ F for $\tau = 0.1$ s with subsequent substitution into a simple analytical model (3). The optimal load value is 0.053 Ω (for $\tau = 0.1$ s).

$Z(\omega)$. The written program can be easily modified to take into account the nonlinear effects of SC. The main result of the study is to demonstrate the simulation of SC directly in time space — this approach is most practical for simulation of real systems. However, the developed technique (in the simplest version) can be applied to optimizing the load of a supercapacitor using only standard impedance measurement (without any measurements in time space). Effective R_{eff} and C_{eff} are found from the impedance $Z(1/\tau) = R_{\text{eff}} + \frac{1}{j\omega C_{\text{eff}}}$ for a given τ, after which the maximum of the analytical solution (3) determines the value of the optimal load R for the discharge of the supercapacitor for a given duration τ. We see the consideration of other types of impulse loads which are more close to a specific practical application as a further development of the study, taking into account nonlinear effects. When considered directly in the time space, this requires minimal changes in the load model, while it is extremely difficult to take into account nonlinear effects based on the impedance approach.

Conflict of interest

The authors declare that they have no conflict of interest.

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