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Structural nature of localized plasticity autowaves dispersion

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The relationships between macroscopic and microscopic deformation methods are considered using comparative characteristics of dislocation systems and the nature of dispersion of localized plasticity autowaves for different stages of the deformation curve. It is established that the form of the dispersion law for each stage of strain hardening is determined by good dislocation ensembles that arise at different stages of the plastic deformation process.

Keywords: plasticity, localization, dislocations, autowaves, dispersion.

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The development of the autowave approach in plasticity physics [1,2] was facilitated by the difficulty of application of dislocation models to severe plastic strain [3]. One of the foundational aspects of the autowave model of localized plasticity is the correspondence rule [1], which stipulates that each stage of the deformation curve corresponds uniquely to its own autowave mode of plastic flow. In turn, the stages of plastic flow are realized sequentially in the process of deformation of materials (see Fig. 1, which is plotted based on the data from [4]).

As is known [5,6], autowave processes are generated only by active deformable media. The activity of a medium is specified by the presence of distributed sources of potential energy associated with dislocations and dislocation ensembles [3], which act as stress concentrators. Their elastic fields [3], which evolve regularly under deformation, make the medium active. It is of interest to compare the macroscopic characteristics of autowave processes of plastic flow [1,2] with available data on the morphology of dislocation ensembles typical of different stages of plastic flow [3].

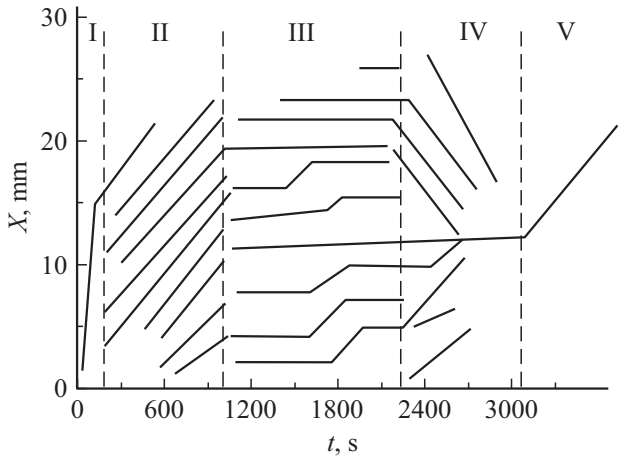

Figure 1. Stages of plastic flow under tension in a single crystal of the Fe–Cr–Ni alloy in the presence of atomic hydrogen (50 ppm). I — Lüders strain, II — stage of linear strain hardening, III — stage of parabolic strain hardening, IV — collapse of the autowave function (pre-fracture), and V — necking.

This comparison may be based on the dispersion laws of localized plasticity autowaves. As was demonstrated, they may be presented in the general form of $\omega(k) \sim k^{\beta}$, where ω is the frequency, k is the wave number, and index β changes discretely in interstage transitions. It follows from Fig. 2 that the stage of Lüders elastoplastic transition has $\beta = 1$ (linear dispersion), the stage of linear strain hardening is characterized by $\beta = 2$ (quadratic dispersion), $\beta = 5/2$ corresponds to the stage of parabolic strain hardening, and the stage of collapse of the localized plasticity autowave (pre-fracture) is characterized by $\beta = 3$ (cubic dispersion).

Using dimensional analysis, one may present the dispersion equation for localized plasticity autowaves in a form that is common to all stages of plastic flow:

$$\omega(k) \sim \left(\frac{\Lambda^{\beta}}{\vartheta}\right) k^{\beta}, \tag{1}$$

where coefficient $\Lambda^{\beta}/\vartheta$ for each stage of the process is specified by spatial (structural) scale Λ and relaxation time ϑ corresponding to this stage. Thus, we obtain the following relations for the Lüders strain stage:

$$\omega(k) \sim (\Lambda/\vartheta)k, \quad \beta = 1, \tag{2}$$

the stage of linear strain hardening:

$$\omega(k) \sim (\Lambda^2/\vartheta)k^2, \quad \beta = 2, \tag{3}$$

the stage of parabolic strain hardening:

$$\omega(k) \sim (\Lambda^{5/2}/\vartheta)k^{5/2}, \quad 2 < \beta = 5/2 < 3 \tag{4}$$

and the pre-fracture (autowave collapse) stage:

$$\omega(k) \sim (\Lambda^3/\vartheta)k^3, \quad \beta = 3. \tag{5}$$

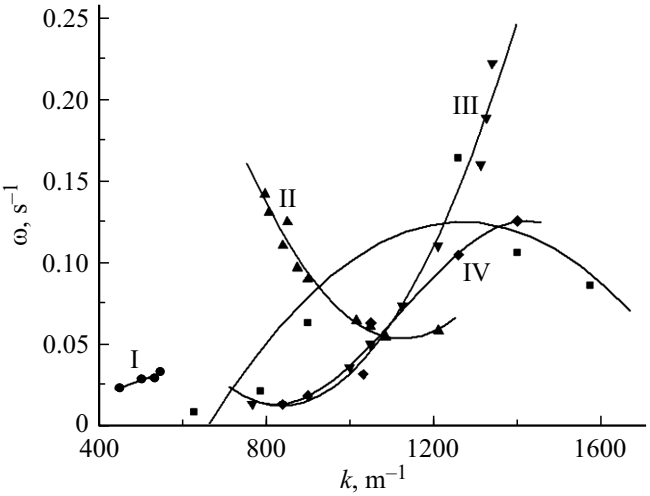

Figure 2. Dispersion curves for different stages. Squares — easy slip [2], circles — Lüders strain (I), triangles — linear strain hardening (II), inverted triangles — parabolic strain hardening (III), and diamonds — pre-fracture (IV).

In relations (3) and (5), $\Lambda^2 = \Sigma$ is area and $\Lambda^3 = \Omega$ is volume, which reveals the geometric nature of this interpretation. The physical meaning of quantities Σ and Ω will be discussed below.

Let us consider the physical nature of relations (2)–(5) under the assumption that changes in the dispersion law are induced by changes in the type of active media during strain hardening [7,8]. Data on dislocation substructures at relevant stages of the process may be needed to interpret the dispersion relations in this case.

At the stage of elastoplastic transition at constant stress $\sigma = \text{const}$, deformation proceeds at the Lüders front moving with a constant velocity [9]. Its phase and group velocities

$$V_{aw}^{(ph)} = \frac{\omega}{k} \quad \text{and} \quad V_{aw}^{(gr)} = \frac{d\omega}{dk} \tag{6}$$

are equal; i.e., $V_{aw}^{(ph)} = V_{aw}^{(gr)} = V_{aw}$. Multiplying the right- and left-hand sides of Eqs. (6) and integrating the products, we obtain

$$\frac{\omega d\omega}{kdk} = \frac{\int \omega d\omega}{\int kdk} = \frac{1/2\omega^2 + c_1}{1/2k^2 + c_2} = V_{aw}^2, \tag{7}$$

where c_1 and c_2 are integration constants. With $c_2 = 0$, relation (7) yields dispersion law $\omega^2 \sim 1 + k^2$ corresponding to the Klein–Gordon equation [8] for displacements u

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u = 0, \tag{8}$$

which is applicable to the propagation of macroexcitations (specifically, solitons in active media). At $k \gg 1$, Eq. (8) is reduced to equation $\partial^2 u/\partial t^2 - \partial^2 u/\partial x^2 = 0$ with linear dispersion $\omega \sim k$, which is typical of elastic strain waves [8].

At the stage of linear strain hardening, when $\sigma \sim \varepsilon$ (ε is strain), the characteristics of elastic strain and plastic flow form an elastoplastic invariant [1,2]:

$$\frac{\lambda V_{aw}}{\chi V_t} = \hat{Z} \approx \frac{1}{2}, \tag{9}$$

where localized plasticity autowaves are specified by their length λ and velocity V_{aw}, while elastic ones are specified by interplanar distance χ and transverse sound velocity V_t, respectively. If one substitutes χ and V_t in invariant (9) with Hartree [10] scales of length $a_0 = \hbar_2/me^2$ and sound velocity $V_s \approx e^2/\hbar(m/2M)^{1/2}$ expressed in terms of Planck constant $\hbar = h/2\pi$, electron charge e, electron mass m, and atom mass M, the following relation is obtained (see [11]):

$$\lambda V_{aw} = \frac{\chi V_t}{2} \approx \frac{\hbar}{2(mM)^{1/2}}. \tag{10}$$

The value of $\lambda V_{aw} \approx 10^{-6}\,\text{m}^2/\text{s}$ calculated using formula (10) is close to the ones obtained experimentally for the studied materials [1]. Equation (10) holds promise for analysis of the nature of the elastoplastic invariant, since it relates the characteristics of autowave plasticity to physical constants.

Let us demonstrate that a quadratic autowave dispersion law at the stage of linear strain hardening follows from Eq. (10). Setting $\lambda V_{aw} \approx \Lambda^2/\vartheta$, we find

$$\lambda V_{aw} = \frac{\Lambda^2}{\vartheta} = \frac{\left(\frac{2\pi}{k}\right)^2}{\frac{2\pi}{\omega}} = 2\pi\frac{\omega}{k^2} \approx \frac{\hbar}{\sqrt{mM}} \approx \text{const}, \tag{11}$$

which yields a quadratic dispersion equation for this stage:

$$\omega = \frac{\hbar}{2\pi\sqrt{mM}}k^2 \sim k^2. \tag{12}$$

The discussed quadratic dispersion law corresponds to nonlinear Schrödinger equation $2i\partial\Phi/\partial t - Q\partial^2\Phi/\partial x^2 + G|\Phi|^2\Phi = 0$ [8] for function Φ in potential $G|\Phi|^2\Phi$. Here, Q and G are coefficients and $i = \sqrt{-1}$. In the general case, this equation characterizes the evolution of the carrier wave envelope in a weakly nonlinear system [6,8]; in the case of plastic deformation, it is applicable to the process of self-organization of a sequence of thermally activated elementary shears. The medium is auto-oscillating at this stage, and it corresponds to a phase autowave with $\omega t - kx = \text{const}$.

At the stage of parabolic strain hardening with $\sigma \sim \varepsilon^{1/2}$, a stationary dissipative structure forms from localized plasticity centers. Varying length λ of the localized plasticity autowave by altering the deformation conditions, we managed to construct a dispersion dependence that is shown in Fig. 2 and follows relation (4): $\omega \sim k^{5/2}$. The stage of parabolic strain hardening may be regarded as a transition from the stage of linear hardening, where $\omega \sim k^2$, to the stage of pre-fracture (collapse of the localized plasticity autowave), where $\omega \sim k^3$.

The dispersion law obtained for this stage of strain hardening, where $\sigma \sim \varepsilon^n$ and $n < 1/2$, by processing the

$X-t$ diagrams for different metals and alloys from [1,2] is presented in Fig. 2. A dispersion relation of the $\omega \sim k^3$ form corresponds to Korteweg–de Vries equation $\partial u/\partial t - \partial^3 u/\partial x^3 = 0$, which characterizes the propagation of excitation pulses in active excitable media [8].

It seems logical to associate the change in dispersion relations in Eqs. (2)–(5) with the evolution of sizes and shapes of dislocation ensembles [3,12] (i.e., the structural part of coefficient Λ^β). Index β should then depend on the configuration of the dislocation ensemble at the corresponding stage of the process. At the yield plateau stage ($\beta = 1$), Lüders strain does indeed transform an elastic medium into a plastically deformable one. The transformation is caused by the avalanche-like release of dislocations from blocking impurities [13]. Coupled with linear dispersion, this allows one to treat the Lüders front as a switching autowave [7] in a medium consisting of interconnected bistable elements (i.e., dislocations that pass from the initial metastable (immobile) state to a stable (mobile) state). As was noted in [14], the transition process kinetics is similar to the kinetics of the first-order phase transition front.

To analyze the role of the dislocation structure at the stages of linear strain hardening and pre-fracture, we take into account that $\Lambda^2 = \Sigma$ and $\Lambda^3 = \Omega$ (see above). Quantities Λ, Σ, and Ω found in expressions (2)–(5) are the geometric characteristics of dislocation ensembles; Σ may be regarded as the surface area of dislocation cells at the stage of linear strain hardening, while Ω is the volume of dislocation tangles forming at the pre-fracture stage [12].

As for the stage of parabolic strain hardening, it is known [3,12] that a cellular dislocation structure emerges at its onset in deformed metals and is replaced by a tangle one during the deformation process. This is consistent with the above reasoning that the stage of parabolic strain hardening, which corresponds to index $2 < \beta = 5/2 < 3$, is a transition from linear hardening to the collapse of the localized plasticity autowave.

The results of the above analysis reveal a connection between dislocation and autowave concepts of plastic flow and verify the notion [1,2] that the physical basis of emergence of different stages in plastic flow is the formation of active media specific to each stage. The dispersion law of localized plasticity autowaves for each stage is related to the parameters of the corresponding dislocation structure. It was demonstrated that the key relations characterizing the autowave nature of plastic flow may be expressed through physical constants.

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Conflict of interest

The authors declare that they have no conflict of interest.

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