Mathematical modeling of influence of localized gravitational noise on propagation of electromagnetic radiation in gravitational field

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Numerical-analytical method of modeling of propagation direction and group delay of electromagnetic radiation in gravitational field of group of astrophysical objects in presence of localized gravitational noise is offered. The method is based on the solution of stochastic Lagrange-Euler differential equations received from Fermat's variation principle. Propagation of radiation in stochastic gravitational field under consideration as process in Euclid space with effective index refraction of vacuum expressed through gravitational potential. Theory of disturbances is used for calculation of fluctuations of propagation direction of radiation and estimation of stochastic Shapiro effect. Results of calculations of side deviations and additional group delay of radiation for various parameters of gravitational noise in gravitational field of group of astrophysical objects are presented.

Keywords: electromagnetic radiation, astrophysical objects, geometrical optics, gravitational lensing, gravitational noise.

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Introduction

It is well known [1–5], that in the gravitational field of a massive object, the direction of propagation, group delay, and spatial attenuation of electromagnetic radiation energy flow are changing. Systems of gravitational objects complicate the process of radiation propagation and lead to the formation of unique distributions of the electromagnetic field in the observer's image plane. Using such distributions, it is possible to reconstruct the parameters and properties of radiation sources and estimate fundamental cosmological constants. Currently, the solution to this problem is critical when detecting the powerful electromagnetic bursts, the nature of which is far from being clear [6]. For a better interpretation of gravitational impact effects during observations of electromagnetic radiation, it is necessary to allow for the gravitational noise generated by stochastic inhomogeneities of the background gravitational field of outer space. In particular, mathematical modeling of the effect of gravitational noise on the formation of lensing pattern and Shapiro time delay for a group of astrophysical objects is of great interest.

1. Numerical Analysis Simulation Apparatus

To analyze the propagation direction and group delay of electromagnetic radiation in a complex gravitational field in presence of localized gravitational noise, Lagrange-Euler differential equations derived from Fermat's variation principle were considered as initial equations [7]:

$$\frac{dR}{d\varphi} = R\,\mathrm{ctg}\beta,$$

$$\frac{d\beta}{d\varphi} = \left(1+\sin^2\beta\,\mathrm{tg}^2\alpha\right)\left(\frac{1}{\tilde{n}}\left(\frac{\partial\tilde{n}}{\partial\varphi}\,\mathrm{ctg}\beta - R\frac{\partial\tilde{n}}{\partial R}\right)-1\right),$$

$$\frac{d\delta}{d\varphi} = \mathrm{tg}\,\alpha, \tag{1}$$

$$\frac{d\alpha}{d\varphi} = \frac{1}{\tilde{n}}\left(1+\cos^2\alpha\,\mathrm{ctg}^2\beta\right)\left(\frac{\partial\tilde{n}}{\partial\delta} - \frac{\partial\tilde{n}}{\partial\varphi}\,\mathrm{tg}\,\alpha\right),$$

$$\frac{d\tau}{d\varphi} = \frac{\tilde{n}R}{c\sin\beta}\sqrt{1+\sin^2\beta\,\mathrm{tg}^2\alpha},$$

where $R(\varphi)$, $\delta(\varphi)$, φ — radial and angular coordinates of the beam, accordingly; $\alpha(\varphi)$, $\beta(\varphi)$ — refraction angle of the beam; $\tau(\varphi)$ — group delay; c — light velocity; $\tilde{n}$ — random effective vacuum refractive index. For the refractive index, a model was used that takes into account the additive contribution of objects to the overall gravitational field:

$$\tilde{n} = n_0 + \tilde{n}_1,$$

$$n_0 = 1 + \frac{R_g}{R} + \sum\nolimits_{i=1}^{N} A_i \exp[-b_{\varphi i}(\varphi-\varphi_{Li})^2 - b_{\delta i}(\delta-\delta_{Li})^2 - b_{Ri}(R-R_{Li})^2], \tag{2}$$

where n_0 — refractive index characterizing regular gravitational field; $\tilde{n}_1$ —describes random localized gravitational noise; R_g — gravitational radius of the major gravitation object; N — number of additional modes of the refractive index; A_i, φ_{Li}, δ_{Li}, R_{Li}, $b_{\varphi i}$, $b_{\delta i}$, b_{Ri} — intensity, coordinates

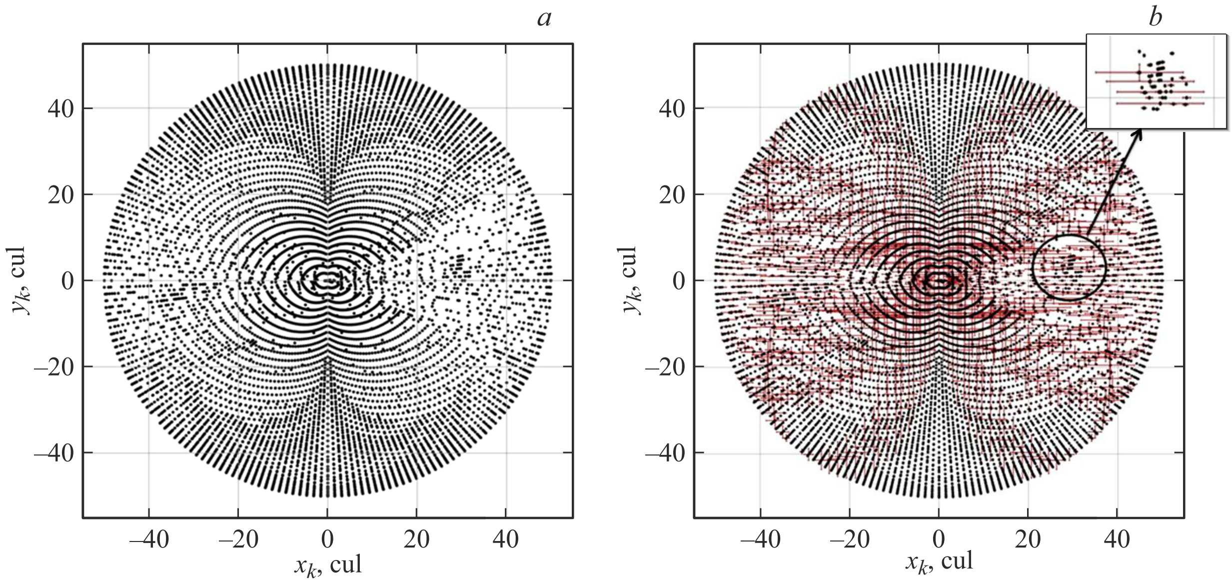

Figure 1. Observer's image plane during propagation of electromagnetic radiation in the gravitational field of three astrophysical objects in the absence of (a) and presence of (b) gravitational noise.

of localization and scale of i-th mode, respectively. The system (1) was solved in approximation of perturbation method provided that $\tilde{n_1} \ll 1$. As a result, a generative system of equations is obtained for calculating the refractive effects of a regular gravitational field (system (1) at $\tilde{n_1} = 0$), as well as a system of equations for calculating the variances of lateral deviations and group delays of beams in the observer's image plane [5]:

$$\frac{d\sigma_\delta^2}{d\varphi} = \frac{\mu}{4}\sqrt{\frac{\pi}{Q}}\Big(\frac{DP^2}{Q} + 16\Big(D - \frac{K}{Q}\Big)(\varphi J_1 - J_2)\Big),$$

$$\frac{dJ_1}{d\varphi} = P^2, \quad \frac{dJ_2}{d\varphi} = \varphi P^2, \tag{3}$$

$$\frac{d\sigma_{\Delta\tau}^2}{d\varphi} = \mu\sqrt{\frac{\pi}{Q}}\,\frac{R_0^2}{c^2\sin^2\beta_0},$$

where

$$P = \frac{1}{\cos^2\alpha_0} + \mathrm{ctg}^2\,\beta_0,$$

$$Q = \frac{1}{\nu_\varphi^2} + \frac{1}{\nu_\delta^2}\,\mathrm{tg}^2\,\alpha_0 + \frac{R_0^2}{\nu_R^2}\,\mathrm{ctg}^2\,\beta_0,$$

$$K = \Big(\frac{1}{\nu_\varphi^2} - \frac{1}{\nu_\delta^2}\Big)^2\,\mathrm{tg}^2\,\alpha_0, \tag{4}$$

$$\mu = \gamma\mu_0,$$

$$\gamma = \exp\Big[-m_R(R - R_L')^2 - m_\varphi(\varphi - \varphi_L')^2 - m_\delta(\delta - \delta_L')^2\Big],$$

R_0, δ_0, α_0, β_0 — beam refraction characteristics at $\tilde{n_1} = 0$; μ_0, ν_R, ν_φ, ν_δ — intensity and extent of correlation of inhomogeneous noise; R_L', φ_L', δ_L', m_R, m_φ, m_δ — coordinates of center and dimensions of gravitational noise localization area.

2. Calculation results and discussion

Fig. 1, 2 shows the results of calculations of axial intensity (1), (3). The initial conditions were as follows: $\varphi_n = 0$, $R_n = 50$ cul (cul — conventional length unit); $\delta_n = 0$. The target angular parameter α_n was in the range of $[-0.94; 0.94]$ rad, and β_n was in the ranges — $[-0.75; -0.03]$ and $[0.03; 0.75]$ rad. The calculation was carried out up to a distance $R_k = 50$ cul, where the observer's image plane was formed with the final angular coordinates of the beam $(\varphi_k; \delta_k)$ marked on it. For gravitational objects the following parameters were set: $R_g = 1$ cul; $A_1 = 0.5$, $R_{L1} = 10$ cul, $\varphi_{L1} = 0.5$, $\delta_{L1} = 0.1$; $A_2 = 0.5$, $R_{L2} = 14$ cul, $\varphi_{L2} = 0.8$, $\delta_{L2} = 0.4$; $b_{R1} = b_{R2} = 1\,\mathrm{cul}^{-2}$, $b_{\varphi 1} = b_{\delta 1} = b_{\varphi 2} = b_{\delta 2} = 6.25$. Parameters of gravitational noise were: $\nu_r = 0.1$ cul, $\nu_\varphi = \nu_\delta = 0.1$, $R_L' = 0$ cul, $\varphi_L' = \delta_l' = 0$, $\mu = 10^{-5}$, $m_R = 25\,\mathrm{cul}^{-2}$, $m_\varphi = m_\delta = 2.56$. Fig. 1 illustrates the observer's image plane with presence of gravitational noise and without it. For clarity, the finite angular values $(\varphi_k; \delta_k)$ are shown here in Cartesian coordinates: $x_k = R_k\cos\varphi_k\cos\delta_k$; $y_k = R_k\sin\varphi_k\cos\delta_k$. Note that for the gravitational field of the selected configuration, a focusing area was formed near the point with coordinates $(x_k; y_k) - (30; 2.5)$ cul (Fig. 1, a). In the presence of gravitational noise, the central part of the distribution of beam points reaching the image plane is noticeably blurred (Fig. 1, b). In general, this pattern corresponds to the disappearance of a clear structure of gravitational lensing. However, analyzing the focusing area, it can be seen that the distribution of points is blurred to a lesser extent, therefore, some enlarged copy of the radiation source can be observed in the image plane.

After the absolute values of the root-mean-square lateral deviations (RMS) of beams had been analyzed for the

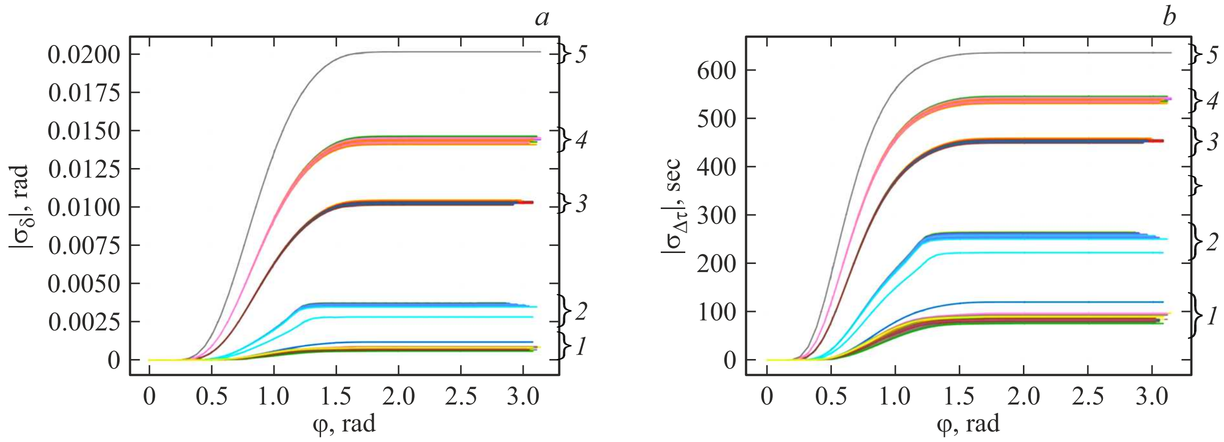

Figure 2. RMS of lateral deviations (a) and delays (b) of beams in the focusing area. *1* — $\alpha_n \in [0, 0.22]$ rad, $\beta_n \in -0.25$ rad; *2* — $\alpha_n \in [0.21, 0.43]$ rad, $\beta_n \in [-0.23, -0.21]$ rad; *3* — $\alpha_n \in [0, 0.25]$ rad, $\beta_n \in [-0.19, -0.17]$ rad; *4* — $\alpha_n \in [0.17, 0.22]$ rad, $\beta_n \in [-0.15, -0.13]$ rad; *5* — $\alpha_n = 0.22$ rad, $\beta_n = -0.11$ rad

image plane in the focusing area (Fig. 2, a) it became clear that the curves split and form the groups with almost identical RMS values. This structure was formed due to the influence of a different number of objects of the gravitational field and the general path of propagation of electromagnetic radiation in gravitational noise. The increase of RMS is associated with an increase in the number of gravitational objects: from zero to three. The largest RMS corresponds to the beams passing near the main gravitating object, which corresponds to a significant radiation propagation path in the gravitational noise area. Small RMS correspond to the beams passing tangentially to the gravitational-noise field. For the selected parameters of models (2), (4) the stochastic Shapiro effects were calculated (Fig. 2, b). The formed structure of values distribution of additional group delay associated with gravitational noise is consistent with the results of lateral deviations RMS modeling.

Conclusion

Based on the beam approximation and perturbation theory, a method for numerical and analytical modeling of electromagnetic radiation refraction in a stochastic gravitational field is proposed. The moments of beams lateral deviations and group delay in the gravitational field of a group of astrophysical objects in presence of localized gravitational noise were calculated. The modeling results showed that radiation is separated in the observer's image plane under the influence of a multicomponent fluctuating gravitational field. A similar separation is also noted in modeling the stochastic Shapiro effect. The proposed method of numerical and analytical modeling can be used to identify gravitational objects based on measurements of the refractive characteristics of received electromagnetic radiation.

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Conflict of interest

The authors declare that they have no conflict of interest

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