Analysis of the structural properties of the Bennu minor planet using harmonic analysis and fractal geometry

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A comprehensive analysis of the structural model of the asteroid Bennu was carried out. An original fractal method was developed for studying small atmosphereless bodies of the Solar System. Self-similar regions on the surface of Bennu were identified that correspond to the same evolutionary processes that had taken place on the celestial object. The developed approach can be applied to study other small bodies in the Solar System.

Keywords: fractal geometry, harmonic analysis, Bennu asteroid.

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Near-Earth asteroid Bennu is now the most mysterious celestial body. After landing of NASA „Osiris-REx" spacecraft on the asteroid, it turned out that the asteroid was either hollow or had a very porous structure. Ejection of matter by the asteroid under the action of meteoroidal impact objects [1], which is observed by ground-based telescopes in the form of meteor showers caused by impact events, is also of particular interest [2]. Note that analysis of the structure and physical properties of various formations on the asteroids is important to describe the asteroid surface formation and damage processes [3]. The study investigates multispectral images ($0.44-0.89\,\mu$m) of asteroid Bennu (Figure 1) using harmonic expansion of data in spherical functions and the proprietary fractal method [4]. Visible color and reflective characteristics of the surface provide important information about the surface evolution aspects.

The investigations used the OLA laser altimeter from the OSIRIS-Rex mission [5]. Thus, the Bennu altimetric model [6] includes altitude measurements (Figure 2) as well as the influence of rotational and gravitational effects. A smoother surface is observed in the equatorial region than towards the asteroid poles. This is explained by the fact that the asteroid is in the rotational Roche lobe [7].

Structural analysis of asteroid Bennu included the following steps. Digital altimetry data of the Bennu surface changes was initially divided into pixels with the mean radius vector of 5 arcseconds. Then, the obtained parameters were used to build a three-dimensional digital model of the asteroid surface.

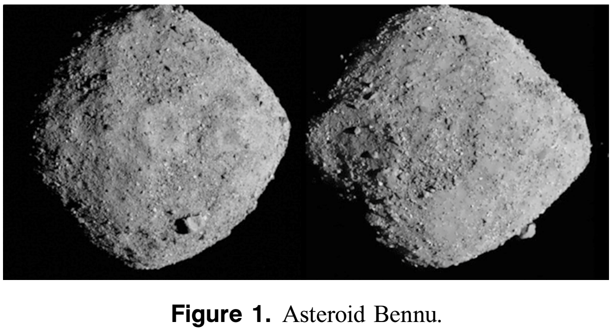

Figure 1. Asteroid Bennu.

Then, the fractal dimension $F_d$was determined for the given regions using expression [7]:

$$\boldsymbol{F}_d = \sum_{\gamma} \frac{\log N_d(n^2_{\gamma+1}) - \log N_d(n^2_{\gamma})}{abs(\log S_{\xi}(n^2_{\gamma+1})) - abs(\log S_{\xi}(n^2_{\gamma}))} \times \left(\frac{\alpha_{\gamma+1}-\alpha_{\gamma}}{N-1}\right), \quad (1)$$

where $N_d(n^2)$ is the number of spheres covering the target area on the Bennu surface. In our case, a restriction of 15 arcseconds was introduced because it is sufficient for modeling. Color dimensions were introduced for more accurate determination of the fractal dimension. Color characterized averaged terrain heights. Color fractal dimensions F_G,F_B, F_R were obtained, where the subscript indicates blue, green and red fractal dimensions, respectively. Then, for the color fractal dimension F_{RGB}, the following can be written

$$F_{RGB} = \frac{1}{2}P[-2(F_R+F_B)+(F_G+F_B)+(F_R+F_G)], \quad (2)$$

where P is the scale parameter.

To determine self-similar areas, the self-similarity coefficient K_f was introduced:

$$K_f = \frac{F^o_d}{F_d}, \quad (3)$$

where F^0_d is the total fractal dimension.

Figure 3 shows the distribution of the self-similarity coefficient K_d on the asteroid surface. Self-similar areas are those that have the same self-similarity coefficients. In our

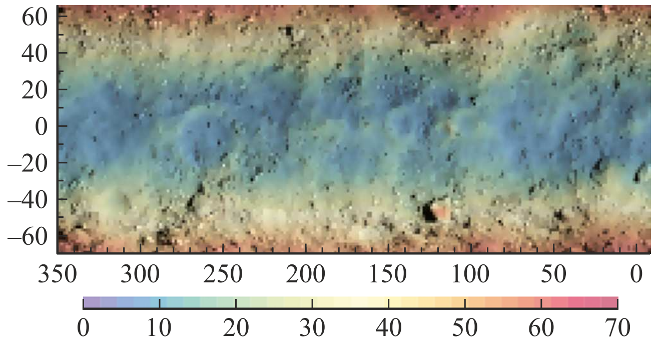

Figure 2. Bennu map model [6]. On the latitude axis, the scale is given in degrees from $+60^\circ$ to -60°, on the longitude axis, the scale is given in degrees from $+0^\circ$ to 350°. Color scale of heights above geopotential in meters is shown at the bottom of the figure.

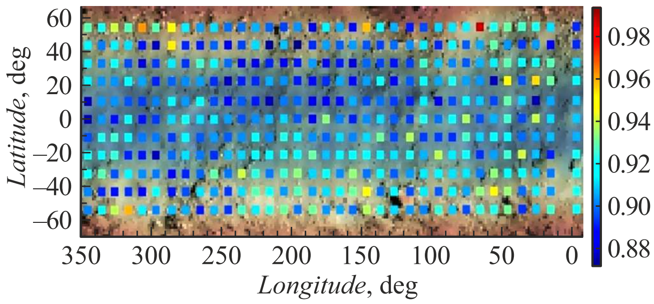

Figure 3. $2D$- distribution model of K_d on the Bennu surface.

case — those that have similar colors. The largest number of coincidences correspond to the values of K_d from -10 to -20. It is considered that these areas evolved in the same physical and chemical processes.

There are currently no studies of multifractal maps of the Bennu surface and its $2D$-model. Note that the findings obtained in this work were averaged in a particular way because the observation data was approximated, but all this doesn't alter the matter of the findings in any significant way.

Finally, self-similar areas and their positions on asteroid Bennu were determined. Conclusions made in [8] regarding an unusual asteroid figure were partially confirmed. As shown in Figure 1, significant thickening of the figure occurs in the Bennu equator direction and, thus, there is a rhomb-shaped object in projection, rather than a sphere. In [8], this effect is explained by the fact that ejection of particles takes place in negative latitudes when exposed to solar radiation and then the particles return to Bennu in the form of re-accumulation of matter at the equator. In our opinion, composition of the asteroidal material itself also plays the leading role here. Detected fractal features may be also explained by the fact that the effect of solar light, gravity and thermal gradients creates a range of stable orbits of particles emitted by Bennu. Small particles 1 cm in size are removed from this Bennu–emitted particles system, and the remaining particles return back and concentrate at the Bennu equator, thus, causing its rhomboid shape [9]. Many other near-Earth asteroids probably have the same features.

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Conflict of interest

The authors declare no conflict of interest.

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