Elasticity of Neutron Star Mantle: Impact of Neutron Adsorption

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Received May 2, 2024
Revised July 17, 2024
Accepted October 30, 2024

The elastic modulus of the neutron star mantle, associated with a change in intercluster distances, is calculated within the framework of the thermodynamically consistent compressible liquid drop model. It is demonstrated that the neutron adsorption on the surfaces of nucleon clusters results in a $\sim 10{-}20\%$ change in the elastic modulus within the typical range of mean nucleon number densities of the neutron star mantle.

Keywords: neutron star, mantle, elastic properties, spaghetti phase, lasagna phase.

DOI: 10.61011/TP.2024.12.60416.387-24

Introduction

Neutron star mantle is a layer that contains nuclear clusters whose energetically favorable shape may take a form of cylinders (spaghetti phase), plane-parallel plates (lasagna phase), inverted cylinders (bucatini phase), inverted spheres (Swiss cheese phase) [1]. The existence of this layer was first predicted in papers [2,3]. Since then, efforts have been made to determine the impact of mantle presence on the evolution and observational manifestations of neutron stars (see, for example, Refs. [4–8]).

In particular, its elastic properties might be important for description of torsional oscillations which serve as an option for explaining quasi-periodic oscillations observed after flares of soft-gamma repeaters associated with neutron stars (see, for example, Ref. [7]). The maximum quadrupole deformation of a neutron star and the corresponding gravitational wave radiation by rotating neutron stars might also depend on the mantle's elastic properties [8].

Elastic properties of the mantle for the spaghetti and lasagna phases were first addressed in Ref. [9] within a liquid-drop model neglecting the neutron adsorption on the cluster surface. Inclusion of the effect of neutron adsorption (deposition) on the nuclear cluster surface into the compressible liquid drop model allows taking into account the difference in rms radii of proton and neutron distributions in the cluster. Moreover, adsorption must be considered for the thermodynamically consistent description of the interface of two-phase system (see, for example, Ref. [10]). Nevertheless, many authors discard it for simplicity. For example, neglecting this effect allows for analytical calculations as described in Ref. [9]. Elastic properties of the mantle were also studied in a computationally demanding relativistic-mean-field model [11]. Another work to be noted is one of Ref. [12] that investigated breaking strain of the lasagna phase in the classical molecular dynamics simulations, and of Ref. [13] that considered the effective shear modulus of the mantle disordered on a hydrodynamic scale.

Here we calculate the mantle's compression modulus for the spaghetti and lasagna phases. We employ a thermodynamically consistent compressible liquid drop model that takes into account the neutron adsorption on the cluster surface. Section 1 describes our model, Section 2 contains our findings and detailed comparison with Refs. [9,11].

1. Physical model

Authors of Ref. [9], whose notations we follow here, showed that the lasagna phase is characterized by two elastic constants B and K_1. The elasticity modulus B describes the response to varying spacing between the plates and K_1 the response to their bending. To describe the elastic properties of the spaghetti, three quantities are required: compression modulus B, the transverse shear modulus C and the elastic modulus associated with bending K_3. Following to Ref. [9], we consider pasta deformations keeping the mean number density of nucleons n_b fixed.

In this work we calculate the elastic modulus B for the spaghetti and lasagna phases within the thermodynamically consistent liquid-drop model, i.e. taking into account the neutron adsorption on the nuclear clusters surface (see, for example, Ref. [14]). Surface energy is calculated using the 2nd-order extended Thomas–Fermi method [15]. Calculations rely on the Wigner–Seitz approximation. Therefore, we consider a cylindrical cell with the circular cross-section, where the cylindrical cluster is located in the center for the spaghetti phase, and a flat layer with a flat cluster in the center for the lasagna phase. Previously, in study [16], we used a similar model to calculate the elasticity modulus C for the spaghetti phase, there the dependence of C on the cluster size and lattice spacing was described analytically.

As noted above, the elasticity modulus B was studied analytically in Ref. [9] within the liquid-drop model that neglected the neutron adsorption effect. In this case, nucleon densities inside and outside the cluster may be assumed constant during deformation, which simplifies the discussion and makes it possible to couple B with the Coulomb energy of the cluster and volume fraction u occupied by the cluster. However, when the adsorption is taken into account, nucleon densities inside and outside the cluster can vary during deformation because a part of nucleons can additionally be adsorbed on the cluster surface or released from it. This leads to explicit dependence of B on the model of nucleon interaction and requires numerical calculation to determine B.

Similar to Ref. [9], to determine B, we calculate the difference in energy densities between the deformed and undeformed states of the mantle at the pre-defined mean nucleon number density n_b and approximate it by the quadratic dependence on $\delta r_\mathrm{c}/r_\mathrm{c}^{eq}$:

$$E\left(n_\mathrm{b}, \delta r_\mathrm{c}\right) - E\left(n_\mathrm{b}, \delta r_\mathrm{c} = 0\right) = \xi B \left(\frac{\delta r_\mathrm{c}}{r_\mathrm{c}^{eq}}\right)^2. \quad (1)$$

Here, r_c^{eq} and δr_c are the equilibrium size of the Wigner–Seitz cell and its variation in deformation, the geometrical parameter $\xi = \frac{1}{2}$ for the lasagna and $\xi = 2$ for the spaghetti (calculations for both phases were conducted independently).

Numerical computations were carried out as follows. For each of the values of n_b from the realistic range of mean nucleon number densities for the mantle, the equilibrium size of the Wigner–Seitz r_c^{eq} cell was determined, then the change in the energy density was calculated with this size being varied within $[0.99\, r_\mathrm{c}^{eq}, 1.01\, r_\mathrm{c}^{eq}]$ where equation (1) is satisfied with good accuracy without introducing next-order corrections. $E\left(n_\mathrm{b}, \delta r_\mathrm{c}\right)$ was calculated in two assumptions corresponding to the limit of very fast and very slow β-reactions. In the former case, β-equilibrium of the matter is maintained during deformation. In the latter case, the fraction of protons in the Wigner–Seitz cell Y_p remains unchanged during deformation.

2. Results and conclusions

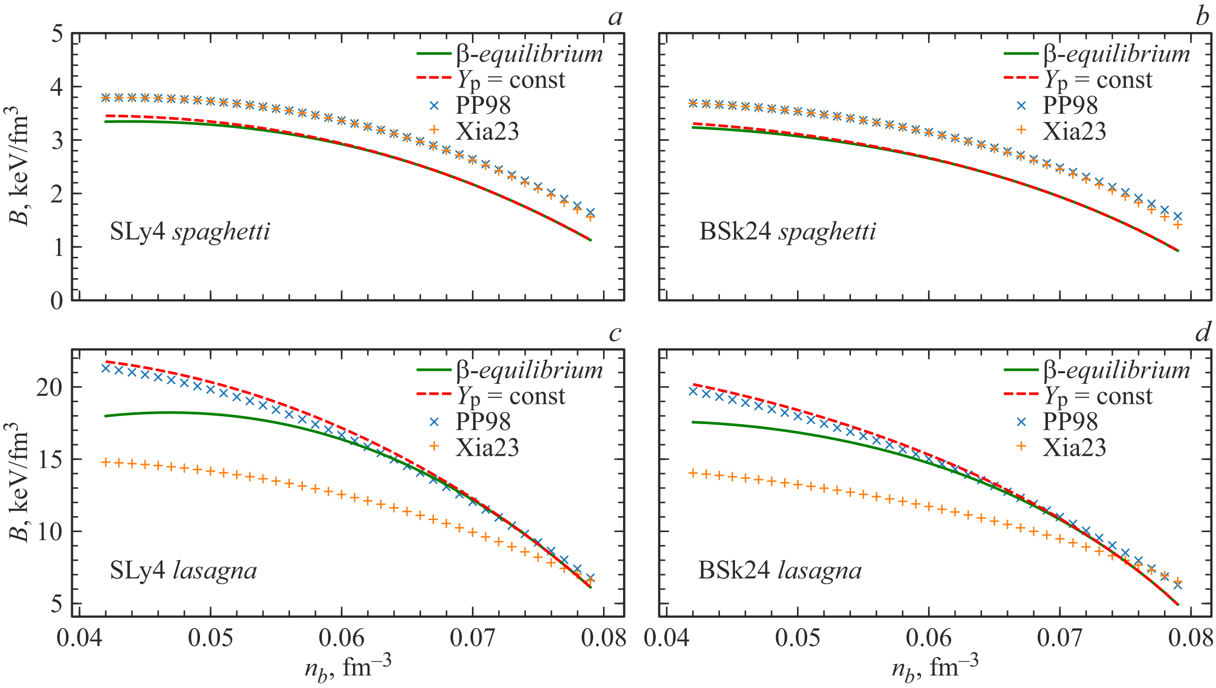
The figure shows B of the spaghetti (Figure, *a,b*) and lasagna (Figure, *c,d*) phases in the mean nucleon number density range n_b typical for the neutron star mantle. Calculation was conducted for the two parameterizations of the Skyrme-type effective interaction: SLy4 [17] (Figure, *a,c*) and BSk24 [18] (Figure, *b,d*). As expected, in the fast β-process limit, relaxation to the β-equilibrated matter leads to energy reduction in the deformed state and, therefore, to decrease of B.

For comparison with the literature, figure also shows B obtained using the expressions derived in Ref. [9] and their updated versions from Ref. [11] presented as analytical approximations of the relativistic mean field calculations (i.e. results in Ref. [11] account for the differences in the neutron and proton distribution profiles in the cluster that are described as the neutron adsorption in the liquid-drop model). Although the first of the works listed above assumed $Y_\mathrm{p} = \mathrm{const}$ and the second one used the β-equilibrium, the fits proposed in Ref. [11] actually represent the equations from Ref. [9] with introduction of additional correction factors that only depend on u.

Figure shows that the expressions from Refs. [9,11] for the spaghetti phase agree well with each other and systematically give the values of B that are by $\sim 10{-}20\%$ higher than those in our calculation. One should also notice a low sensitivity of B to the assumed rate of the β-processes during deformation, which follows both from our calculations and comparison of findings of Refs. [9,11].

In turn, for the lasagna phase, results of our calculations with the assumption of $Y_p = \mathrm{const}$ agree well with Ref. [9]. However, in the case of the fast β-processes at $n_\mathrm{b} \lesssim 0.07\,\mathrm{fm}^{-3}$ the obtained results predict a higher value of B by $\gtrsim 10\%$ (up to $\sim 30{-}40\%$) than the approximation from Ref. [11]. Nevertheless, at higher densities that are more typical for the lasagna phase, all calculations agree well with each other. Note a higher sensitivity of the lasagna compression modulus to the β-process rate during deformation than that of the spaghetti compression modulus.

As mentioned above, the difference of our results from those in Ref. [9] is explained by the fact that the neutron adsorption is accounted for, which changes the surface energy description significantly (see, for example, Ref. [10]) and leads to a complex dependence of B on the chosen nuclear model. The approximated equations proposed in Ref. [11] differ from the expressions in Ref. [9] by introduction of the correction factors that are equal to 10^{-3u^4} and $10^{0.55u-10u^8-0.19}$ for the spaghetti and lasagna, respectively. These factors were derived for one particular nucleon interaction model in [11] (for the second model applied in that paper, the mantle is energetically unfavorable and the authors did not calculate the elastic properties). For the spaghetti, this factor gives a correction of about 5% at $u \sim 0.3$ that is typical for the spaghetti phase (for example, [3]), and this correction grows dramatically with further growth of u. For the SLy4 and BSk24 models applied here, $u \lesssim 0.3{-}0.35$, the difference between the approximations [9,11] for the spaghetti does not exceed 10% (see the figure), while, according to our calculations, inclusion of the neutron adsorption leads to a somewhat higher effect. It is difficult to find the exact reason, but, in our view, this may be caused by the following factors: (1) as mentioned above, the neutron adsorption effect on the compression modulus depends on the particular nuclear model and probably for one applied in Ref. [11] it turns to be very low or does not appear at all due to the numerical effects; (2) in Ref. [11], the moduli of elasticity were determined by fitting the energy calculated not only for the small deformations, but also for the large deformations (as specified in Ref. [16], this may affect the obtained quantitative results); (3) during approximation of their numerical results, the authors of Ref. [11] probably



Dependence of elastic constant B on n_b. *a, b* — spaghetti phase, *c, d* — lasagna phase. *a, c* — for SLy4, *b, d* — for BSk24. Solid line („β-equilibrium“) shows our calculations with the assumption that the nuclear matter stays β-equilibrated during deformation, dashed line („Y_p = const“) shows our calculations with the assumption of the constant proton fraction, crosses („PP98“) show the analytical expression from [9], pluses („Xia23“) show the approximation expression from [11]. For details see the text.

made the main focus on the high filling factors u, where the introduced correction was very significant, but did not pay much attention to the approximation uncertainty at small u, where the correction was small; (4) it cannot be ruled out that the compressible liquid drop model does not have sufficient accuracy and overestimates the neutron adsorption effect on the elastic properties, for example, due to neglecting the corrections associated with the cluster surface curvature (see, for example, [14] that shows the importance of the last effect for the equilibrium mantle structure). This assumption may be verified by the direct comparison of the elastic properties with the calculations using the extended Thomas–Fermi method. However, such a study is more computationally expensive, and we are planning to undertake it in the future.

Finally, it should be noted that, although this study shows the importance of taking into account the neutron adsorption, nevertheless, the quantitative description of this effect depends on the chosen nuclear model and this dependence requires additional investigations that we plan to do in future.

Acknowledgments

The authors are grateful to the reviewers for their useful comments.

Funding

N.A. Zemlyakov's and A.I. Chugunov's work was performed within the scope of the topic of state assignment No.FFUG-2024-0002 of Ioffe Institute. N.N. Shchechilin's work was supported by the grant of FWO (Belgium) and Fonds de la Recherche Scientifique (Belgium) under the Excellence of Science (EOS) program (project №40007501).

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by E.Ilinskaya