The influence of space curvature on the moment of inertia tensor of axisymmetric magnetic field of radiopulsar

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The influence of space curvature on the input of the star precession due to moment of inertia of magnetic field outside neutron star on radiopulsar braking indexes is considered.

Keywords: radiopulsar, neutron star, precession.

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Introduction

Radio pulsars — are spinning neutron stars surrounded with extremely strong magnetic field $B \sim 10^{11} - 10^{14}$ Gs [1]. This magnetic field has energy density $\sim \frac{B^2}{8\pi}$ and, accordingly, mass density $\sim \frac{B^2}{8\pi c^2}$. Inside a light cylinder this field spins together with the star. Its mass inside the light cylinder may be assumed „attached" to the neutron star, roughly speaking. Accordingly, one can assume that the mass and impulse of the field contribute to the effective tensor of inertia of the star, making it different from the spherical one, which results in a precession even in isolated pulsars [2]. Another interpretation of this process is also possible [1]. According to this interpretation, the impact at the star and the magnetosphere spinning together with it is described as an action of a certain additional „abnormal braking torque" of forces applied to the surface of the neutron star [3]. This precession is potentially responsible for the appearance of a cyclic component in the evolution of the radio emission parameters in pulsars with the specific time scale $T \sim 10^3 - 10^4$ year and a low-frequency component of „red noise" [4]. It is possibly also related to the repeatability of the flashes in the sources of fast radio bursts (FRB) [5]. The impact of OTO effects at the magnetic field of pulsars was considered, for example, in papers [6–8]. In this paper we consider the impact of the space curvature around the neutron star at the addition to the pulsar braking index $n = \ddot{P}P/(\dot{P})^2$, related to the precession of the star caused by the contribution to the moment of inertia of its magnetic field beyond the star itself.

1. Model

In this paper we will use the results of papers [8], which considered the impact of the space curvature at the pulsar magnetic field, and [9], which considered the impact of the space curvature near the neutron start at the moment of inertia δI^f of the magnetic field beyond the star. Let the magnetic field beyond the neutron star be described with a single harmonic with numbers l and m [2] and axis of symmetry $\vec{e}_{lm}$. In this paper we will limit ourselves to the case $m \neq \pm 1$, therefore, the moment of impulse $\vec{L}^f_{lm}$ of such magnetic field beyond the neutron star is [2]:

$$\vec{L}^f_{lm} = I^f_{lm}\,\vec{\Omega} + \delta I^f_{lm}\,\vec{e}_{lm}\,(\vec{e}_{lm}\cdot\vec{\Omega}), \quad (1)$$

where $\vec{\Omega}$ — angular speed of rotation of the neutron star, $\Omega = 2\pi/P$, P — pulsar period. Harmonics with values l, differing by more than 3, do not interfere, and their contributions to the moment of inertia of the magnetic field may be simply added up. Let us consider the simplest model of the pulsar magnetic field. Let it consist of the harmonic $(lm) = (10)$, which describes the dipole field of the pulsar, and the harmonic (lm) with $l > 5$, describing the contribution of the small-scale component of the magnetic field. Let $< B^2_{10} >$ and $< B^2_{lm} >$ — be the average values of the field intensity for the corresponding harmonics on the surface of the neutron star, then we will introduce the parameter $\nu = \sqrt{< B^2_{lm} > / < B^2_{10} >}$, describing how much the small-scale field exceeds the dipole field on the surface of the neutron star [9]. Then the moment of impulse $\vec{L}^f$ of the magnetic field beyond the star is equal to

$$\vec{L}^f = (I^f_{10} + I^f_{lm})\,\vec{\Omega} + \delta I^f_{10}\,\vec{e}_{10}\,(\vec{e}_{10}\cdot\vec{\Omega})\delta I^f_{lm}\,\vec{e}_{lm}\,(\vec{e}_{lm}\cdot\vec{\Omega}). \quad (2)$$

For simplicity, let us limit ourselves to an axisymmetric case, when $\vec{e}_{lm} = \vec{e}_{10}$, then

$$\vec{L}^f = I^f\,\vec{\Omega} + \delta I^f\,\vec{e}_{10}\,(\vec{e}_{10}\cdot\vec{\Omega}), \quad (3)$$

where $\delta I^f = \delta I^f_{10} + \delta I^f_{lm}$ and $I^f = I^f_{10} + I^f_{lm}$. Member δI^f describes the difference of the star moment of inertia from the spherical one and brings it to precession. The precession period of the neutron star T_{pr} may be roughly assessed as $T_{pr} = K_{pr}\,P\,I_{ns}/\delta I^f$, where K_{pr} — coefficient of order of

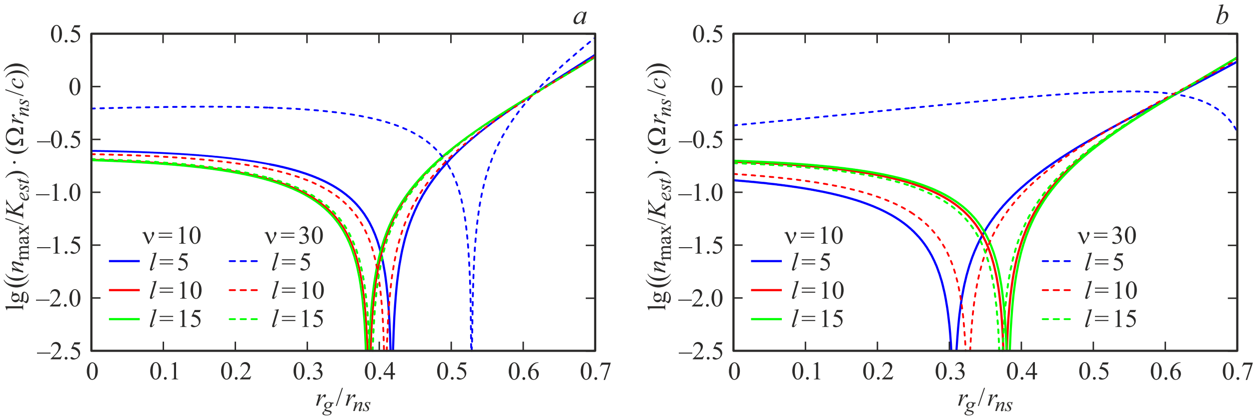

Figure 1. Dependence of estimate of maximum braking index $n_{\max}$ on the value of ratio r_g/r_{ns} for several values l at $\nu = 10$. *a* — complies with case $m = 0$, *b* — $m = l$.

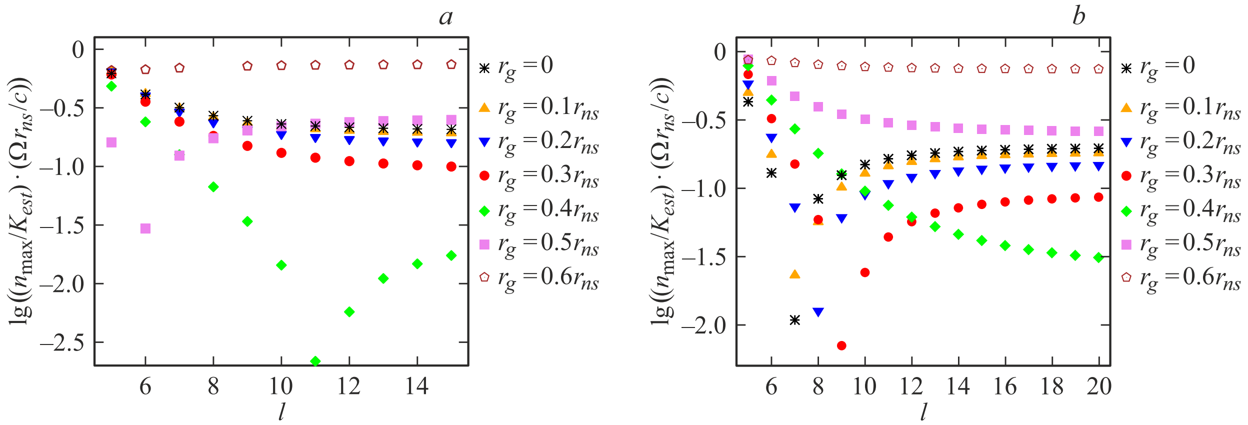

Figure 2. Dependence of estimate of maximum braking index $n_{\max}$ on parameter l for several values r_g/r_{ns} at $\nu = 30$. *a* — $m = 0$, *b* — $m = l$.

unity. In the considered case $\vec{e}_{lm} = \vec{e}_{10}$ it is simply equal to $K_{pr} \approx \cos\chi$, where χ — angle between the vector of the angular speed of the pulsar $\vec{\Omega}$ and vector $\vec{e}_{10}$. Within the considered model [2] the period of precession of the neutron star is $T_{pr} \sim 10^3 - 10^4$ years, which is much longer than the period observed in the pulsar B1828-11 $T_{pr} \approx 468$ days [10], but agrees more or less with the results of paper [4]. During the precession the electric current flowing through the internal clearances changes, which causes cyclic changes of current losses and the braking rate of the pulsar [11], the period of which is equal to the precession period T_{pr}. The latter is reflected in the value of the pulsar braking index $n = \ddot{P}\,P/(\dot{P})^2$. According to [4], the maximum possible value of the braking index $n_{\max}$, caused by precession of the neutron star with the period T_{pr}, may be assessed as $n_{\max} = K_n \cdot 2\tau/T_{pr}$, where $\tau = P/(2\dot{P})$ — characteristic age of the pulsar and K_n — coefficient of the order of unity. Coefficient K_n depends on the structure of the magnetic field, the location of pulsar clearances, where the particles are accelerated, on the structure of the currents flowing through the clearances [11]. We will assess the braking rate of the pulsar as $\dot{\Omega} = K_{br} \cdot m_{10}^2\,\Omega^3/(I_{ns}\,c^3)$, where m_{10} — dipole magnetic moment of the pulsar corresponding to the harmonic $(lm) = (10)$, I_{ns} — moment of inertia of the neutron star and K_{br} — coefficient of the order of unity [1,12]. Coefficient K_{br} depends first of all on the angle of inclination χ. Within the model of the magnetic dipole losses we have $K_{br} \approx (2/3) \cdot \sin^2\chi$, within the current losses model - $K_{br} \approx (2/3) \cdot \cos^2\chi$ [1]. According to paper [13] one can write $K_{br} \approx (2/3) \cdot (1 + \sin^2\chi)$. Accordingly, the maximum possible value of the braking index $n_{\max}$ may be assessed as [9]:

$$n_{max} = \frac{K_{est}}{4\pi^2} \cdot \frac{\delta I^f}{m_{10}^2} \cdot c^3\,P \qquad (4)$$

where r_{ns} — radius of neutron star and $K_{est} = K_n/(K_{br}\,K_{pr})$ — coefficient of the order of unity.

Conclusion

Fig. 1 and 2 show the dependence of the value $n_{\max}$, estimated using formula (4), on the ratio r_g/r_{ns}, where $r_g = 2GM_{ns}/c^2$ — Schwarzschild radius of the neutron star, and M_{ns} — mass of the neutron star, and on the number l, describing the small-scale component of the magnetic field. The feature on the curves at $r_g \approx (0.3 - 0.4)\,r_{ns}$ is related

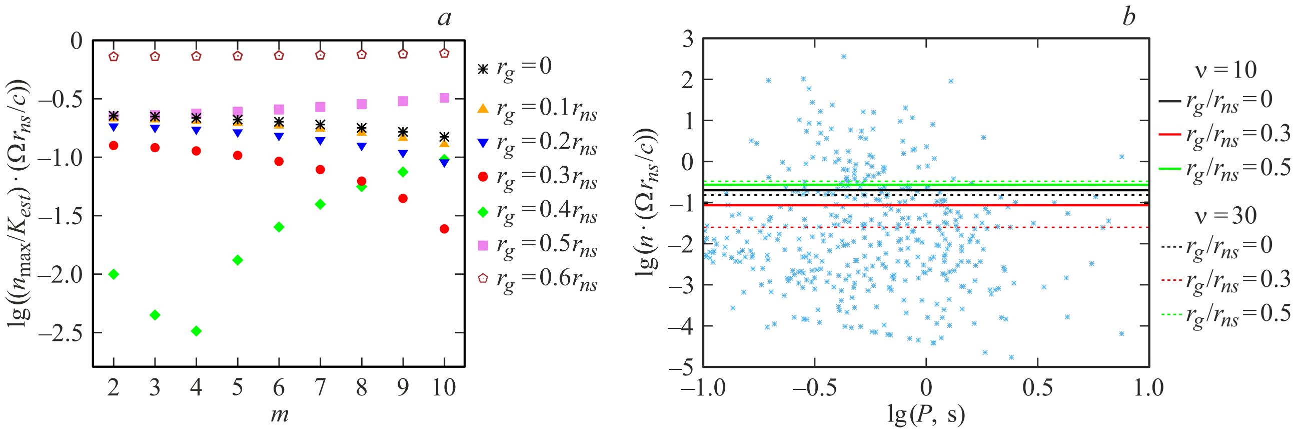

Figure 3. *a* — dependence of the estimate of the maximum braking index $n_{\max}$ on the number of m at $l = 10$ and $\nu = 30$ for several values r_g/r_{ns}. *b* — asterisks show the observed values of the pulsar braking index *n*, taken from [14]. The period of pulsar *P* is plotted on the X axis. Horizontal lines comply with the estimate $n_{\max}$ at $K_{est} = 1$ and $m = l = 10$.

to the fact that we use a logarithmic scale along the Y axis, and in this region the value δI^f passes through zero and changes the sign [9]. Fig. 3, *a* shows the dependence of estimate $n_{\max}$ on the azimuthal number *m*. Fig. 3, *b* provides the comparison of the produced estimates for the maximum value of the braking index $n_{\max}$ with the observed values taken from [14]. The fact that some pulsars have much higher braking indices may be related to the fact that the main contribution in them, in contrast to the tensor of inertia from the spherical one is given by the neutron star strain. At least in the isolated radio pulsar B1828-11 the precession is seemingly caused by the difference of the neutron star form on the spherical one [10]. Besides, a certain increase in the braking indices may be related to the current decomposition of the magnetic field [12]. It is also possible that in some pulsars the values of the braking indices given in [14] rather reflect the specific nature of the radio emission of the pulsars, while their real values may turn out to be much lower $n \sim 1 - 4$ [15]. It should also be noted that in the paper, when we assessed the precession period, we considered for the simplicity only the „coaxial" configuration $\vec{e}_{lm} = \vec{e}_{10}$. At the same time, in case of a strictly axisymmetric magnetic field of the pulsar, the star precession within the used model [11] will not impact the currents flowing through the internal clearances and, accordingly, the pulsar braking rate [11]. However, it would be sufficient to divert the vector $\vec{e}_{lm}$ by $5^\circ - 10^\circ$, so that at the accepted values $\nu \sim 10 - 30$ the field in the vicinity of the internal clearance would become substantially not axisymmetric, and the current flowing through the clearance would start changing during the precession. Besides, for the pulsars close to coaxial ones $\chi \lesssim 30^\circ$, the error in the estimation of the braking index $n_{\max}$, except for the region of „cusps", where the precessio stops, will not exceed $30 - 50\%$, which is acceptable within the precision of other estimates.

Conflict of interest

The authors declare that they have no conflict of interest.

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