Dynamics of the Earth's ionosphere in spherical functions

© D.S. Trofimov, S.D. Petrov, I.V. Chekunov, N.V. Shcherbakova

St. Petersburg State University,
199034 St. Petersburg, Russia
e-mail: d.trofimov@spbu.ru

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This paper examines the time series of spherical decomposition coefficients of global maps of the total electron content of the Earth's ionosphere. It has been demonstrated that zonal harmonics correlate well with the 11-year cycle of solar activity; tesseral harmonics of the 1st order have a pronounced daily periodicity. The resulting series of spherical expansion coefficients will be used to construct a model of the ionosphere.

Keywords: total electron content, spherical functions, ionosphere .

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Introduction

Currently the global navigation satellite systems (GNSS), such as GPS, GLONASS, Galileo or BeiDou have become not only the source of precise coordinate-time information, but an important global tool for research of various geophysical processes. In particular, the contribution of GNSS to the research of the processes in the Earth ionosphere is huge. Physical properties of the medium, through which the navigation signal of GNSS passes, impacts the signal propagation (first of all, the time of its propagation), which affects the final coordinates setting. On the other hand, the presence of the models for accounting of correction data, and knowledge of the specific coordinates of a receiver, makes it possible to precisely determine the parameters of the medium, through which the signal passed. One of the determined parameters is the total electron content (TEC) of the ionosphere. This paper is the logical continuation to papers [1,2] for determination of ionosphere parameters based on GNSS observations. In these papers TEC was determined in one point in the vicinity of the south geomagnetic pole using the observations at one observation point at the Vostok antarctic station. To compare and control the correctness of the obtained results, the global charts of ionopshere TEC were used. The presence of the wide networks of GNSS stations and multiple centers for the processing of GNSS observations makes it possible to build the global charts of ionosphere. The examples of such charts are the charts provided by the Center for Orbit Determination in Europe (CODE) [3] or the Information-Analytical Center of Coordinate-Time and Navigation Support (IAC CTNS) [4]. These charts contain ionosphere TEC values in the nodes of the spatial grid with the corresponding time step (1 or 2 h). It is assumed that all free electrons are present in the thin layer at the height of 450 km. The main practical use of these charts is the accounting for the ionosphere delay in high-precision processing of GNSS measurements. However, the combination of these charts, despite the restrictions (such as a single-layer model), displays the state of the ionosphere in dynamics for decades already and may be used to research the processes happening in the ionosphere and to research the relation of these processes to other geophysical processes caused by the solar activity, human operations or processes in the Earth's interior. But the 2D distribution is not always convenient for analysis, especially analysis of the accumulated long-term data. Analysis of periodical patterns in it is complicated. Therefore, it seems necessary to change from the 2D distribution in space to the presentation of the information in the form of time series. The most acceptable method to implement such transition would be the decomposition of the global TEC charts by spherical functions.

1. Problem formulation

Zonal (order $k = 0$) harmonics contain a marked seasonal component, with the period equal to the period of the tropical year, and a periodical component that shows the dependence on the eleven-year cycle of the solar activity (fig. 1). Tesseral coefficients, especially of the order $k = 1$, demonstrate the presence of a marked daily periodicity (fig. 2), and seasonal periodicity with the period equal to the tropical year. The zonal harmonic of degree $n = 0$ reflects the global electron content of the ionosphere, i.e. the average TEC for the entire ionosphere [9]. This characteristic complies well with the solar activity. The coefficients a_{nk} and b_{nk} at $k = 1$ are usually sinusoids displaced by phase for $\pi/2$. The main development of this paper is building a long-term model of spherical decomposition coefficients, which will make it possible to do the long-term confirmed forecast of the ionosphere, which may be used for the purposes of precise development of GNSS observations. The current ionosphere forecasting services [10,11] provide the forecast at the intervals from several hours to five days. There are papers [12], where the ionosphere forecasting is done for

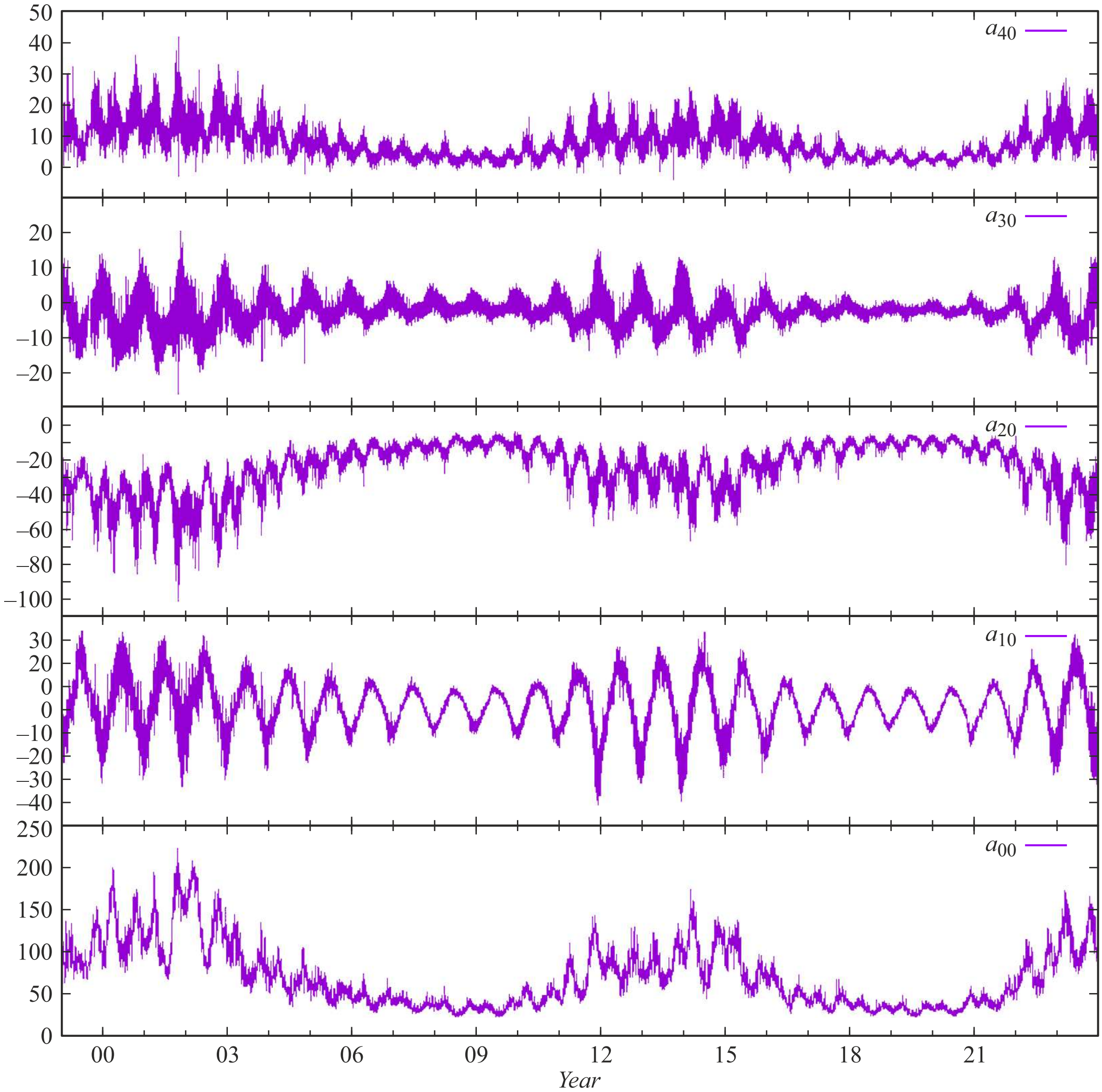

Figure 1. Zonal decomposition coefficients of a_{n0} degrees from 0 to 4 for 1999−2023

the longer intervals of time, they use the machine training methods.

The main problem of this paper is to determine time characteristics of the TEC spherical decomposition coefficients. The main end objective is the development of a model for precise and long-term forecasting of TEC for use in high-precision processing of GNSS. However, the achievement of the forecasting objective requires definition of how the ionosphere behaves, which factors impact the TEC. It is necessary to separate the ionosphere electron content into a well-modeled component, which has a strictly periodical nature, and a component that is strongly dependent on some external effects relative to the ionosphere, such as solar activity or some processes on the surface or Earth's interior. Therefore, the analysis of the spherical decomposition coefficients seems to be a good tool to research the processes in the Earth ionosphere and its connections to other geophysical processes. It is necessary to note that the use of the decomposition as such by spherical functions in respect to the ionosphere and global TEC charts is not novel. The TEC charts themselves are built on the basis of TEC decomposition by spherical functions [5–7] up to the 15th degree (previously, in the first years of the service, decomposition to the 12th degree and 8th order was used). The source TEC is determined on the basis of the observations at the permanent GNSS stations included in the global network that are located rather unevenly. Using TEC determined for these unevenly located points, the spherical decomposition coefficients are determined, on the basis of which the TEC values in the nodes of the spatial grid are calculated. However, this paper investigates specifically the long-term behavior of the

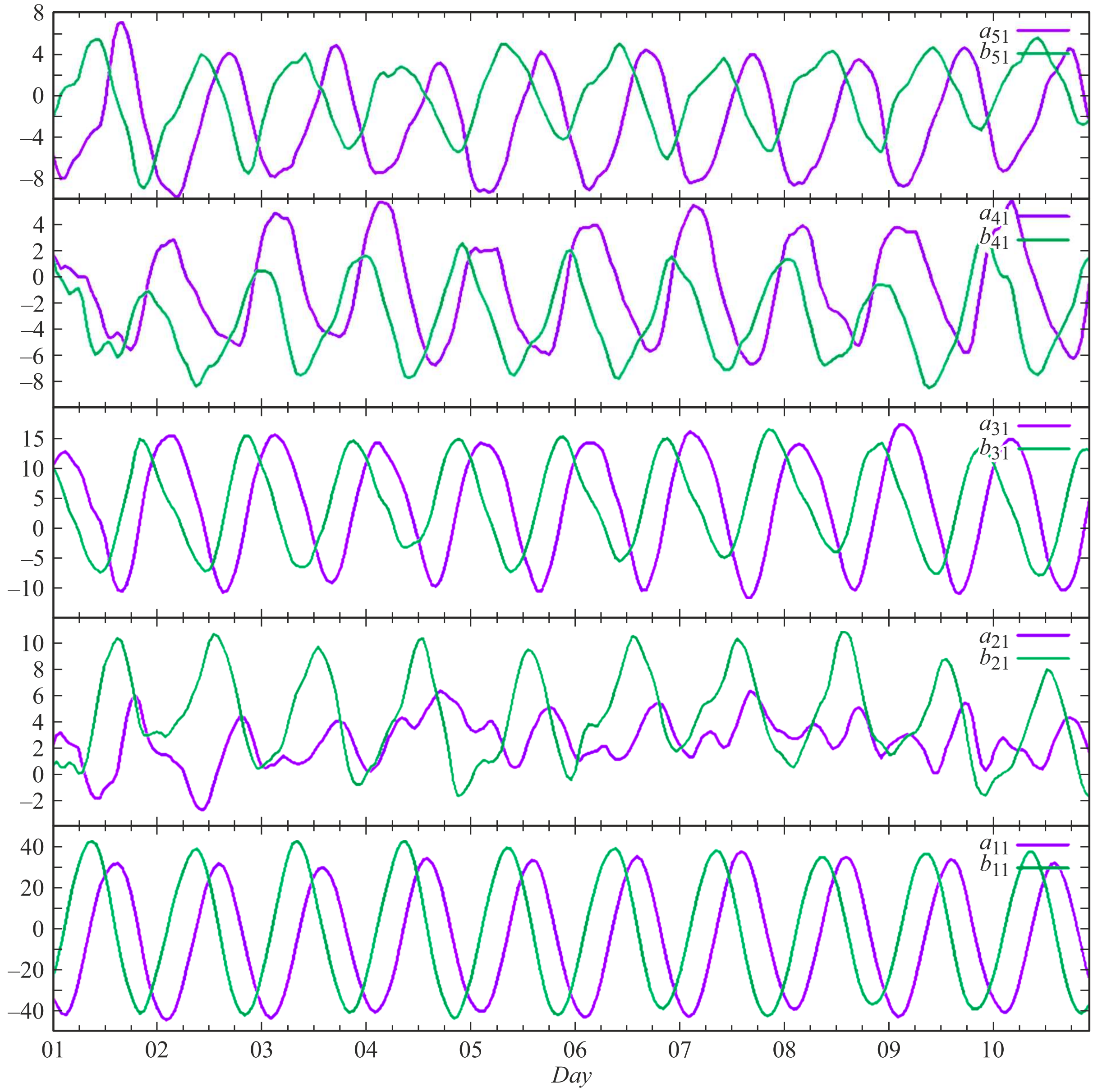

Figure 2. Decomposition coefficients a_{nk} and b_{nk} for degrees $n = 1 - 5$ and order $k = 1$.

spherical decomposition coefficients, and not the proximity of the 2D TEC distribution produced on its basis to the observational data.

2. Main mathematical tool

The mathematical tool applied in this paper is standard and is described in paper [8]. However, let us briefly provide the main ideas of the method. Any value y on the surface of the sphere with the coordinates of λ — longitude and ϕ — latitude may be presented in the form of a sum:

$$y(\lambda, \varphi) = \sum_{n=0}^{n_{\max}} \sum_{k=0}^{n} \bar{P}_{nk} \sin\phi (a_{nk} \cos k\lambda + b_{nk} \sin k\lambda), \quad (1)$$

where $\bar{P}_{nk}$ — normalized Legendre polynomials of degree n and kind k, a_{nk} and b_{nk} coefficients of spherical decomposition, $n_{\max}$ — maximum degree of decomposition. In its turn the normalized Legendre polynomials are determined as

$$\bar{P}_{nk} = N_{nk} P_{nk}, \quad (2)$$

where N_{nk} — normalizing function, and P_{nk} — regular associated Legendre polynomials. Spherical decomposition coefficients a_{nk} and b_{nk} are determined by the least-square method. For example, we have a vector of observations Y of size N:

$$Y = (y_1, y_2, \ldots, y_N), \quad (3)$$

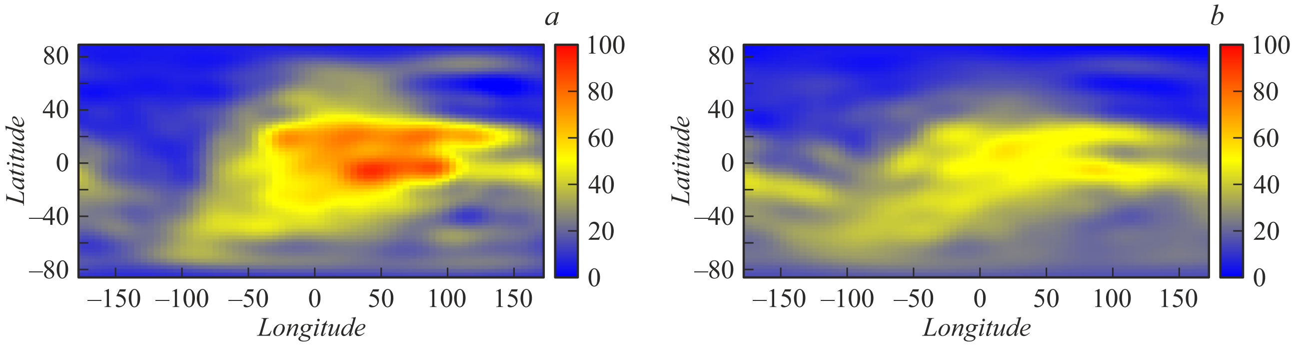

Figure 3. TEC charts as of 12:00 UTC on January 31, 2024 *a* — built by CODE using the results of the observations, *b* — forecast for the 30 days ahead made using data of December 2023

we have a vector of unknowns X, comprising coefficients a_{nk} and b_{nk}:

$$X = (a_{00}, a_{10}, a_{11}, b_{11}, \ldots, a_{n_{\max}n_{\max}}, b_{n_{\max}n_{\max}}), \quad (4)$$

the size of this vector is determined as $(n_{\max}+1)^2$. The matrix of coefficients A of size $((n_{\max}+1)^2 \times N)$ is determined as values $\bar{P}_{nk}\sin\phi\cos k\lambda$ and $\bar{P}_{nk}\sin\phi\sin k\lambda$ in the points with the coordinates (λ, ϕ), for which the observations Y were given. Assessment of the vector of unknowns, if all observations have the same weight

$$\hat{X} = (A^T A)^{-1} A^T Y. \quad (5)$$

3. Source data

The source data for the analysis are the global TEC charts of the ionosphere, presented by IAC CTNS and CODE. The IAC CTNS charts are available starting from January 1, 2018, the CODE data are available starting from January 1, 1999. The time resolution of the data is 2 h (time resolution of IAC CTNS charts is 2 h, of CODE charts in the beginning of 2000s was 2 h, now it is 1 h, for succession these data were rarefied to 2 h). The grid pitch by longitude is 5°, by latitude — 2.5°. The data are given with the precision of up to 0.1 TECU. IAC CTNS data were used to finalize the method, the final analysis was made on the basis of the CODE data for the period of time from January 1, 1999 to December 31, 2023, in total 25 full years, 109 572 TEC charts. The charts are calculated on the basis of the observations made on the permanent GNSS stations. Initially for CODE the data of approximately 150 stations were used, currently the number of stations, the data of which are used to analyze the observations, is around 300. Delay in the provision of data for CODE is around five days.

4. Results

During the analysis the TEC was decomposed up to the 15th degree and 15th order, i.e. to the maximum degree used to build the charts. The total number of the produced coefficients was 256. We received full 25-year series of spherical decomposition coefficients covering 2 eleven-year periods of solar activity.

Currently the autoregression model of order $p = 600$ was developed as a test for all 256 coefficients of spherical decomposition. For the degrees of decomposition starting from $n = 2$ it may provide the forecasting for one month ahead with the precision higher than the Klobuchar model [13], which is used to process the single-frequency GPS-observations. The example of the chart built on the basis of this model is given in fig. 3 compared to the observational data. The chart built on the basis of the forecast repeats the main morphological features of the chart built using real observations, while the charts built on the basis of the Klobuchar model, the region of higher ionization, are modeled with the regular geometric figure. On the other hand, there is „smoothing“ of TEC values, and the longer is the forecasting term, the stronger would the maximum TEC values differ to the lower side compared to the maximum values obtained on the basis of the real observations. This model is planned for further improvement due to the replacement of the autoregression to the trigonometric polynomial.

The precise definition of the periodic dependence of all harmonics and the transfer function between the spherical decomposition coefficients and indices of solar activity (for zonal coefficients with a marked relationship with the solar activity) will make it possible to define the off-model behavior of the spherical decomposition coefficients, which may be related to certain geophysical events, not related to the solar activity [14], and will also make it possible to build the charts of the abnormal TEC behavior.

Conflict of interest

The authors declare that they have no conflict of interest.

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