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# Pulse excitation of quantum systems: specific features and general patterns

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The features of pulse excitation of a quantum oscillator are investigated both without and with damping. Expressions are obtained for the duration and carrier frequency of exponential and double exponential pulses, which describe the main features of the process in weak and strong excitation modes. The boundaries of these modes are established in terms of the Rabi frequency for the time and spectral dependences of the excitation probability.

**Keywords:** quantum oscillator, pulse excitation, saturation effect.

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Intensive development of the technique for generation of ultrashort laser pulses with the specified parameters makes the improvement of the theoretical description of their interaction with the substance relevant for the features of the ultrashort electromagnet interaction. This paper is dedicated to this problem using the example of excitation of a quantum oscillator by two types of pulses.

## Main formulae

The basic formula for the probability of transition of a quantum oscillator (QO) between stationary states $|n\rangle$ and $|m\rangle$ $(m > n)$ was obtained by J. Schwinger in his theory of quantized electromagnetic field [1]:

$$W_{n\to m} = \frac{n!}{m!}(\nu)^{m-n}\exp(-\nu)|L_n^{m-n}(\nu)|^2, \tag{1}$$

where $L_m^k(x)$ — generalized Laguerre polynomials, $\nu$ — key parameter, which, when the QO is excited with an electromagnetic pulse, may be presented [2] as

$$\nu = \Omega_0^2|\tilde{E}(\omega_0, \tau, \omega_c)|^2, \tag{2}$$

where $\omega_0$ — QO own frequency, $\tilde{E}(\omega_0, \tau, \omega_c)$ — Fourier transform normalized by amplitude $E_0$ by the time of electric field intensity in the pulse with duration of $\tau$ and carrier frequency $\omega_c$,

$$\Omega_0 = \frac{d_0E_0}{\hbar} = \frac{qx_0E_0}{\hbar} = \frac{qE_0}{\sqrt{2m\hbar\omega_0}} \tag{3}$$

— Rabi frequency, $q$, $m$ — charge and weight of the oscillator, $x_0 = \sqrt{\hbar/(2m\omega_0)}$ — specific length of the quantum oscillator. The key parameter $\nu$ may be expressed via the energy of excitation of the associated classic oscillator (the associated classic oscillator is the oscillator with the same parameters as its quantum analogue) [3]:

$$\nu = \frac{\Delta\varepsilon_{\text{clas}}}{\hbar\omega_0}. \tag{4}$$

One of the methods to describe the excitation of the quantum oscillator by electromagnet pulses is the calculation of the average number of the excited quanta. Let us consider for simplicity the excitation of the oscillator from the main state, when the probability of excitation is given by the Poisson's distribution. By definition, the average number of the excited quanta is equal to

$$\bar{n} = \sum_n nW_{0\to n}(\nu) = \sum_n \frac{1}{(n-1)!}\nu^n\exp(-\nu) = \nu. \tag{5}$$

Therefore, the average number of the excited quanta is equal to parameter $\nu$. With account of the equation (4) we have the following relation to the excitation energy of the associated classic oscillator:

$$\bar{n} = \frac{\Delta\varepsilon_{\text{clas}}}{\hbar\omega_0} = \nu. \tag{6}$$

## Probability of QO excitation with various pulses

Let us consider the QO excitation from the main state with pulses having an exponential envelope:

$$\tilde{E}_{\exp}(t, \tau) = \theta(t)\exp(-t/\tau)\cos(\omega_c t), \tag{7}$$

where $\theta(t)$ — Heaviside function, and with the double exponential envelope:

$$\tilde{E}_{2\exp}(t, \omega_c, \tau) = \exp(-|t|/\tau)\cos(\omega_c t). \tag{8}$$

Note that formulae (7) and (8) describe the pulses that are asymmetrical and symmetrical in time and, therefore, differ qualitatively. It is important that the analytical description of the quantum oscillator excitation may be obtained for them. Thus, the key parameter $\nu$ in case of the exponential pulse is equal to

$$\nu_{\exp} = \bar{n}_{\exp} \cong \frac{1}{4}\frac{\Omega_0^2\tau^2}{1+(\omega_0-\omega_c)^2\tau^2}. \qquad (9)$$

Similarly for the double exponential pulse one may get

$$\nu_{2\exp} = \bar{n}_{2\exp} \cong \frac{1}{4}\frac{\Omega_0^2\tau^2}{[1+(\omega_0-\omega_c)^2\tau^2]^2}. \qquad (10)$$

Formulae (9), (10) are made with account of the condition $\omega_c\tau \gg 1$.

From (1), (9) it follows that the dependence of the QO excitation probability on the pulse duration in case of an exponential pulse may either be monotonic increasing, or have one maximum. The position of this maximum for the function $W_{0\to n}(\tau)$ is given with the formula

$$\tau_{\max}^{(\exp)} = \frac{1}{\sqrt{\Omega_0^2/4n-(\omega_c-\omega_0)^2}}. \qquad (11)$$

Since the radical expression in the right part of this equation must be positive, it follows that the maximum occurs, if the following inequation is met

$$\Omega_0 > 2\sqrt{n}|\omega_0-\omega_c|. \qquad (12)$$

Otherwise, the function $W_{0\to n}(\tau)$ is monotonic increasing.

Inequation (12) may be named the condition for the mode of intense disturbance of the quantum oscillator with the exponential pulse. If the opposite inequation is met, the excitation is weak. Note that in resonance $\omega_c = \omega_0$ any value of the Rabi frequency is compliant with the intense excitation.

In case of a double exponential pulse in the mode of weak disturbance, which is given by the inequation

$$\Omega_0 < 4\sqrt{n}|\omega_0-\omega_c|, \qquad (13)$$

there is one maximum in $\tau$-dependence of the excitation probability. The position of this maximum is defined with the formula

$$\tau_{\max}^{(2\exp)} = \frac{1}{|\omega_c-\omega_0|}. \qquad (14)$$

It is evident that this maximum disappears at the resonant carrier frequency $\omega_c = \omega_0$.

The simple analysis shows that in the mode of intense disturbance, $\Omega_0 > 4\sqrt{n}|\omega_0-\omega_c|$, maximum (14) changes into minimum, and two new maxima appear, to which the following pulse durations correspond:

$$\tau_{\max,1,2}^{(2\exp)} = \frac{\Omega_0 \pm \sqrt{\Omega_0^2-16n|\omega_c-\omega_0|^2}}{4\sqrt{n}|\omega_c-\omega_0|^2}. \qquad (15)$$

You can see that in the resonance the high value of the pulse duration in the maximum changes into infinity $\tau_{\max,2}^{(2\exp)}(\omega_c=\omega_0)\to\infty$, and the lower one is equal to $\tau_{\max,1}^{2\exp}(\omega_c=\omega_0)=2/\Omega_0$.

Let us consider the spectrum of QO excitation with the exponential and double exponential pulses.

In case of the exponential pulse, using the above formulae, you can obtain the following expression for the spectral maxima:

$$\omega_{\max}^{(\exp)} = \omega_0 \pm \sqrt{\Omega_0^2/4n-\tau^{-2}}. \qquad (16)$$

These frequencies are the solutions to equation $d\bar{n}/d\omega_c = 0$, which defines the spectral extrema of probability. From (16) it follows that these maxima are implemented only in the mode of intense disturbance, which is described by inequation

$$\Omega_0 > 2\sqrt{n}/\tau. \qquad (17)$$

Otherwise, in case of weak disturbance, there is one spectral maximum in the own frequency of oscillator $\omega_c = \omega_0$.

Similarly, you can get an expression for the spectral maxima when the QO is excited from the main state by the double exponential pulse:

$$\omega_{\max}^{(2\exp)} = \omega_0 \pm \frac{1}{\tau}\sqrt{\frac{\Omega_0\tau}{2\sqrt{n}}-1}. \qquad (18)$$

Hence it appears that the condition for the mode of intense disturbance when excited with a double exponential pulse is also given by inequation (17).

## Model accounting of quantum oscillator damping

The main assumption of our model consists in the following. In expressions (1), (4) we will be using the excitation energy of the classic oscillator with nonzero damping

$$\Delta\varepsilon_{\text{clas}}(\gamma=0) \to \Delta\varepsilon_{\text{clas}}(\gamma\neq 0). \qquad (19)$$

With account of damping for the excitation energy of the associated classic oscillator we have

$$\Delta\varepsilon_{\text{clas}} = \frac{q^2E_0^2}{2m}\int_0^\infty d\omega|\tilde{E}(\omega)|^2\frac{4\Omega^2\gamma/\pi}{(\omega^2-\omega_0^2)^2+4\omega^2\gamma^2}. \qquad (20)$$

Then in case of the exponential pulse (7) for the average number of the excited quanta (assume that $\omega_c\tau \gg 1$) we obtain

$$\bar{n}_{\exp}(\gamma) = \frac{1}{4}\Omega_0^2\tau\frac{\gamma+1/\tau}{(\omega_c-\omega_0)^2+(\gamma+1/\tau)^2}. \qquad (21)$$

From here you can find the position of the spectral maxima when excited by the exponential pulse of QO with damping in transition $0\to n$:

$$\left|\Delta_{\max}^{(\exp)}(\gamma)\right| = \sqrt{(1+\gamma\tau)\left(\frac{\Omega_0^2}{4n}-\frac{1+\gamma\tau}{\tau^2}\right)},$$

$$\Delta = \omega_c - \omega_0. \tag{22}$$

From this formula it follows that the Rabi saturation frequency with account of damping is equal to

$$\Omega_0^{(\mathrm{sat})}(\gamma) = \frac{2\sqrt{n}}{\tau}\sqrt{1+\gamma\tau}. \tag{23}$$

Thus, the side spectral maxima arise, if the following inequation is met

$$\Omega_0 > \Omega_0^{(\mathrm{sat})}(\gamma). \tag{24}$$

Otherwise, there is one maximum at the own frequency of QO.

To conclude, note that within the theory of disturbances ($\nu \ll 1$) the probability of QO excitation in transition $0 \to 1$ matches the probability of excitation of the double-level system with the oscillator force equal to one, and is given with expression (2). Therefore, there is a relation seen between the description of the pulse excitation of the substancer within two fundamental quantum models.

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### Conflict of interest

The authors declare that they have no conflict of interest.

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