Optics and Spectroscopy, 2024, Vol. 132, No. 12

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Local angular spectrum of the disturbance of a monochromatic wave field

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Received September 12, 2024
Revised November 26, 2024
Accepted December 03, 2024

A theoretical study of the influence of the spatial position of an observation point on the angular spectrum of the disturbance of a scalar monochromatic light wave field which source has a finite linear aperture is carried out. The concept of a local angular spectrum of a wave field disturbance is introduced. It is shown that, in contrast to the angular spectrum classically defined in the infinite plane of the field cross-section, and which for this reason can be called the total angular spectrum of the wave field, the local angular spectrum of the wave field disturbance is different for different observation points and is determined by both the size and shape of the linear aperture of the source and the coordinates of the position of the observation point. The expressions for determining the local angular spectrum, the law of change of this spectrum during the propagation of an optical wave field in free space, the connection of the local angular spectrum of a separate disturbance with the total angular spectrum of the wave field are obtained.

Keywords: angular spectrum, Fourier transform, spatial frequencies, scalar monochromatic wave field, extended light source.

DOI: 10.61011/EOS.2024.12.60450.7050-24

Introduction

One of the methods to study the properties of scalar monochromatic wave fields in the optics, first of all the features of their propagation in homogeneous and heterogeneous media, is the method of wave field presentation in the form of integral decomposition by angular spectrum of plane waves — spectrum of spatial harmonics [1–6]. The concept of the angular spectrum of the wave field in the optics is fundamental in definition of its spatial correlation properties, both transverse [1,3–5] and longitudinal relative to the field propagation direction [4,5,7–9]. The study of the properties of statistically heterogeneous spatially limited fields is of special interest, since the real sources of light that we come across in the practice and that have the finite linear dimensions of the aperture create exactly such fields. In particular, the transformation of spatially frequency and correlation properties of such fields is interesting when they pass through the optical systems or interact with the objects of complex stochastic structure, and when images of such objects are generated [1,3,5], for example, in microscopy [10–17], digital methods of image restoration, such as digital optical holography [18–20] and ptychography [21], at the numerical modeling of electromagnetic fields propagation [22–31], in the methods of phase recovery from the spatial distribution of wave field intensity [32,33], int he study of light scattering by micro- and nanoparticles [34–36], in design of meta-optic elements [37].

There are two approaches to definition of the angular spectrum of the scalar wave field. In the first approach, being in fact a convenient mathematical representation, the angular spectrum of monochromatic wave field is defined via the Fourier transform of the spatial distribution of the complex amplitude of this field disturbances in a certain plane of observation, perpendicular to the axial direction of the field propagation — optical axis [1–6]. The complex amplitude angular spectrum of the monochromatic wave field determined in this manner is the density of amplitude distribution for the angular components — plane waves, into which the wave field is decomposed, by transverse spatial frequencies, which actually depend only on the angles of propagation of these angular components.

Further we will call this approach in the definition of the angular spectrum the classic one, and the angular spectrum determined by the above method, — the full angular spectrum of the wave field, since it is defined via integration along all local disturbances of the wave field in the infinite observation plane. The classic approach makes it possible to describe the propagation of the monochromatic wave field in the free space, and also to define the patterns of such field diffraction in various fine amplitude or phase random and determined screens.

A version of the classic definition of the monochromatic wave field angular spectrum is its definition through the inverse Fourier transform of the transverse spatial correlation function of the wave field [4,5]. In this case the angular spectrum is the angular spectral power (intensity) density, which is the middle square in the assembly of realizations for the amplitude angular spectrum module.

Therefore, in any case in the classic representation the angular spectrum of the wave field in virtue of its definition via integration in the infinite plane of this field cross section is its certain integral characteristic. For this reason and in virtue of the laws of conservation the module and width of the full angular spectrum do not change when the wave field propagates in free space from one plane of observation to another [1–6].

The second approach in the definition of the angular spectrum which is intuitive and proceeds from the natural optic experiment, assumes that the angular spectrum of the wave field is the angular distribution of the complex amplitude (or intensity) of angular spatial harmonics (plane waves), constituting this wave field in a certain point of observation [7–9]. This approach may also be called radiometric [3], since it may be reduced to consideration of the energy propagation along the light beams, emitted from quasi-zero-dimensional areas in the plane of the field source and crossing in a certain field of observation.

This approach — the summation of the contributions to the disturbance of the wave field in a certain point of observation from the individual quasi-zero-dimensional areas in the plane of the source, has historically been the basis for solving the objectives of diffraction and methods for determination and measurement of coherence in the wave fields [1,3], which is clearly seen in transition from Cartesian coordinates in the source plane to the related angles of beam distribution (in transition to the angular distribution of radiation from the source to the observation point) [38]. In this approach the width of the angular spectrum of the wave field in the point of observation will mostly be determined by the combination of angles, at which the light beams arrive from the source. This combination of angles forms a solid angle, under which the linear aperture of the source is seen from the observation point, and determines the width of distribution by angles of wave field intensity in the observation point. Since the value of the solid angle depends on the position of its top, which is located in the point of observation, the width of the angular spectrum of wave field disturbance in this point in the considered approach will depend on its position in the space. The angular spectrum of wave field disturbance determined in the considered manner will be called a local angular spectrum of such disturbance.

The objective of this paper consists in combination of two approaches in determination of the angular spectrum of the scalar monochromatic wave field within a single mathematical description based on Fourier transforms, establishment of the connection between the local and full angular spectra and the law of change of the local angular spectrum during propagation of the scalar monochromatic wave field along the main direction of its propagation in the free homogeneous and isotropic space.

1. Classic representations on angular spectrum of monochromatic wave field

Spatial distribution of the complex amplitude $U(x, y, z)$ of monochromatic wave field disturbances in a certain plane (XY), perpendicular to the main direction of the

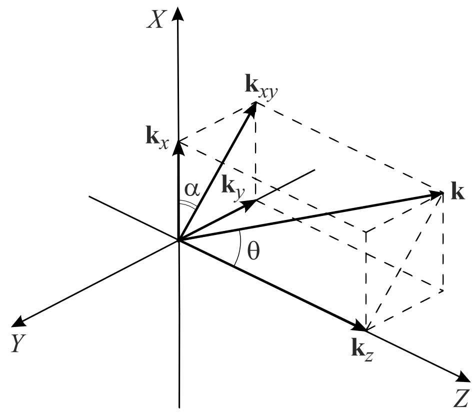

Figure 1. Wave vector **k** of plane wave incident at angle θ to axis Z, and its components in 3D space.

Z field propagation (crossing this axis in a certain point with coordinate $z = \text{const}$), may be presented in the form of a superposition of plane waves with various spatial frequencies k_x, k_y and density of complex amplitude distribution $W(k_x, k_y, z)$ of these waves in the considered plane with the following expression [2,6]:

$$U(x, y, z) = \iint_{-\infty}^{+\infty} W(k_x, k_y, z) \exp\{i(k_x x + k_y y)\} dk_x dk_y, \tag{1}$$

where $k = 2\pi/\lambda$ — wave number corresponding to wavelength λ of monochromatic wave field; k_x, k_y and k_z — spatial frequencies — projections of wave vector **k** on axis X, Y and Z accordingly, which may be expressed as follows:

$$k_x = k \cos\alpha \sin\theta,$$

$$k_y = k \sin\alpha \sin\theta,$$

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} = k \cos\theta, \tag{2}$$

where α and θ — angles that specify the direction of the wave vector **k** in space (fig. 1): θ — angle between axis Z and wave vector **k** (zenith angle), α — angle between axis X and projection $\mathbf{k}_{xy}$ of vector **k** on plane (XY) (azimuthal angle).

Hereinafter we will limit ourselves in the description of the complex amplitude of monochromatic wave field disturbances by the consideration of only homogeneous, non-decaying, plane waves [3,4] and their impact at generation of wave disturbances of the field in the area of the space that is of interest for us.

In Fourier optics [2,6] the density of complex amplitude distribution $W(k_x, k_y, z)$ is called the angular spectrum of wave field (1) in a certain plane (XY), since in the case of the monochromatic field (at $k = \text{const}$) this value is actually only the function of the propagation angles θ and α of the plane waves.

A law [2,6] is known on the change of the angular spectrum of the monochromatic wave field in the process of its propagation in the free homogeneous and isotropic space from plane (ΞH), crossing the axis Z in a certain reference point $z = 0$, to the observation plane (XY) $(z = \text{const})$:

$$W(k_x, k_y, z) = W_0(k_x, k_y) \exp\left\{ i\sqrt{k^2 - k_x^2 - k_y^2}z \right\}$$
$$= W_0(k_x, k_y) \exp\{ik_z z\}, \quad (3)$$

where $W_0(k_x, k_y) = W(k_x, k_y, z = 0)$ — angular spectrum of the wave field in plane (ΞH). Expression (1) in this case will look as follows

$$U(x, y, z) = \iint\limits_{-\infty}^{+\infty} W(k_x, k_y) \exp\{ik_z z\}$$
$$\times \exp\{i(k_x x + k_y y)\} dk_x dk_y. \quad (4)$$

Angular spectrum $W_0(k_x, k_y)$ of wave field in plane (ΞH) is related by inverse Fourier transform with spatial distribution $U_0(\xi, \eta)$ of complex amplitude of monochromatic wave field disturbances in this plane [2,6]:

$$W_0(k_x, k_y) = \iint\limits_{-\infty}^{+\infty} U_0(\xi, \eta) \exp\{-i(k_x \xi + k_y \eta)\} d\xi d\eta. \quad (5)$$

Plane (ΞH), corresponding to $z = 0$, shall be deemed to be the plane of the wave field disturbances source for all other planes with $z > 0$. To determine the angular spectrum $W_0(k_x, k_y)$ of the wave field in the plane $z = 0$, spatial distributions of amplitude $A_0(\xi, \eta)$ and phase $\varphi(\xi, \eta)$ of wave disturbances of this field in the considered plane are deemed to be known (including the statistics of these values determining the spatial correlation properties of the wave field in the source plane [4,5]):

$$U_0(\xi, \eta) = A_0(\xi, \eta) \exp\{i\varphi(\xi, \eta)\}.$$

Integration in (5) is carried out in all points of the plane (ΞH), however, in practice the sources of the wave fields (both primary and secondary) have limited transverse spatial dimensions. For this reason the complex amplitude $U_0(\xi, \eta)$ in the source plane (ΞH) may be deemed the finite function, i.e. different from zero only in a certain limited area Σ of this plane:

$$\begin{cases} U_0(\xi, \eta) \neq 0 \text{ at } (\xi, \eta) \in \Sigma, \\ U_0(\xi, \eta) = 0 \text{ at } (\xi, \eta) \notin \Sigma. \end{cases} \quad (6)$$

Considering (6) the expression (5) may be rewritten as follows:

$$W_{0\Sigma}(k_x, k_y) = \iint\limits_{\Sigma} U_0(\xi, \eta) \exp\{-i(k_x \xi + k_y \eta)\} d\xi d\eta \quad (7)$$

or as

$$W_{0\Sigma}(k_x, k_y) = \iint\limits_{-\infty}^{+\infty} U_0(\xi, \eta) t_\Sigma(\xi, \eta) \exp\{-i(k_x \xi + k_y \eta)\} d\xi d\eta, \quad (8)$$

where $t_\Sigma(\xi, \eta)$ — amplitude aperture function:

$$t_\Sigma(\xi, \eta) = \begin{cases} 1 \text{ at } (\xi, \eta) \in \Sigma, \\ 0 \text{ at } (\xi, \eta) \notin \Sigma. \end{cases} \quad (9)$$

Expression (8), according to the inverse convolution theorem may be written in the form of angular spectrum convolution $W_0(k_x, k_y)$ of wave field $U_0(\xi, \eta)$ and angular spectrum $T_\Sigma(k_x, k_y)$ of linear aperture of the field source [2,6]:

$$W_{0\Sigma}(k_x, k_y) = W_0(k_x, k_y) \otimes T_\Sigma(k_x, k_y), \quad (10)$$

where

$$T_\Sigma(k_x, k_y) = \iint\limits_{-\infty}^{+\infty} t_\Sigma(\xi, \eta) \exp\{-i(k_x \xi + k_y \eta)\} d\xi d\eta.$$

Expression (10) shows [2,6] increase in the width of the wave field angular spectrum in the source plane (and directly in all other planes $z > 0$ of free space) with the reduction of transverse dimensions of the source and is a mathematical description of the wave field diffraction phenomenn at the hole in the nontransparent screen.

Therefore, the complete angular spectrum of monochromatic wave field disturbances in the source plane $(z = 0)$ is determined by both spatial distributions of amplitude $A_0(\xi, \eta)$ and phase $\varphi(\xi, \eta)$ of wave disturbances and statistics of these values in the considered plane, as well as the dimensions of area Σ — transverse dimensions (linear aperture) of the source. The complete angular spectrum of the wave field in the source plane is a complex value in the general case:

$$W_{0\Sigma}(k_x, k_y) = |W_{0\Sigma}(k_x, k_y)| \exp\{i\Phi_{0\Sigma}(k_x, k_y)\},$$

where $\Phi_{0\Sigma}(k_x, k_y) = \arg(W_{0\Sigma}(k_x, k_y))$ — initial phase of angular component of the field with spatial frequencies k_x, k_y. This initial phase is formed by all points (ξ, η) of the source aperture Σ. Besides, depending on the characteristics of the source (distributions $A_0(\xi, \eta)$ and $\varphi(\xi, \eta)$, dimensions and shape of the aperture Σ) such distribution of initial phases may be formed for $\Phi_{0\Sigma}(k_x, k_y)$ angular components of monochromatic wave field, that these angular components will come to a certain point of observation (x, y, z) in the phase, which will focus the field in the area around with point with maximum of disturbances amplitude in it. Therefore, representation of the wave field in the form of a plane wave spectrum does not prevent the cases of such field focusing.

Expression (3) relates to each other the angular spectra of the wave field in the source plane $(z = 0)$ and random observation plane $z > 0$ and shows that in process of wave field distribution in the free space the complete angular spectrum of the field will not change in its cross sections, and only phase incursions change between the plane waves — angular components of the considered wave field [2,6].

However, the photodectors that exist in the nature or in the engineering have the finite physical dimensions of the reception aperture and for this reason may only cover the limit area of the plane [22–31], where the angular spectrum of the wave field is determined. Within the limit this limited area may be pulled together in a dot. It is evident that in this situation the angular spectrum of wave field disturbances, „perceived" with a small-size receiver, may differ from the complete angular spectrum of the wave field. The cause for such difference as shown below are the finite dimensions of the real wave field sources.

2. Angular spectrum of disturance of monochromatic wave field in a certain observation point

At the finite linear aperture Σ of the monochromatic wave field source the propagation of angular components of this field in space $z > 0$ has, according to (10), the diffraction nature: each of the angular components acquires its own propagation angles α and θ, amplitude $|W_{0\Sigma}(k_x, k_y))|$ and initial phase $\Phi_{0\Sigma}(k_x, k_y) = \arg(W_{0\Sigma}(k_x, k_y))$ with account of the impact of this aperture dimensions and shape. Besides, the angular components of the field will have the wave fronts limited in transverse dimensions by projections of the linear aperture Σ of the source on the plane, perpendicular to the direction of propagation of each angular component. Therefore, the amplitude of the wave field angular components is zero outside these projections. Each of such angular components of the wave field will have the finite area of crossing Σ' with the selected observation plane, perpendicular to the main direction of this field propagation.

When the wave field spreads in the free space, its diffraction broadening occurs — angular components of the field spreading at different angles θ to axis Z, are spatially separated (diverge) the further they are from the source plane. This results in the fact that the areas of crossing Σ' of aperture-limited angular components of the wave field with the observation plane will be imposed upon each other differently depending on the selection of this plane. As a result — not all angular components of the considered wave field may arrive to a certain dedicated limited area in the observation plane (limit case of such area — dot). This effect is especially noticeable for a spatilly periodical wave field with the discrete angular spectrum.

The example of such field may be the propagation of the monochromatic wave field disturbances in the free space, its source being the sinusoid amplitude diffraction lattice, illuminated with a plane wave of monochromatic color and covered with a screen that has a hole to determine size D of this secondary source — linear aperture Σ of diffraction lattice (fig. 2).

For clarity we will consider the unidimensional transverse distribution of complex wave field disturbances amplitude

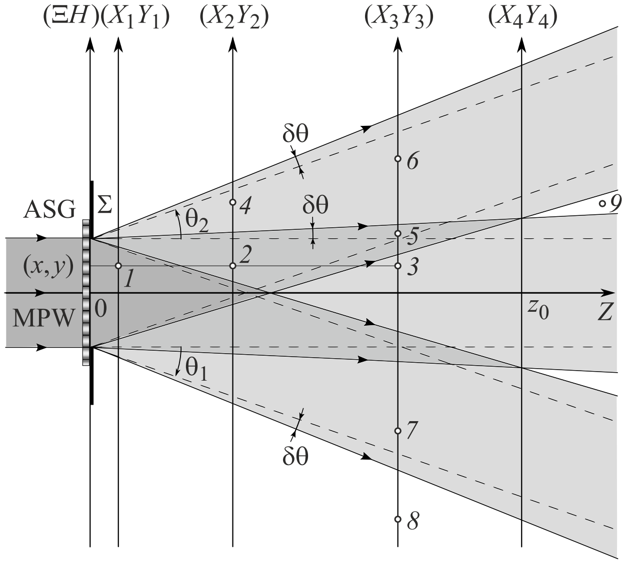

Figure 2. Angular components of the wave field of sinusoid amplitude diffraction lattice ASG with the finite linear aperture Σ when illuminated by monochromatic plane wave (MPW). For details see the text.

that in the plane of the source $(z = 0)$ is [6]

$$U_0(\xi) = 1 + q\cos\left(\frac{2\pi}{d}\xi\right), \quad (11)$$

where d — spatial period of the diffraction lattice, $q < 1$ — depth of amplitude modulation of the field with the lattice. Field amplitude $A_0(\xi)$ (11) varies in accordance with the cosinusoid law, and phase $\varphi(\xi, \eta) = \text{const} = 0$. Without account of the finite dimensions of the lattice linear aperture the field (11) may be presented as the sum of complex amplitudes of three plane waves:

$$U_0(\xi) = U_{00}(\xi) + U_{01}(\xi) + U_{02}(\xi)$$
$$= 1 + \frac{q}{2}\exp\left\{-i\frac{2\pi}{d}\xi\right\} + \frac{q}{2}\exp\left\{+i\frac{2\pi}{d}\xi\right\}. \quad (12)$$

For plane wave $U_{00}(\xi)$ of single amplitude the angle of propagation θ_0 relative to axis Z is equal to zero, and for plane waves $U_{01}(\xi)$ and $U_{02}(\xi)$ with amplitudes $q/2$ the angles of propagation are equal to accordingly $\theta_1 = -\theta$ and $\theta_2 = +\theta$, where

$$\theta = \arcsin\{\lambda/d\}. \quad (13)$$

Taking into account of the dimensions D of linear aperture of the lattice, the full angular spectrum of the wave field of the considered source may be defined using (7) as

$$W_{0\Sigma}(k_x) = D\frac{\sin\left(k_x\frac{D}{2}\right)}{k_x\frac{D}{2}} + D\frac{q}{2}\frac{\sin\left(\left(k_x + \frac{2\pi}{d}\right)\frac{D}{2}\right)}{\left(k_x + \frac{2\pi}{d}\right)\frac{D}{2}}$$
$$+ D\frac{q}{2}\frac{\sin\left(\left(k_x - \frac{2\pi}{d}\right)\frac{D}{2}\right)}{\left(k_x - \frac{2\pi}{d}\right)\frac{D}{2}}. \quad (14)$$

According to (14), sinusoidal amplitude diffraction grating with linear aperture of finite dimensions D, illuminated with a plane wave of monochromatic light creates a wave field in the form of three beams (fig. 2), propagating at angles θ_0, θ_1 and θ_2 to axis Z and having angular divergence, determined by angle $2\delta\theta$, where

$$\delta\theta = \arcsin\{\lambda/D\} \approx \lambda/D. \quad (15)$$

In fig. 2 the areas of space occupied by each of these light beams separately are shown with light grey color; areas where two beams cross — with a darker (medium) grey tone, and the area where all three beams cross is shown with a dark grey tone. Accordingly, the areas where the wave field is absent, are shown with white background.

The simple geometric assumptions may be used to find a coordinate z_0 of plane $((X_4Y_4)$ in fig. 2), where one can deem that these three beams stop crossing each other,

$$z_0 \approx D\frac{\cos(\theta - \delta\theta)\cos(\delta\theta)}{\sin\theta}. \quad (16)$$

Dimensions D of the linear aperture of the grating determine the divergence of beams, making the wave field in question: the larger D, the smaller the divergence angle $2\delta\theta$ (according to (15)), and the stronger the wave fronts of these beams approach the plane ones.

Therefore, at $D \gg \lambda$ one may approximately assume that the wave field of the amplitude diffraction grating is a sum of three quasiplane aperture-limited waves, which at $z > z_0$ do not cross each other. Besides, the full angular spectrum of such wave field disturbances in each of the transverse planes $((X_1Y_1)$, (X_2Y_2), (X_3Y_3), fig. 2) remains same and consists of three angular components (see expression (12)) — quasiplane waves propagating at angles θ_0, θ_1 and θ_2 to axis Z. However, for the observers located in the points with the identical transverse coordinates (x, y) in these planes (points *1*, *2* and *3* in fig. 2) or different points of one plane (points *2*, *4* or points *3, 5, 6, 7* and *8* in fig. 2), the angular spectrum of wave field disturbances in the selected point of observation (local angular spectrum) will be limited only by those angular components of the field that pass through this point. Thus, the point *1* (dark-grey area) will be passed by all three angular components of the wave field with propagation angles θ_0, θ_1 and θ_2. The points *2* and *5* (medium-grey areas) will be passed by two components with propagation angles θ_0 and θ_2. The points *4* and *6*, point *3* and point *7* (in the light-grey areas) will be passed only by one angular component with propagation angles θ_2, θ_0, and θ_1 accordingly. And the points *8* and *9* (white background) will not be passed by any angular component of the wave field in question.

Modeling using formula (16) shows that at $D > 5\lambda$ the dependence of value z_0 on D becomes linear, i.e. at $D \to \infty$ value z_0 also goes to infinity. Besides, linear apertures of wave fronts in angular components of the total wave field also tend towards infinite dimensions, which in the end results in the fact that any observation point will be passed by all angular components of the wave field, and local angular spectra for all the observation points will be same and equal to the full angular spectrum of the wave field.

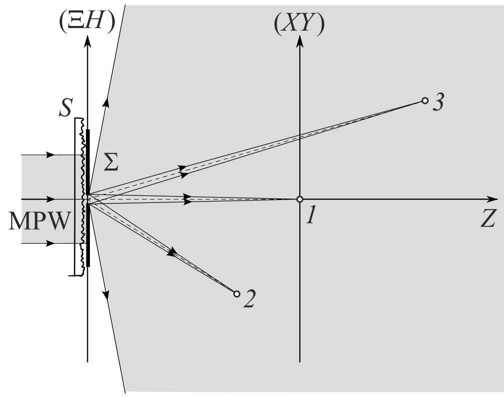

Figure 3. Field of quasi-point source of light in the form of quasi-Lambertian diffuser S with small aperture Σ, illuminated with a plane monochromatic wave MPW, and body angles determining local angular spectra in three different points of space *1, 2* and *3*. For details see the text.

Therefore, the cause for the difference in the local angular spectra of wave field disturbances in various spatial points of observation from each other and from the full angular spectrum of the wave field is the finite linear aperture of the source of this wave field.

Another indicative example of the difference in the local angular spectrum of wave field disturbance in a certain point of observation from the full angular spectrum is the field of quasi-point $(D \to \lambda)$ monochromatic source of light emitting practically evenly into the entire half-space $z > 0$ $(2\delta\theta \to \pi)$ (fig. 3, for simplicity a 2D case is considered; the area of the space occupied by the wave field is shown with grey color). From fig. 3 you can see that at extremely wide full angular spectrum of the wave field of such source the local angular spectrum of this field disturbances in any point of observation is very narrow, actually consisting of only one angular component with spatial frequency corresponding to the direction from this source to observation point.

3. Expression for local angular spectrum

To determine the local angular spectrum of the wave field in a certain point of observation $(x, y, z > 0)$, an expression is necessary, which binds the complex amplitude $U(k, x, y, z)$ of the wave field in the considered point with the distribution of the complex amplitude $U_0(\xi, \eta)$ of this wave field in the plane of source $(z = 0)$. Such binding is provided by Green's formula [1,3,6], which with account of the finiteness of the linear aperture dimensions Σ of the field source may be written as in (7) and (8) in the following form

$$U(k,x,y,z) = -\frac{1}{2\pi}\iint_{\Sigma} U_0(\xi,\eta)\frac{d}{dz}\left(\frac{\exp\{ikR\}}{R}\right)d\xi d\eta$$

$$= -\frac{1}{2\pi}\iint_{-\infty}^{+\infty} U_0(\xi,\eta)t_\Sigma(\xi,\eta)\frac{d}{dz}\left(\frac{\exp\{ikR\}}{R}\right)d\xi d\eta, \quad (17)$$

where

$$R = \sqrt{(x-\xi)^2+(y-\eta)^2+z^2},$$

and $t_\Sigma(\xi,\eta)$ — amplitude aperture function determined by (9). Provided that $R \gg \lambda$ (asymptotic approximation) [1,3,6]

$$\frac{d}{dz}\left(\frac{\exp\{ikR\}}{R}\right) = \left(ik - \frac{1}{R}\right)\frac{\exp\{ikR\}}{R}\cos\theta_L$$

$$\approx ik\frac{\exp\{ikR\}}{R}\cos\theta_L, \quad (18)$$

where $\theta_L = \theta_L(x,y,z;\xi,\eta)$ — angle between the normal line to infinitely small emitting site $d\xi d\eta$ (with center in point $S(\xi,\eta)$) of the source plane and the direction towards the observation point $P(x,y,z)$, i.e. angle between axis Z and radius-vector $\mathbf{R}$, connecting the points $S(\xi,\eta)$ and $P(x,y,z)$ (or between axis Z and wave vector $\mathbf{k}$, since vectors $\mathbf{R}$ and $\mathbf{k}$ — are collinear (fig. 4)),

$$\cos\theta_L = \frac{z}{R} = \frac{z}{\sqrt{(x-\xi)^2+(y-\eta)^2+z^2}}. \quad (19)$$

When substituting (18) in (17) the Green's formula starts looking like Fresnel–Kirchhoff expression serving as the mathematical representation of the Huygens–Fresnel principle [6],

$$U(k,x,y,z) = \frac{1}{i\lambda}\iint_{\Sigma} U_0(\xi,\eta)\frac{\exp\{ikR\}}{R}\cos\theta_L d\xi d\eta$$

$$= \frac{1}{i\lambda}\iint_{-\infty}^{+\infty} U_0(\xi,\eta)t_\Sigma(\xi,\eta)\frac{\exp\{ikR\}}{R}\cos\theta_L d\xi d\eta, \quad (20)$$

where the complex amplitude of the spherical wave propagating from the point source in point $S(\xi,\eta)$ of the source plane (infinitely small emitting site $d\xi d\eta$ with the center in this point) and reaching the point of observation $P(x,y,z)$, depends not only on the complex amplitude $U_0(\xi,\eta)$ of wave field disturbances in point $S(\xi,\eta)$ and distance R between points $S(\xi,\eta)$ and $P(x,y,z)$, but on the area of elemental emitter $d\xi d\eta\cos\theta_L$ seen from the point of observation $P(x,y,z)$. The last circumstance means that the points of observation are exposed only to some energy of the considered spherical wave (some energy emitted by the elemental site $d\xi d\eta$). Besides, for the fixed point of observation $P(x,y,z)$ each point $S(\xi,\eta)$ of the source has the only angle θ_L assigned. Therefore, the combination of angles θ_L from all points $S(\xi,\eta)$ of aperture Σ in the source forms the body angle, at which this aperture is seen from the point of observation $P(x,y,z)$.

Expression (19) may be transformed to

$$\cos\theta_L = \frac{z}{\sqrt{x^2+y^2+z^2+\xi^2+\eta^2-2\sqrt{x^2+y^2}\sqrt{\xi^2+\eta^2}\cos\varphi}}, \quad (21)$$

where φ — angle between radii-vectors $\mathbf{r}(x,y)$ and $\boldsymbol{\rho}(\xi,\eta)$ (fig. 4),

$$\varphi = \arccos\left(\frac{x}{\sqrt{x^2+y^2}}\right)+\gamma = \arcsin\left(\frac{y}{\sqrt{x^2+y^2}}\right)+\gamma, \quad (22)$$

γ — angle between axis Ξ and radius-vector $\boldsymbol{\rho}(\xi,\eta)$ (fig. 4).

With account of (22) the expression (21) is transformed to

$$\cos\theta_L = \frac{z}{\sqrt{x^2+y^2+z^2+\rho^2-2\rho[x\cos\gamma - y\sin\gamma]}}, \quad (23)$$

where $\rho = (\xi^2+\eta^2)^{1/2}$.

From the formal point of view one can assume that for any geometric form of the linear aperture Σ of the wave field source the angle γ changes in the range from 0 to 2π, and value ρ for each angle γ varies within $\rho_{\min}(\gamma)$ - $\rho_{\max}(\gamma)$. For example, for the aperture in the form of a circle with diameter D $\rho_{\min}(\gamma) = 0$ and $\rho_{\max}(\gamma) = D/2$ for all γ (hereinafter axis Z is assumed to be passing through the geometric center of the aperture). For the rectangular aperture with sides a and b (along axes Ξ and H, fig. 5)

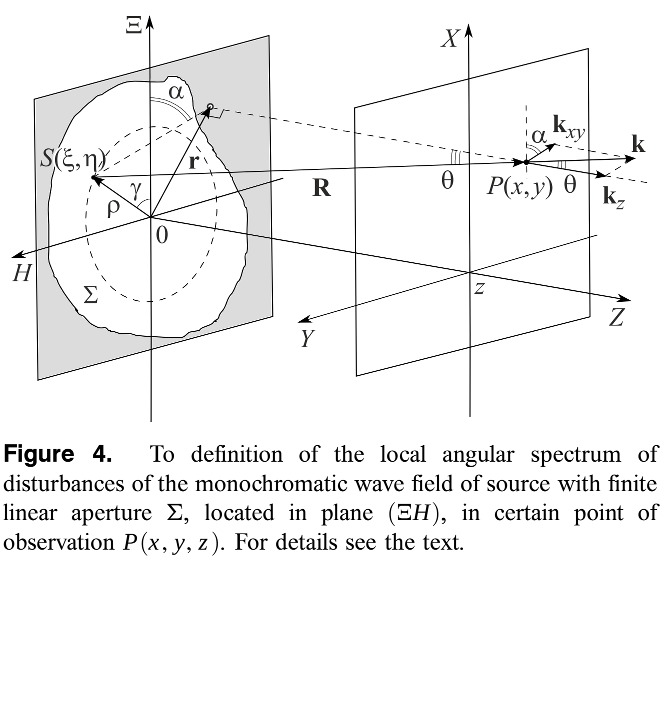

Figure 4. To definition of the local angular spectrum of disturbances of the monochromatic wave field of source with finite linear aperture Σ, located in plane (ΞH), in certain point of observation $P(x,y,z)$. For details see the text.

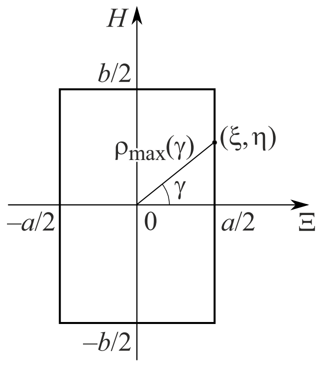

Figure 5. To definition of value $\rho_{\max}(\gamma)$ for rectangular linear aperture of the source. For details see the text.

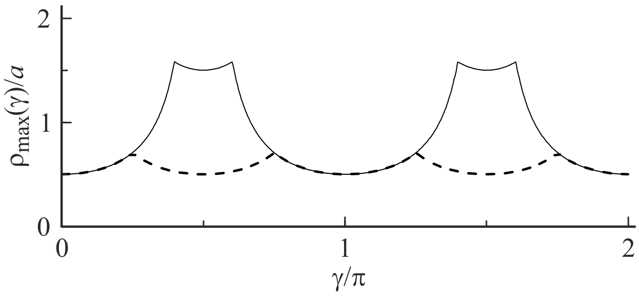

Figure 6. Values $\rho_{\max}(\gamma)$ normalized to length of side a for sources with linear aperture of rectangular ($b = 3a$, solid line) and square ($b = a$, dashed line) form.

$\rho_{\min}(\gamma) = 0$ for all γ, and $\rho_{\max}(\gamma)$ is defined as follows:

$$\rho_{\max}(\gamma) = \begin{cases} \frac{a}{2\cos\gamma} \text{ at } 0 \le \gamma \le \gamma_1 \text{ at } (2\pi - \gamma_1) < \gamma \le 2\pi, \\ \frac{b}{2\sin\gamma} \text{ at } \gamma_1 < \gamma \le (\pi - \gamma_1), \\ \frac{-a}{2\cos\gamma} \text{ at } (\pi - \gamma_1) < \gamma \le (\pi + \gamma_1), \\ \frac{-b}{2\sin\gamma} \text{ at } (\pi + \gamma_1) < \gamma \le (2\pi - \gamma_1), \end{cases}$$

where

$$\gamma_1 = \arcsin\left(\frac{b}{\sqrt{a^2 + b^2}}\right).$$

Fig. 6 shows the curves of dependence for the value $\rho_{\max}(\gamma)$ normalized by value a for the rectangular ($b = 3a$, solid line) and square ($b = a$, dashed line) apertures of the wave field source.

It is evident that in virtue of the finiteness of the dimensions of linear aperture Σ for the source of the wave field the range of variation of angles θ_L for each angle γ will be finite. This range, based on expression (23), is determined as follows:

$$\begin{cases} \min\{\theta_1(\gamma), \theta_2(\gamma)\} \le \theta_L(\gamma) \le \max\{\theta_1(\gamma), \theta_2(\gamma)\}, \text{ at} \\ \theta_1(\gamma) = \arctan\left(\frac{z}{\sqrt{x^2+y^2+z^2+\rho^2_{\min}(\gamma)-2\rho_{\min}(\gamma)[x\cos\gamma - y\sin\gamma]}}\right) \\ = \arctan\left(\frac{\sqrt{x^2+y^2+\rho^2_{\min}(\gamma)-2\rho_{\min}(\gamma)[x\cos\gamma - y\sin\gamma]}}{z}\right), \\ \theta_2(\gamma) = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2+\rho^2_{\max}(\gamma)-2\rho_{\max}(\gamma)[x\cos\gamma - y\sin\gamma]}}\right) \\ = \arctan\left(\frac{\sqrt{x^2+y^2+\rho^2_{\max}(\gamma)-2\rho_{\max}(\gamma)[x\cos\gamma - y\sin\gamma]}}{z}\right), \\ 0 \le \gamma \le 2\pi, \end{cases} \tag{24}$$

where $\rho_{\min}(\gamma)$ and $\rho_{\max}(\gamma)$ are determined for each angle γ by geometric shape of linear aperture Σ of the source of wave field, and functions min and max determine the minimum and maximum values of two values $\theta_1(\gamma)$ and $\theta_2(\gamma)$.

For instance, for the previously considered unidimensional ($y = 0$) case of transverse distribution of the complex amplitude of wave field disturbances developed by the amplitude diffraction grating with the finite linear aperture, angle γ actually takes only two values: $\gamma = 0$ and $\gamma = \pi$ and $\rho_{\min}(\gamma) = 0$ and $\rho_{\max}(\gamma) = D/2$ for these two values of angle γ. Ranges of angles variation $\theta_L(\gamma)$ for a certain point of observation (x, z), according to expression (24), will be determined by the following inequalities:

$$\begin{cases} \arctan\left(\frac{x - D/2}{z}\right) \le \theta_L(x, z) \le \arctan\left(\frac{x}{z}\right), & \text{at } \gamma = 0, \\ \arctan\left(\frac{x}{z}\right) < \theta_L(x, z) \le \arctan\left(\frac{x + D/2}{z}\right), & \text{at } \gamma = \pi. \end{cases} \tag{25}$$

To determine the transverse spatial frequencies k_x, k_y of angular components in the 3D wave field, which may potentially pass through the point of observation (x, y, z), it is necessary, according to (2), apart from the zenith angle value θ_L to determine the value of the azimuthal angle α_L for each of these components. Since the wave vector **k** of each of these angular components is collinear to the radius-vector **R**, connecting a certain point $S(\xi, \eta)$ of the source with the point of observation $P(x, y, z)$, in this case each zenith angle θ_L will have the only matching azimuthal angle α_L. I.e. each point of the source has the only matching pair of transverse spatial frequencies k_x, k_y of the angular component of the wave field and accordingly the only angular component, which may potentially pass through the fixed point of observation [3].

The connection of the azimuthal angle α_L with γ and $\rho(\gamma)$ may be found from geometrical constructions and is

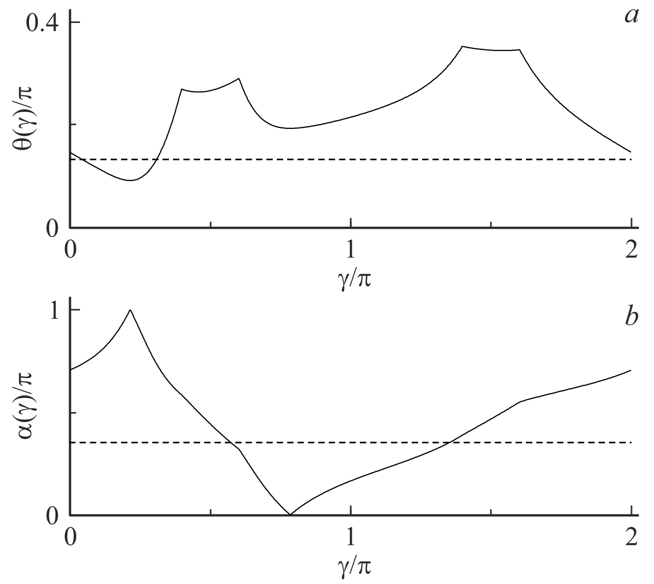

Figure 7. The reduced values of the zenith angle (a) and the corresponding reduced value of the azimuthal angle (b) of the angular components of the monochromatic wave field in the source with aperture of rectangular shape $a \times b$ $(b = 3a)$ at $\rho(\gamma) = \rho_{\min}(\gamma)$ (dashed line) and $\rho(\gamma) = \rho_{\max}(\gamma)$ (solid line) for the point of observation with coordinates $x = 0.2a$, $y = 0.4a$, $z = a$.

determined by expression

$$\alpha_L(\gamma) = \arccos \frac{x - \rho(\gamma)\cos\gamma}{\sqrt{x^2 + y^2 + \rho^2(\gamma) - 2\rho(\gamma)[x\cos\gamma - y\sin\gamma]}}$$

$$= \arcsin \frac{y - \rho(\gamma)\sin\gamma}{\sqrt{x^2 + y^2 + \rho^2(\gamma) - 2\rho(\gamma)[x\cos\gamma - y\sin\gamma]}}. \quad (26)$$

Fig. 7 shows as an example the curves of the reduced values of the zenith angle $\theta_L(\gamma)/\pi$ (fig. 7, a) and curves of the corresponding reduced values of the azimuthal angle $\alpha_L(\gamma)/\pi$ (fig. 7, b) of the angular components of the monochromatic wave field of the source with the aperture of rectangular shape $a \times b$ $(b = 3a)$ at $\rho(\gamma) = \rho_{\min}(\gamma)$ (dashed line) and $\rho(\gamma) = \rho_{\max}(\gamma)$ (solid line) for the point of observation with coordinates $x = 0.2a$, $y = 0.4a$, $z = a$. Fig. 7, a shows that the values of angles $\theta_L(\gamma)$ at $\rho_{\min}(\gamma)$ and $\rho_{\max}(\gamma)$ may correlate to each other in a different way depending on angle γ, which finds reflection in condition (24). As fig. 7, b shows, a similar situation is true for angle $\alpha_L(\gamma)$.

Taking into account the relation determined by expression (8) between the aperture-limited complex amplitude $U_0(\xi, \eta)t_\Sigma(\xi, \eta)$ of the field in the plane of the source and the full angular spectrum $W_{0\Sigma}(k_x, k_y)$ of the wave field in this plane and applying the inverse Fourier transform, expression (20) is transformed as follows when this complex amplitude is substituted

$$U(k, x, y, z) = \iint\limits_{-\infty}^{+\infty} W_{0\Sigma}(k_x, k_y) \left(\frac{1}{i\lambda} \iint\limits_{-\infty}^{+\infty} \exp\{i(k_x\xi + k_y\eta)\} \times \frac{\exp\{ikR\}}{R} \cos\theta_L d\xi d\eta \right) dk_x dk_y. \quad (27)$$

When substituting

$$kR = \tilde{k}_z z + \tilde{k}_x(x - \xi) + \tilde{k}_y(y - \eta),$$

where sign „tilde“ means dependence of values on pairs of angles $(\theta_L(\gamma); \alpha_L(\gamma))$, expression (27) can be also written as follows:

$$U(x, y, z) = \iint\limits_{-\infty}^{+\infty} W_{0\Sigma}(k_x, k_y) P(k_x, k_y; x, y, z) \times \exp\{ik_z z\} \exp\{i(k_x x + k_y y)\} dk_x dk_y, \quad (28)$$

where

$$P(k_x, k_y; x, y, z) = \frac{1}{i\lambda} \iint\limits_{-\infty}^{+\infty} \frac{\cos\theta_L}{R} \delta(\tilde{k}_x - k_x, \tilde{k}_y - k_y, \tilde{k}_z - k_z) \times \exp\{-i[(\tilde{k}_x - k_x)\xi + (\tilde{k}_y - k_y)\eta]\} \times \exp\{i[(\tilde{k}_z - k_z)z + (\tilde{k}_x - k_x)x + (\tilde{k}_y - k_y)y]\} d\xi d\eta.$$

Function $P(k_x, k_y; x, y, z)$ plays the role of the limiting aperture function for the angular spectrum $W_{0\Sigma}(k_x, k_y)$ of the monochromatic wave field in the plane of source $(z = 0)$ depending on the position of the observation point (x, y, z), i. e. „cuts“ from this angular spectrum the angular components (plane waves), which will pass through this point of observation when the wave field propagates in the free space $z > 0$. Function $P(k_x, k_y; x, y, z)$ is different from zero only for angular components of the wave field, propagating at angles θ to axis Z from the range determined by (24), and azimuthal angles α (each corresponding to the certain value of angle θ), determined by (26). Therefore for function $P(k_x, k_y; x, y, z)$, using the filtering action of δ-function, you can record the following expression:

$$P(k_x, k_y; x, y, z) = \begin{cases} \frac{1}{i\lambda}\frac{\cos\theta}{R} = \frac{1}{i2\pi}\frac{k^2 - k_x^2 - k_y^2}{zk}, \\ \text{at } k_x = k_x(\theta_L(\gamma); \alpha_L(\gamma)), \\ \quad k_y = k_y(\theta_L(\gamma); \alpha_L(\gamma)); \\ 0, \\ \text{at } k_x \neq k_x(\theta_L(\gamma); \alpha_L(\gamma)), \\ \quad k \neq k_y(\theta_L(\gamma); \alpha_L(\gamma)). \end{cases} \quad (29)$$

Expression (28) may then be written in the form similar to expression (1),

$$U(x, y, z) = \iint\limits_{-\infty}^{+\infty} W_L(k_x, k_y; x, y, z) \exp\{i(k_x x + k_y y)\} dk_x dk_y, \quad (30)$$

where $W_L(k_x, k_y; x, y, z)$ — local angular spectrum of excitation of the monochromatic wave field in the point of observation (x, y, z),

$$W_L(k_x, k_y; x, y, z) = W_{0\Sigma}(k_x, k_y) P(k_x, k_y; x, y, z) \times \exp\{ik_z z\} = W_\Sigma(k_x, k_y, z) P(k_x, k_y; x, y, z), \qquad (31)$$

where, in its turn, $W_\Sigma(k_x, k_y, z)$ — full angular spectrum of monochromatic wave field developed by a source with the finite linear aperture Σ, in plane $z > 0$, where the point of observation lies (x, y, z), i.e. the angular spectrum of the wave field in its classic sense.

Expression (31) determines the connection of the local angular spectrum of monochromatic wave field excitation in a certain point of observation with the full angular spectrum of the wave field. This expression shows that same as in the case with the full angular spectrum, the propagation of the wave field in space causes phase incursions between the angular components of the field. However, this changes the quantitative and qualitative composition of the angular components in the wave field disturbances when changing from one point of observation to another.

Such local change of the quantitative and qualitative composition of the angular components in the wave field disturbances is determined by aperture function $P(k_x, k_y; x, y, z)$, playing the role of the angular receiving aperture of the point of observation (x, y, z). The range of spatial frequencies, for which in the specified point of observation the function $P(k_x, k_y; x, y, z)$ is different from zero, is determined by the body angle, at which the linear aperture of the field source is seen from this point of observation. The qualitative composition of the local angular spectrum of monochromatic wave field disturbance in the specified point of observation will be determined by the crossing of the variation range of the full angular spectrum $W_{0\Sigma}(k_x, k_y)$ spatial frequencies in the wave field in the source plane with the range of the spatial frequencies, for which the aperture function $P(k_x, k_y; x, y, z)$ in this point of observation is different from zero. Besides, function $P(k_x, k_y; x, y, z)$ will also change, as per (29), the weight coefficients (amplitude) of the wave field angular components reaching the point of observation, thus varying the quantitative composition of the local angular spectrum in this point.

For the previously considered unidimensional $(y = 0)$ case of the transverse distribution of the complex amplitude of wave field disturbances, the source of which is the amplitude diffraction grating with the finite linear aperture of size D, the aperture function $P(k_x, x, z)$ in accordance with inequalities (25) will differ from zero for spatial frequencies k_x, meeting the condition

$$k \sin\left(\arctan\left(\frac{x - D/2}{z}\right)\right) \le k_x \le k \sin\left(\arctan\left(\frac{x + D/2}{z}\right)\right). \qquad (32)$$

Expression (32) shows that as distance z increases from the plane of the source to the plane where the point of observation is located, the range Δk_x of spatial frequencies k_x, for which the aperture function $P(k_x, x, z)$ differs from zero, narrows down. Accordingly, this causes narrowing of the local angular spectrum $W_L(k_x, x, z)$ of the wave field as the point of observation is removed from the plane of the source. From (32) it also follows that the increase in the dimensions of the linear aperture D of the field source with fixation of the point of observation, on the contrary causes the increased range of the spatial frequencies Δk_x, for which the aperture function $P(k_x, x, z)$ is different from zero, which in its turn causes the expansion of the local angular spectrum $W_L(k_x, x, z)$. While the change of the transverse coordinate of the point of observation x, as per (32), causes in the general case the change in the range of spatial frequencies Δk_x, for which the aperture function $P(k_x, x, z)$ differs from zero, and to the change of the value of the average spatial frequency of this range. Expression (32) is received for the 2D distribution of the monochromatic wave field disturbances in plane (XZ), but the conclusions made on its basis may cover the 3D space, too.

Fig. 8 shows the curves of absolute values of the full angular spectrum in the plane of source $W_{0\Sigma}(k_x)$ (thin solid line), aperture function $P(k_x, x, z)$ (dashed line) and local angular spectrum $W_L(k_x, x, z)$ (thick solid line) for the considered case of the monochromatic wave field developed by the amplitude diffraction grating with the linear aperture of size D, for conventional points *1, 2* and *3* in fig. 2.

Modeling parameters for fig. 8: $\lambda = 0.55\,\mu$m, $D = 10\,\mu$m, $d = 1\,\mu$m, $q = 0.8$, $x = 0.25D$, $z = 0.25D$ (fig. 8,*a*), $z = 0.75D$ (fig. 8,*b*), $z = 1.75D$ (fig. 8,*c*). Modeling of the angular spectrum in the plane of source $W_{0\Sigma}(k_x)$ was carried out using formula (14), and for convenience this spectrum was normalized to maximum value. Side maxima in curves $|W_{0\Sigma}(k_x)|$ of fig. 8 correspond to the beams with the angles of propagation $\theta_1 = -\theta$ and $\theta_2 = +\theta$, where θ is determined by expression (13). Asymmetry of curve $|P(k_x, x, z)|$ relative to zero spatial frequency is caused by shift of the point of observation from the optical axis.

From fig. 8 you can see that as expected, as the point of observation is removed from the point of observation from the source, the aperture function narrows down, and its amplitude decreases, which decreases the width of the local angular spectrum and amplitudes of its components, which, in its turn, characterizes the change of the local spatial properties of considered monochromatic wave field disturbances.

Conclusion

This paper introduces the concept of the local angular spectrum of scalar monochromatic wave field disturbance in a certain point of observation as a set of angular components of the wave field passing through this point. The main difference of the local angular spectrum from the full angular spectrum of the wave field determined in Fourier optics in a certain plane [1–6], perpendicular to the

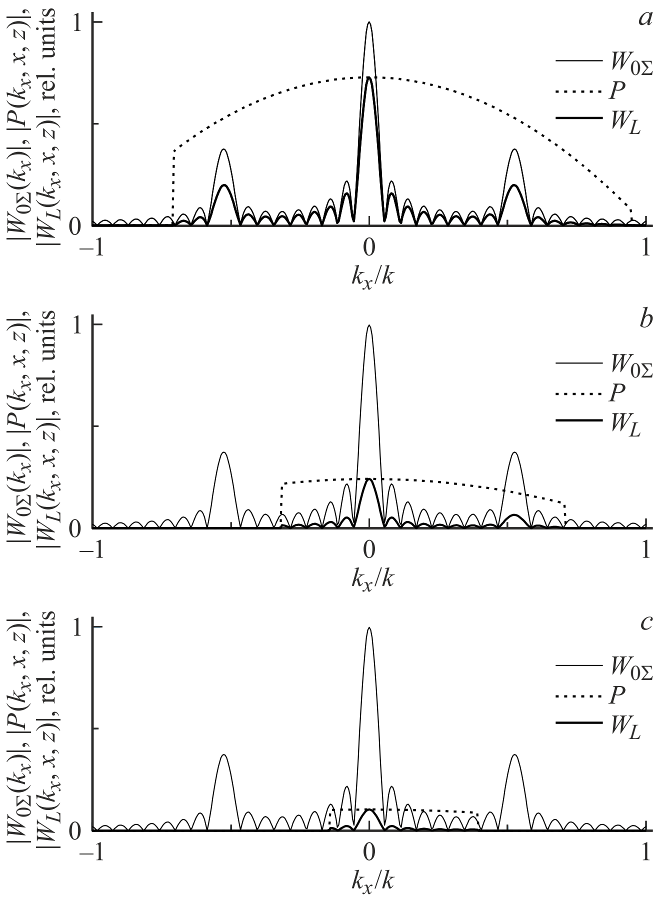

Figure 8. Curves of absolute values of the full angular spectrum $W_{0\Sigma}(k_x)$, aperture function $P(k_x, x, z)$ and local angular spectrum $W_L(k_x, x, z)$ of the monochromatic wave field developed by the amplitude diffraction grating with finite linear aperture of size D, for conventional points of observations in fig. 2: (a) point 1, (b) point 2, (c) point 3.

main direction of the field propagation, consists in changing the local angular spectrum as the point of observation changes in space. The reason for such change are the finite dimensions of the aperture in the real sources.

The finite aperture of the wave field source forms an angular aperture of reception in a certain point of observation that is determined by a body angle, at which the source aperture is seen from this point. The local angular spectrum of monochromatic wave field disturbance with the finite linear aperture is determined by crossing of the range of spatial frequencies of the full angular spectrum of the wave field and the range of spatial frequencies corresponding to the angular aperture of reception in the point of observation. Therefore, it is actually shown that the wave field disturbance from the source with the finite linear aperture is determined in the specified point of observation not by the full angular spectrum, but by its certain part only.

The fact that the local angular spectrum depends on the body angle, at which the source aperture is seen from the point of observation, leads to another difference of this spectrum from the full angular spectrum of the wave field: width of the local spectrum decreases when the linear aperture of the source decreases.

In our opinion, the introduction of the concept of the local angular spectrum makes it possible to more fully understand the properties of the statistically heterogeneous spatially limited scalar monochromatic wave field and reflects these properties more visually. For example, a fact known well theoretically and experimentally that the lengths of the spatial correlation of the monochromatic wave field extend both in transverse direction [1,3–5] and in the longitudinal one (along the direction of the field propagation) [7–9,39] at the distance from the source plane is explained by the narrowing, depletion of the local angular spectra of this field disturbances. The change of the local angular spectrum of wave field disturbances when changing from one spatial point to another, is, in our opinion, one of the reasons for the complex amplitude-phase structure and statistical heterogeneity of the spatially limited wave fields, which manifests itself in the form of various features and dislocations in interference of such fields [5,40,41].

This paper considers the propagation of the spatially limited monochromatic wave field in the free homogeneous and isotropic space and, actually, finds the cause for the statistical spatial heterogeneity of such field. However, the process of distribution of the wave fields with different spectra and nature (visible optical, terahertz, X-ray, acoustic, etc.) through the locally heterogeneous and anisotropic media [28,42,43] is of great interest both from theoretical and practical points of view, the example being biological tissues or various composites. Modeling of such process is a non-trivial task, requiring significant computing capacities even with involvement of the state-of-the-art computing technology, logics and simplifying models of medium and wave field interaction. The concept of the local angular spectrum introduced in this paper for the disturbances of the scalar monochromatic wave field makes it possible to reduce the time of computing procedures for determination of the local parameters of such field as it propagates in heterogeneous anisotropic media at the expense of the local reduction of the band of spatial frequencies that generate a wave disturbance in the point of observation, compared to the full angular spectrum of the wave field.

The results obtained in this paper in our opinion may be generalized both for scalar broadband frequency wave fields and vector wave fields.

Funding

The work was carried out within the framework of the State Assignment of the Ministry of Science and Higher Education of the Russian Federation (theme №. 121022000123-8 Precision diagnostics, sensors and process control in technical and living systems based on photonic technologies, including the solution of thermophysical problems).

Acknowledgments

The authors are grateful to the journal reviewer for valuable comments on the manuscript of the article.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by M.Verenikina