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The influence of the cathode field enhancer on the conditions for the transition of electrons to the runaway mode

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The influence of field enhancers (pointed protrusions on the cathode) on the conditions for the transition of electrons emitted from them to the runaway mode in high-pressure gas is studied analytically. It is shown that the classical directly proportional dependence of the critical runaway field on pressure is replaced by the weaker root dependence in the presence of protrusions of sufficient height. A simple criterion for electron runaway, taking into account the field distortion near the enhancers, is formulated. The minimum height of the protrusions required for a noticeable reduction in the runaway threshold is determined depending on the gas pressure.

Keywords: runaway electrons, subnanosecond gas breakdown, inhomogeneous electric field, field enhancers.

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Introduction

The phenomenon of electron runaway in gases — their continuous acceleration under conditions when they receive more energy from an applied electric field than they lose as a result of inelastic collisions [1–3] — plays an important role in the development of pulsed breakdown of gases [4–9]. The runaway electrons (RE) carry out preionization of the gas within the times comparable to the time of light propagation through the interelectrode gap, which determines the subsequent breakdown dynamics [10–14]. A sharp heterogeneity of the distribution of the electric field in the gas gap generally characterizes laboratory studies of this phenomenon. Such heterogeneity can be related both to the use of field amplifiers — pointed protrusions on the cathode with various shape [12,15–20], and to the natural microrelief of the cathode surface — the presence of various defects on it [21–23]. The presence of macro- and micropoints ensures a local field amplification to the values necessary for the initiation of field electron emission and the subsequent transition of free electrons to runaway mode under conditions of a relatively low average field [24,25].

The transition of initially low-energy electrons to the runaway mode in a homogeneous field requires that its intensity exceed a certain threshold E_c, depending on the type of gas and its density (pressure) [10,11,26]. The situation is more complicated in a nonuniform field. On the one hand, it is required that the field strength exceed the critical value E_c at the electron launch site — near the cathode tip. On the other hand, it is necessary for the electron to continue runaway at the periphery, where the field is below the runaway threshold for thermal electrons.

A situation was studied in Refs. [17,27–29] when the field is heterogeneous throughout the interelectrode gap — its intensity decreases with the distance z from the cathode according to the power law $z^{-\gamma}$, where the exponent γ characterizes the degree of heterogeneity of the field distribution. The value of the exponent depends on the specific geometry of the electrodes. It refers to the interval $0 < \gamma < 1$ for the conical cathode and to the interval $0 < \gamma < 0.5$ for the wedge-shaped cathode; $\gamma = 1$ for the needle cathode and $\gamma = 0.5$ for the blade cathode. It has been demonstrated that the dynamics and conditions of electron runaway are qualitatively different for the cases of weakly inhomogeneous $(0 < \gamma \leq 0.5)$ and strongly inhomogeneous $(0.5 < \gamma \leq 1)$ fields. The effect of electron ionization multiplication in the gaps with a heterogeneous field distribution ($\gamma = 1$ and 2 — cylindrical and spherical geometries) on the conditions of RE generation was discussed in Ref. [30], based on the approach from Refs. [31,32]: the authors of Refs. [31,32] suggested using the criterion of the absence of Townsendian electron multiplication $\alpha_i d \leq 1$ as a runaway criterion for the case of a homogeneous fields (here α_i is an impact ionization coefficient, d is an interelectrode distance).

A qualitatively different situation, which will be analyzed in this paper, was realized in the experiments [33,34]. The field distribution in most of the gas-discharge gap was close to uniform; the region of sharply heterogeneous field was concentrated near field amplifiers with a size significantly smaller than the interelectrode distance. In this case, field amplifiers play the role of „springboards", on which the electron gains the energy necessary for its runaway after entering a weak quasi-homogeneous field. The energy loss of an electron decreases with the increase of the energy, and a fast enough electron is able to runaway in a subcritical field. An example of this is the runaway of electrons in lightning discharges [26,35]. We would also like to note the series of papers [23,36–38], where the motion of field emission electrons emitted by natural microprotrusions on the cathode surface was studied numerically. It was found that in conditions of high-pressure gas (tens of atmospheres), the passage of a small area of an enhanced field near the micropoint significantly reduces the threshold of electron runaway.

Understanding the conditions of electron runaway is crucial when developing fast electron sources based on the RE use. For example, it is necessary to ensure the synchronicity of the emission of RE flows with picosecond accuracy from a set of concentric field amplifiers on the cathode for generating a disk bunch of electrons [39]. This necessitates the formulation of criteria for the generation of REs that are convenient for practical use and applicable to a wide variety of configurations of field amplifiers. This study is aimed at obtaining such criteria.

1. Necessary information about the REs

The friction (deceleration) force of a non-relativistic electron (kinetic energy ε is less than the rest energy ~ 510 keV) in a gas as a result of collisions with its molecules can be estimated using the Bethe formula [40]:

$$F(\varepsilon) = \frac{2\pi Z e^4 n}{\varepsilon} \ln\left(\frac{2\varepsilon}{I}\right), \tag{1}$$

where Z is the number of electrons in a neutral gas molecule, e is the elementary charge, n is the concentration of gas molecules, I is the average energy of inelastic losses. The dependence of this force on energy is non-monotonic. It has a maximum $F_{\max} = 4\pi Z e^4 n/\mathrm{e}I$ attributable to energy $\varepsilon_c = \mathrm{e}I/2$, where $\mathrm{e} \approx 2.718$ is the base of the natural logarithm. If an electron is affected by a force exceeding the value $F_{\max}$ from the electric field, it will accelerate regardless of its initial energy. This means with respect to a homogeneous electric field of magnitude E_0 that formally all free electrons will enter the runaway mode if

$$E_0 > E_c, \quad E_c \equiv F_{\max}/e = 4\pi Z e^3 n/\mathrm{e}I, \tag{2}$$

that is, the strength of the external field E_0 exceeds the critical value E_c [5,10,11,41]. For the sake of brevity, we will call condition (2) the field condition of electron runaway.

Electrons can also run away in a subcritical field, i.e. at $E_0 < E_c$ [10,26,42]. This happens in a situation where the initial energy of an electron ε_0, with which it will enter a homogeneous field E_0, is high enough. The friction force rapidly decreases with the increase of electron energy at $\varepsilon > \varepsilon_c$ according to (1). Then, for a given E_0, it is always possible to specify a critical value ε_r for the initial energy of the electron ε_0, above which the electrons will continuously accelerate (note that this statement is incorrect in the relativistic domain due to the appearance of a minimum in the dependence $F(\varepsilon)$ at an energy of the order of 1 MeV [26,35]). The corresponding runaway condition, which we will call the energy condition, is written as

$$\varepsilon_0 > \varepsilon_r, \quad F(\varepsilon_r) = eE_0. \tag{3}$$

Due to the non-monotonic nature of the dependence of F on ε, the equation $F(\varepsilon_r) = eE_0$ has two roots. For runaway conditions (3) only the root $\varepsilon_r > \varepsilon_c$ makes sense.

2. Derivation of the runaway criterion

It is clear that a low-energy thermal electron is unable to gain energy exceeding the threshold ε_r in a subcritical field. Then the realization of the energy runaway condition (3) assumes that an electron, previously accelerated in a stronger supercritical electric field, enters the region of a relatively weak homogeneous field with a strength of $E_0 < E_c$.

We assume that the area of the enhanced electric field is caused by the presence of a field amplifier — a pointed protrusion with a height of h on a flat cathode (Fig. 1), which causes a local distortion of the field distribution. The field strength will significantly exceed the average value E_0 in the electrode gap at the tip of the protrusion. It is customary to use the field gain factor $\beta = E_{\max}/E_0$, where $E_{\max}$ is the field reached at the tip of the cathode protrusion [21,24]. A distinction is made between field amplification due to the cathode geometry used — the presence of a macroscopic scale field amplifier (units — tens of millimeters) and the microrelief of the cathode surface — the presence of micron-scale protrusions on it. We will not separate these scales in Sec. 2; we will formulate an electron runaway criterion applicable over the entire size h range and return to the discussion of the features of electron runaway in the presence of macro- and microprotrusions in Sec. 5.

Let us consider the behavior of a free electron emitting from the top of the protrusion of a — field amplifier. For

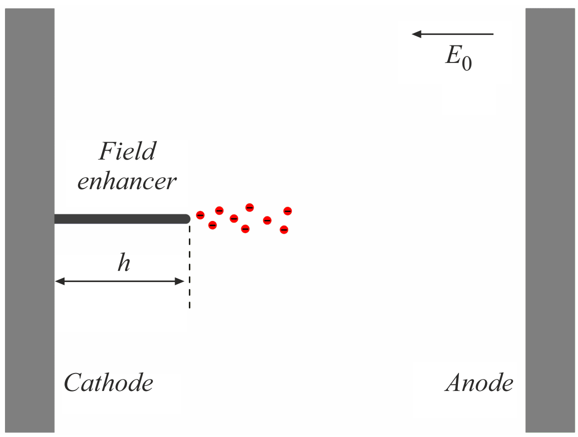

Figure 1. Schematic representation of the geometry of the interelectrode gap.

such an electron to enter runaway mode, it is necessary that the field near the tip of the protrusion exceeds a critical value, i.e. $E_{max} > E_c$. This condition will be written as $\beta > E_c/E_0$ using the field gain factor β. From general considerations, it is clear that to implement this condition, a protrusion with a sufficiently high aspect ratio h/r is required, where r is the radius of its vertex. For example, the estimate is $\beta \approx 2 + h/r$ for a cylindrical protrusion with a rounded top [21]. In the future, we will consider the condition $\beta > E_c/E_0$ fulfilled and will not return to its analysis. The energy condition (3) will then determine the transition of the field emission electrons to the runaway mode.

The energy gained by an electron in the vicinity of the cathode field amplifier can be estimated from above by the potential difference it passes through (such a vacuum approximation corresponds to neglecting energy losses in inelastic collisions with gas molecules). The latter is calculated with acceptable accuracy as the product of the average field in the gap E_0 by the height of the protrusion h, which gives $\varepsilon_{vac} \approx eE_0h$. It was shown in Refs. [17,43] that in an heterogeneous field decreasing with a distance z from the cathode according to the inverse root law $E \propto z^{-1/2}$, under runaway threshold conditions, an electron loses half of its energy in inelastic collisions. This field distribution in the axisymmetric formulation corresponds to a cone with an opening angle of $98.6°$ (the so-called Taylor angle [28.44]). The energy gained by the electron near the field amplifier is then estimated as $\varepsilon_0 \approx 0.5\varepsilon_{vac} \approx 0.5eE_0h$. By analogy with this ratio, we assume that the following is valid for protrusions of arbitrary shape near the runaway threshold, i.e. when ε_0 only slightly exceeds the threshold ε_r

$$\varepsilon_0 \approx \kappa e E_0 h, \tag{4}$$

where κ is a dimensionless parameter that is obviously less than one. It is clear that κ depends on the geometry of the protrusion. Its value will increase with an increase in the degree of sharpening of the cathode protrusion, i.e., in fact, with an increase of the value of the field gain factor β. It is natural to expect that the parameter κ is in the range $0.5 < \kappa < 1$. The lower limit corresponds to the above-mentioned cone with the Taylor opening angle, i.e. a relatively blunt protrusion, the height of which is less than the radius of the base. The upper limit corresponds to the „ideal" protrusion, which formally ensures the acceleration of the electron without energy loss. If, by definition, an electron passes through a potential difference ε_{vac}/e in a homogeneous field, i.e. in the absence of a protrusion, moving along the field lines at a distance h, then the electron moves in an amplified field in the presence of a protrusion and, consequently, the required distance decreases. The reduction of the path of an electron leads to a decrease of energy losses as a result of interactions with gas molecules. The condition $\kappa = 1$ corresponds to the limit when the path is zero (it should be noted that formally this situation is realized for a protrusion in the form of an infinitely thin needle due to the logarithmic divergence of the field potential at a singular point). Below, we will use the intermediate value $\kappa = 0.7$ for estimates without focusing on the geometry features of specific amplifiers. This approach will allow ensuring the universality of the desired runaway criterion.

Using the relation (4), it is possible to formulate the following runaway condition for electrons starting from a cathode protrusion with a height h with the help of the energy criterion (3):

$$E_0 > E_r, \quad F(\kappa e E_r h) = eE_r. \tag{5}$$

The threshold runaway field E_r here is related to the threshold energy ε_r by the ratio $\varepsilon_r = \kappa e E_r h$ resulting from (4). Electrons gaining such energy near the field amplifier (Fig. 1) will, according to (3), continue to runaway after entering the region of a relatively weak homogeneous field.

Due to the non-monotonous dependence of the function F on the argument y there may be several roots of equation $F(\kappa e E_r h) = eE_r$, which defines the threshold runaway field for criterion (5). We are only interested in the root having the property $E_r > \varepsilon_c/(\kappa e h)$, i.e. belonging to the region $\varepsilon > \varepsilon_c$, where the braking force decreases with an increase of the electron energy according to the Bethe formula (1). Such a root does not exist if the height of the protrusion is less than a certain value $h_{min} = \varepsilon_c/(\kappa e E_c)$. The presence of this limitation reflects the fact that the electron is unable to gain energy exceeding ε_c near low-altitude protrusions, and thus pass through the region of maximum braking force at $E_0 < E_c$. Thus, the presence of protrusions with a height $h < h_{min}$ does not affect the nature of electron runaway in the framework of the discussed model. The classical field condition (2) will be the condition for the runaway of electrons in such a situation. In the end, the general condition for generating REs will be

$$E_0 > \begin{cases} E_c, & h < h_{min}, \\ E_r(h), & h \geq h_{min}. \end{cases} \tag{6}$$

Both criteria give the same value of the runaway field equal to E_c at $h = h_{min}$, i.e. $E_r(h_{min}) = E_c$. It is always $E_r < E_c$ at $h > h_{min}$, i.e. the electron is able to runaway in a subcritical (in terms of the runaway field condition (2)) electric field.

Equation (5) defining the threshold runaway field E_r, using the Bethe formula (1) takes the following form

$$\frac{2\pi Z e^2 n}{\kappa h} \ln\left(\frac{2\kappa e E_r h}{I}\right) = E_r^2. \tag{7}$$

It is convenient to normalize the values E_r and h to their characteristic values E_c and h_{min}. This corresponds to the introduction of the dimensionless variables $E_n \equiv E_r/E_c$ and $h_n \equiv h/h_{min}$. Then the relationship between the parameters of the problem will have a universal form, independent of the properties of the gas

$$E_n^2 h_n = 1 + \ln(E_n h_n). \quad (8)$$

The formula (8) is not resolved with respect to the runaway threshold field E_n. We can use the following approximate expression for practical needs which gives an explicit dependence of E_n on h_n:

$$E_n \approx \sqrt{\frac{1 + \delta \ln h_n}{h_n}}. \quad (9)$$

Here δ is a constant belonging to the interval $0.5 \le \delta \le 1$. δ should be taken closer to the upper part of the specified range for small (close to unity) values of h_n and closer to the lower part for $h_n \gg 1$ when using an approximation (9). It should be noted that the formula (9) gives the runaway threshold with high accuracy at $\delta = 0.65$ for $1 < h_n < 10000$ (this includes the entire range of parameters we are interested in (see Sec. 5)).

3. Dependence of the threshold field on the gas pressure

The critical strength E_c for the runaway of electrons in a homogeneous electric field is directly proportional to the concentration of gas molecules n according to (2). We have $E_c \propto p$ since for a gas $n \propto p$, where p is the pressure. This corresponds to the well-known laws of similarity for electrical discharges in gases [45]. However, the threshold runaway field E_r will no longer be related to pressure by such a simple linear relationship for the heterogeneous field provided by the field amplifier (Fig. 1). It would be realized if, with increasing pressure, there was a multiple decrease in the scale of the cathode protrusion. The presence of a fixed protrusion h leads to a different dependence of E_r on p (see results of numerical simulation of [23,36–38], which show that it is not linear in nature). We obtain this dependence analytically.

Let's introduce the reduced critical runaway field $\tilde{E}_c \equiv E_c/p$, which depends only on the type of gas [45]. Then the expression (7) defining the runaway threshold field E_r takes the form

$$E_r^2 = \frac{\varepsilon_c \tilde{E}_c p}{\kappa e h} \ln\left(\frac{\mathrm{e}\kappa e E_r h}{\varepsilon_c}\right). \quad (10)$$

This transcendental equation gives (implicitly) the dependence of the threshold field E_r on the height of the field amplifier h and the gas pressure p. The relationship of the minimum tip height with the pressure required for applicability of (10) is given by the formula

$$h_{\min} = \frac{\varepsilon_c}{\kappa e \tilde{E}_c p}. \quad (11)$$

It shows that the height $h_{\min}$ decreases with the increase of the pressure as $1/p$ in accordance with the laws of similarity [45].

The formula (10) is not resolved with respect to the runaway threshold field E_r. We can use the approximation (9) for practical needs, which in terms of the reduced critical field $\tilde{E}_c$ is written as

$$E_r^2 \approx \frac{\varepsilon_c \tilde{E}_c p}{\kappa e h}\left[1 + 0.65 \ln\left(\frac{\kappa e \tilde{E}_c p h}{\varepsilon_c}\right)\right]. \quad (12)$$

This expression gives an explicit dependence of the desired threshold E_r on p and h.

4. Analysis of the electron runaway criterion

Let us consider the dependence of the threshold runaway field E_r on the pressure p and the height of the cathode protrusion h for nitrogen. The reduced critical electric field strength ($\tilde{E}_c$) at a temperature of 300 K is ~ 590 V/(cm·Torr) or ~ 450 kV/(cm·atm) for it according to [11.45], which corresponds to $Z = 14$ and $I = 80$ eV. The maximum friction force falls on $\varepsilon_c \approx 109$ eV.

Figs. 2 and 3 show the dependences of the threshold field E_r described by the formula (10) on the height of the protrusion h on a flat cathode (Fig. 1) at fixed pressures of 1 and 10 atm, respectively. It can be seen from the figures that the field decreases monotonously with the growth of h. The estimate $E_r \propto h^{-1/2}$ is obtained if the weak logarithmic dependence on h is neglected in (10) or in the approximation (12). Then, the electron gains energy $\varepsilon_0 \propto E_r h \propto h^{1/2}$ near the field amplifier, i.e. it grows in a root manner with h. Thus, the runaway of an electron upon entering a homogeneous field with a strength of E_r, which decreases with the increase of the height of the protrusion, is ensured by an increase of its energy ε_0.

Figs. 2, *b* and 3, *b* show in detail areas of relatively small — comparable to the magnitude of $h_{\min}$ — values of h ($h_{\min} \approx 3.5\,\mu$m and $h_{\min} \approx 0.35\,\mu$m, for pressures 1 and 10 atm, respectively). The runaway energy criterion (5) does not work at $h < h_{\min}$: the energy gained by the electron near the protrusion is not enough to pass through the maximum friction force in the Bethe formula (1) at $E_0 < E_c$. In this case, the runaway threshold is given by the classical field criterion (2), according to which the field should exceed the threshold $E_c \approx 0.45$ MV/cm for $p = 1$ atm and $E_c \approx 4.5$ MV/cm for $p = 10$ atm. The runaway criterion we obtained is applicable at $h > h_{\min}$. Let's pay attention to the fact that it is necessary to have protrusions with a height of $10\,\mu$m at atmospheric pressure for the cathode protrusions to begin to have a noticeable effect on runaway conditions, for example, to reduce its threshold by 20%. Micron and submicron protrusions will significantly affect the runaway process in the area of high pressures (10 atm and higher). Taking into account the presence of natural micron-scale protrusions on the surface of any real cathode (see, for example, the photographs in Refs. [23,46] of the surface of the cathode used in experiments [47]), the runaway threshold for a high-pressure

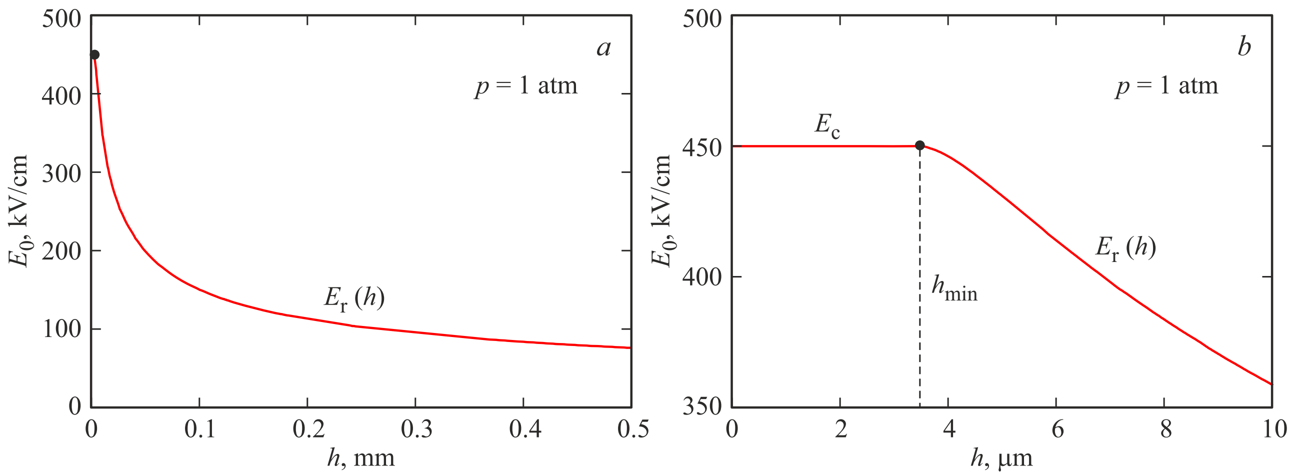

Figure 2. Dependence of the threshold field strength for electron runaway on the height of the protrusion on two different scales (gas—nitrogen, $p = 1$ atm, $\kappa = 0.7$).

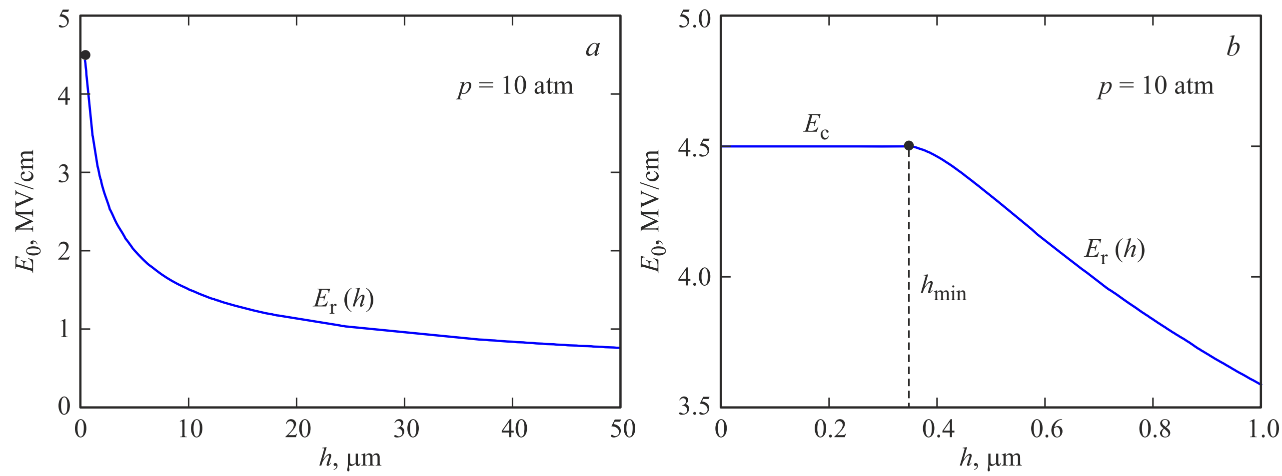

Figure 3. Dependence of the threshold field strength for electron runaway on the height of the protrusion on two different scales (gas—nitrogen, $p = 10$ atm, $\kappa = 0.7$).

gas will always be lower than the classical E_c. This circumstance was pointed out in Ref. [37] where the effect of micropoints on the cathode on the probability of electron transition to the runaway mode was studied. Returning to the case of atmospheric pressure nitrogen, we would like to note that macroscopic field amplifiers should be used to reduce the electron runaway threshold (see Sec. 5) since the effect of the microrelief of the cathode surface is negligible.

Figs. 2, *a* and 3, *a* show that the threshold runaway field E_r significantly decreases with the growth of h. For instance, it decreases in three times at $p = 10$ atm for $h = 10\,\mu$m in comparison with E_c. It should be noted that the shapes of the curves in Figs. 2 and 3, adjusted for the different scales used, coincide, which is attributable to the invariance of the expression (10) with respect to the substitutions $p \to Cp$, $h \to C^{-1}h$, and $E_r \to CE_r$, where C is the constant.

Fig. 4 shows the dependence of the threshold runaway field on the pressure of a gas (nitrogen) at various fixed values of the height of the cathode protrusion in the range from 0 to $40\,\mu$m. The case $h = 0$ corresponds to the classical field runaway criterion $E_0 > E_c = \tilde{E}_c p$. It can be seen that the presence of micron-scale protrusions on the cathode leads to a significant decrease of the runaway threshold, and the difference between the values of E_r and E_c increases with the increase of the pressure. For instance, the value E_c is 20 MV/cm at $p = 50$ atm, while $E_r \approx 2$ MV/cm for $h = 40\,\mu$m, i.e. an order of magnitude lower. We get the estimate $E_r \propto p^{1/2}$ if the weak logarithmic dependence on E_r is neglected in the right-hand side of (10) (or on p in (12)), i.e. the threshold runaway field increases for $h \neq 0$ with the increase of the pressure according to a much weaker (compared to the classical linear dependence $E_c \propto p$) root law. This situation is illustrated in Fig. 5, which shows the dependence of the runaway field on pressure at its relatively small values up to 2 atm for $h = 0$ (ideally smooth cathode) and for $h = 10\,\mu$m. It is possible to specify a threshold (minimum) pressure value at which the new runaway criterion should be applied for a given h:

$$p_{\min} = \frac{\varepsilon_c}{\kappa e \tilde{E}_c h}.$$

It should be noted that this relation is an analogue of the formula (11) for determining $h_{\min}$. We have $p_{\min} \approx 0.35$ atm for $h = 10\,\mu$m and $\kappa = 0.7$, which corresponds to the situation shown in Fig. 5. The presence of a microprotrusion at $p < p_{\min}$ has virtually no effect on the transition of electrons to the runaway mode. The runaway threshold will be determined in this case, like in case of a homogeneous field ($h = 0$), by the classical criterion $E_0 > E_c = \tilde{E}_c p$. The presence of a microprotrusion is already beginning to affect the dynamics of free electrons at $p > p_{\min}$. The root law $E_r \propto p^{1/2}$ will give radically lower runaway fields for a given surface roughness at $p \gg p_{\min}$ than the classical linear law $E_c \propto p$.

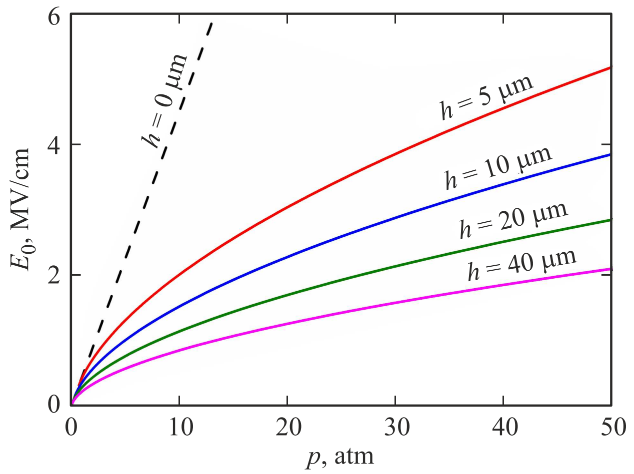

Figure 4. Dependence of the threshold field strength for electron runaway on the gas pressure for cathode protrusions of different heights (gas—nitrogen, $h = 0, 5, 10, 20, 40\,\mu$m, $\kappa = 0.7$).

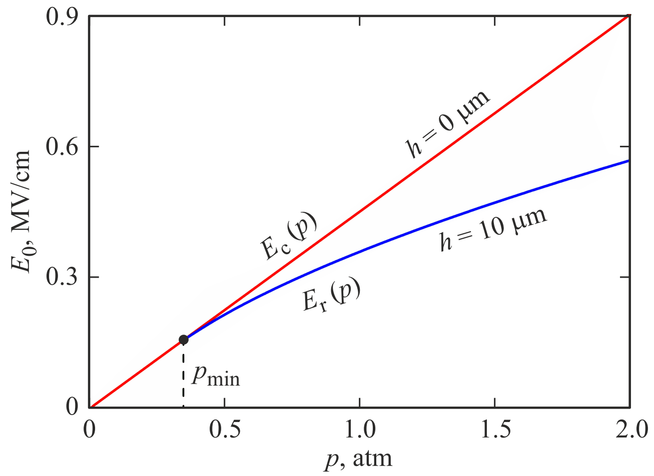

Figure 5. Dependence of the threshold field strength for electron runaway on gas pressure at close to atmospheric values in the absence ($h = 0$) and the presence ($h = 10\,\mu$m) of micropoints (gas—nitrogen, $\kappa = 0.7$).

5. Discussion of the results

It follows from the runaway criterion formulated in this paper that at high (tens of atmospheres) pressure of the working gas, micron-scale protrusions have a significant effect on the transition of electrons to runaway mode. This result is consistent with the results of numerical calculations in Refs. [23,36–38], where it was found that passing a small area of an enhanced field near the micropoint can significantly facilitate the runaway of an electron at gas pressures above 10 atm. The possible role of this effect was pointed out in Ref. [37] in explaining the experimental results [47], in which REs were recorded at nitrogen pressures up to 40 atm under conditions of a homogeneous electric field in the interelectrode gap. Its intensity was ~ 1.1 MV/cm at $p = 40$ atm, i.e. it was more than an order of magnitude lower than the critical one for the generation of REs (~ 18 MV/cm). However, as noted in Ref. [37], the effect of lowering the runaway threshold becomes quite noticeable with relatively long micropoints (at $h > 10\,\mu$m), which are very rarely observed on the surface of electrodes. A mechanism was proposed in Ref. [48] for initiating explosive electron emission at the cathode-dense gas interface based on the accumulation of positive ions at the natural protrusions with the size of $\sim 1\,\mu$m produced as a result of gas ionization by field emission electrons. The distance at which ions are generated decreases with the increase of gas density, which leads to an increase of their Coulomb field on the emitting surface. As a result, an explosive increase of the emission current density occurs for a gas of high (tens of atmospheres) pressure according to estimates in Ref. [48], leading to the formation of an explosive emission center in tens of picoseconds. It gives rise to the development of a plasma channel penetrating towards the anode. It can be assumed that REs are generated at the top of the plasma tip when its height reaches a value of tens of microns (see also [49,50]). This scenario of RE generation is close to that discussed in Ref. [37], except that long micropoints on the cathode were considered in Ref. [37], and plasma protrusions of similar geometry developing from explosive emission centers were considered in Ref. [48].

Let us compare the analytical dependences of the threshold runaway field on gas pressure obtained in this work at various values of h with the results of numerical simulation in Ref. [37]. First of all, we would like to note that the root dependency $E_r \propto p^{1/2}$ that we have deduced describes the results qualitatively correctly [37]. A quantitative comparison reveals that the specific values E_r given by the expressions (10) or (12) are approximately 2 times higher than those calculated in Ref. [37]. The reason for this discrepancy is obvious. The criterion of electron runaway (5) formulated by us, like the classical criterion (2), is deterministic in nature. Formally, when it is fulfilled, all electrons emitted by the cathode protrusion become runaway; if the field is below the threshold, then runaway is impossible. It is clear that in reality the runaway process is probabilistic. The runaway of a certain part of electrons is possible even at lower fields. When simulating the motion of electrons by the Monte Carlo method, it was assumed in Ref. [37] that the runaway condition was fulfilled if only 1% of the total number of electrons entered the runaway mode. In our opinion, this circumstance leads in Ref. [37] to a twofold decrease of the threshold relative to the values of E_r and E_c in the analytical model.

Micron-scale protrusions caused by the natural microrelief of the cathode surface will not significantly affect the conditions of RE generation at the gas pressure comparable to the atmospheric pressure. It is necessary to have macroscopic field amplifiers of various shapes of pointed protrusions of millimeter scale on the cathode for a noticeable decrease of the runaway threshold. Let us discuss the conditions for the runaway of electrons in atmospheric pressure air (nitrogen) for several cathode configurations used in laboratory studies of pulsed gas breakdown, when the distribution of the electric field was close to uniform, with the exception of a relatively small neighborhood of field amplifiers [33,34].

Let's estimate the threshold of generation of REs in case of a pulsed breakdown of an air coaxial line by a traveling voltage wave [34]. The field amplifier was a 2.5 mm protruding disk insert into the central electrode of the line, providing spatial reference of the radial breakdown. The interelectrode distance (d) was 12.5 mm (the inner and outer radii of the electrodes were 11 and 23.5 mm, respectively); the characteristic potential difference (U), at which a breakdown involving REs occurred was 85 kV. The average field in the radial gap is estimated as $E_0 = U/d \approx 68$ kV/cm, which is almost 7 times lower than the classical critical runaway field for nitrogen (air) at atmospheric pressure $E_c \approx 450$ kV/cm [11.45]. However, if we take into account the presence of a field amplifier, then we find $E_r \approx 38$ kV/cm using formula (12) for nitrogen (air) with $p = 1$ atm for the runaway threshold, which turns out to be less than the average field of ~ 68 kV/cm. Thus, the condition $E_0 > E_r$ we derived for the RE generation was fully fulfilled in case of a radial breakdown of the coaxial line.

Let us now discuss the conditions for generating REs in the gas gap in the form of a gap with a width of d at the end of the coaxial transmission line. A cathode configuration with a gradient screen was used in Ref. [33] to study the possibility of generating RE avalanches [26,51], ensuring a field distribution close to uniform over the gap with $d = 20$ mm. Various forms of cathode field amplifiers with $0.5 \leq h \leq 1.5$ mm were used to create the initial RE flow. We find the following range of values of the threshold runaway field using the formula (12) for atmospheric air ($p = 1$ atm): $E_r \approx 48-77$ kV/cm, which is almost an order of magnitude lower than the critical field $E_c \approx 450$ kV/cm. It should be noted that the characteristic value of the voltage at the gap (in idle mode) was ~ 500 kV, which corresponds to the average field of ~ 250 kV/cm, i.e. the runaway condition $E_0 > E_r$ was obviously fulfilled wherein the average field was significantly less than the critical value E_c.

Thus, using several examples, we have shown how the threshold field of electron runaway is determined for rather complex electrode configurations. In the framework of our approach, the threshold E_r which takes into account the field distortion near the cathode field amplifiers, for gas with given characteristics is estimated using a single parameter — height of amplifiers h. The condition $E_0 > E_r$ ensures the electron runaway at a distance from the field amplifier in a relatively weak field ($E_0 < E_c$). It should be recalled that it is necessary that the field near the tip exceeds a critical value for an electron starting from a protrusion to continuously accelerate: $E_{max} > E_c$, or, using the field gain factor, $E_0 > E_c/\beta$. Analyzing the possibility of runaway of free electrons on the periphery, we assumed that the latter condition was obviously fulfilled, i.e. the value of the geometric parameter β for the analyzed field amplifiers is quite high.

Conclusion

In this paper, an analytical study of the effect of local distortion of the electric field near the protrusions on the cathode on the conditions of transition of electrons starting from them to the runaway mode is carried out. It is shown that the classical, following from the laws of similarity for electric discharges in gases, directly proportional dependence of the critical runaway field on the gas pressure $E_c \propto p$ is replaced by a weaker root dependence $E_r \propto p^{1/2}$ in the presence of protrusions of sufficient height (11). The obtained simple expressions (10) and (12), which relate the runaway field to the parameters of the gas and the cathode protrusion, easily allow evaluating how the presence of a field amplifier reduces the runaway threshold relative to the value E_c, corresponding to an ideally uniform field distribution in the entire interelectrode gap.

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Conflict of interest

The authors declare that they have no conflict of interest.

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