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Analysis of the spectral dependence of the transmission coefficient of the system titanium dioxide film–glass substrate

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Received June 20, 2024
Revised September 23, 2024
Accepted October 16, 2024

Weakly absorbing titanium dioxide (anatase) films of different thicknesses on glass substrate have been obtained. The spectral dependence of the transmission coefficients of both the glass substrate alone and the glass substrate with the titanium dioxide film was measured over a wavelength range of 350–900 nm. The spectral dependence of the transmission coefficient of the film–substrate system on the wavelength of the spectrophotometer light has pronounced alternating maxima and minima caused by interference. The thicknesses of one, three, and five layer anatase films and the refractive index dispersion for the five-layer film were determined. Calculations of the spectral dependence of the transmission coefficient were performed, showing that the smaller the ratio of titanium dioxide film thickness to the wavelength of light, the better the agreement between the calculated and experimental transmission coefficient dispersion. The spectral dependence of the absorption coefficient for a five-layer titanium dioxide film is determined.

Keywords: titanium dioxide (anatase), refractive index, dispersion, transmission and absorption coefficients.

DOI: 10.61011/TP.2025.01.60522.210-24

Introduction

The optical, electrophysical, and photocatalytic properties of nanoscale films of titanium dioxide TiO_2, as well as titanium dioxide films doped with impurity atoms, have been intensively studied in recent years due to its unique characteristics. Impurity atoms are known to change the electrophysical and optical properties of semiconductors and dielectrics, for example, the spectral characteristics of photoconductivity, reflection and absorption coefficients [1]. This allows directional control of the optical, electrophysical, and photocatalytic properties of titanium dioxide films.

It was found in [2] that TiO_2 doping with carbon and vanadium impurities leads to the appearance of C and V impurity states in the band gap, resulting in an increase of the photocatalytic activity of anatase. Ref. [3] studied the properties of TiO_2 thin films doped with silver nanoparticles for translucent coatings and transparent contacts for fabrication of solar cells. It was shown in Ref. [3] that the concentration of silver nanoparticles affects the refractive index, conductivity, and thickness of TiO_2–Ag films. Titanium dioxide films modified with cobalt exhibit magnetic properties according to Ref. [4]. It was shown in Ref. [5] that the doping of titanium dioxide films with manganese oxide makes it possible to directionally change the optical properties of TiO_2. The effect of annealing in argon on the electrical and photoelectric characteristics of the structure TiO_2–silicon substrate was studied in [6,7].

A number of technological methods are used for the fabrication of TiO_2 thin films, in particular, reactive magnetron sputtering, electron beam evaporation, vapor-phase deposition, pulverization followed by pyrolysis, and thermal decomposition of the organotitanium precursor. Moreover, the optical properties of titanium dioxide films depend on the method of their preparation [2–12]. Experimental band gap width E_g of titanium dioxide (TiO_2, anatase) lies in the range of 3.18–3.43 eV [2,5,8], therefore titanium dioxide has high transparency in the visible region of the spectrum. The dependence of the refractive index and the band gap on the thickness of the titanium dioxide film is shown in Ref. [10]. The mismatch of the crystal lattices (atoms, ions) at the interface between the titanium dioxide film and the substrate may be one of the reasons for this. The mismatch of crystal lattices in the heterocontact creates mismatches at the interface of voltage and dislocation, which leads to the formation of surface levels at the boundary of the heterocontact of „surface levels“ and a change of the properties of the dielectric — the structure of energy bands, the band gap width [13].

The purpose of this study is to obtain crystalline titanium dioxide films of various thicknesses by thermal decomposition of an organotitanium precursor on a glass substrate for use in photocatalysis, and to determine the dispersion of the refractive index and absorption coefficient of the film.

1. Experimental results

Thin layers of titanium dioxide on a transparent substrate were obtained by the method of activated chemical decomposition of a film of an organotitanium precursor

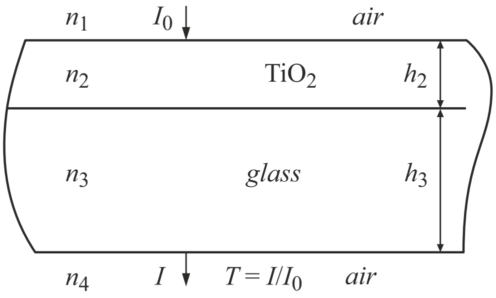

Figure 1. Air–glass substrate–titanium oxide–air system.

under the heat impact. A microscope cover glass with a thickness of $h = 0.18$ mm was a transparent substrate. Films of polyethoxytitane, which was prepared from 5–7% titanium chloride solution, were used as a precursor. The titanium dioxide films was formed on a glass substrate by heating of polyethoxytitane film to a temperature of 773 K in SNOL 15/1300 muffle furnace, followed by complete cooling of the furnace. After the thermal decomposition of the organotitanium precursor, the resulting titanium dioxide film was firmly fixed on the surface of the substrate. The characteristic heating time of, for example, a glass substrate was estimated using the expression $t_0 \sim h_3^2/a_3$, where $a_3 \approx 0.38 \cdot 10^{-6}\,\mathrm{m^2/s}$ — the thermal conductivity coefficient of the glass is $\sim 10^{-1}$ s. Therefore, no thermoelastic stresses occur in the titanium dioxide film–glass substrate system.

The film thickness was increased by re-applying the organotitanium precursor to the resulting titanium oxide film and subsequent heating in a muffle furnace to 773 K and cooling [11]. Thus, three-layer and five-layer titanium dioxide films on a glass substrate were obtained. X-ray phase analysis has shown that a homogeneous crystalline phase of anatases is formed using this method of producing titanium dioxide films of various thicknesses [11,12], i.e. there are no boundaries between the applied layers.

An attempt to obtain titanium dioxide films by ultraviolet photolysis of polyethoxytitane films according to X-ray phase analysis showed that TiO_2 is formed only in the amorphous state. Moreover, TiO_2 layers are not strong without heat treatment and repeated application of the polyethoxytitane film dissolves the previous layer. The transmittance spectra of the substrate and the substrate–titanium dioxide system were recorded using two-beam UV-2550 spectrophotometer from Shimadzu in the wavelength range of 190–900 nm [11].

Figure 1 shows the air–glass substrate–titanium dioxide–air system. The following notations are introduced here: n_1, n_4 — refractive indices of air equal to one; n_2 — refractive index of titanium dioxide film (anatase phase); n_3 — refractive index glass plates.

The spectral dependences of the transmittance coefficients $T(\lambda)$ of the glass substrate–titanium dioxide system are shown in Fig. 2–4. Fig. 2 (curve *3*) also shows the spectral dependence of the transmittance coefficient of a glass substrate.

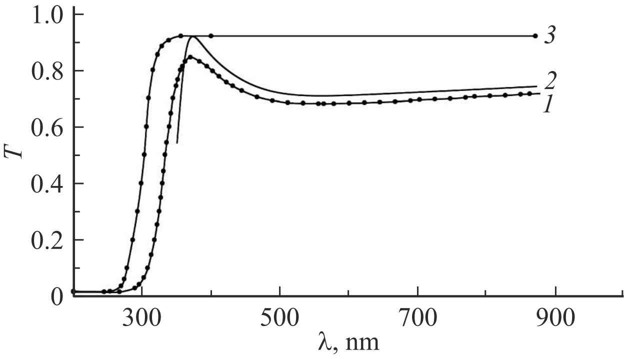

Figure 2. Spectral dependence of the transmittance coefficients of the glass substrate–TiO_2 single-layer film system (curves *1* and *2*) and glass substrate (curve *3*): *1* — experimental curve; *2* — theoretical curve calculated by formula (3) [12].

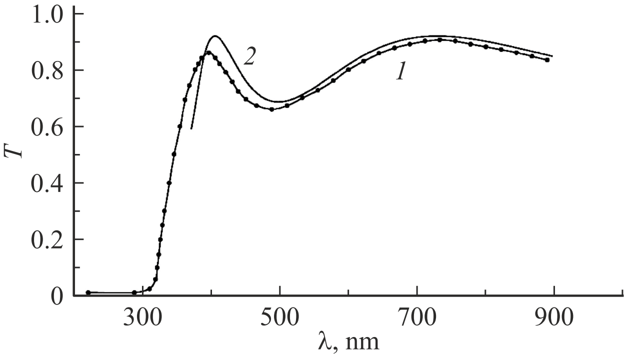

Figure 3. Spectral dependence of the transmittance coefficients of the glass substrate–TiO_2 three-layer film system (curves *1* and *2*): *1* — experimental curve; *2* — a theoretical curve calculated using the formula (3) [12].

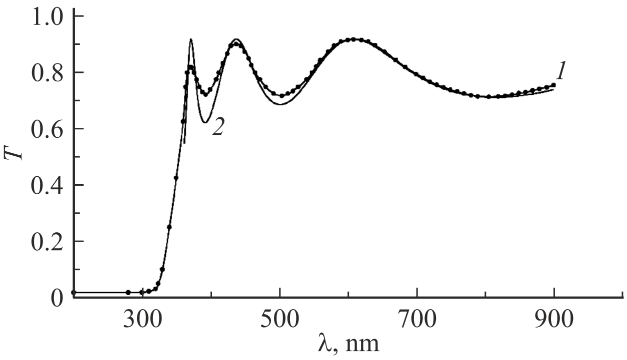

Figure 4. Spectral dependence of the transmittance coefficient of the glass substrate– TiO_2 five-layer film system (curves *1* and *2*): *1* — experimental curve; *2* — a theoretical curve calculated using the formula (3) [12].

The transmittance spectrum of the glass substrate–TiO_2 single-layer film system (Fig. 2, curves *1* and *2*) has one maximum and one minimum in the wavelength range of $340.0 \leq \lambda \leq 900.0$ nm. The transmittance of the glass substrate in a given spectral range $T_0 = 0.919$.

Fig. 3 shows the transmittance coefficient spectrum of the glass substrate–three-layer titanium dioxide film system. The transmittance spectrum of this system (curves *1* and *2*) already has two maxima and one minimum in the wavelength range of $340.0 \leq \lambda \leq 900.0$ nm.

Fig. 4 shows the transmittance spectrum of a glass substrate–five-layer titanium dioxide film system. The experimental spectral curve of the transmittance coefficient (1) already has four distinct alternating highs and lows as can be seen from Fig. 4. The maxima and minima of the transmittance coefficient of the transparent substrate–TiO_2 system are attributable to the interference of light on the titanium dioxide film. As can be seen from Fig. 2–4, an increase of the thickness of TiO_2 film leads to an increase of the number of maxima and minima on the spectral dependences of the transmittance coefficient.

2. Calculation results

Let us analyze the experimental dependences of the transmittance coefficients shown in Fig. 2–4 using the paper of A.S. Valeev [14]. A.S. Valeev considered a case that is often implemented in practice when a weakly absorbing film is deposited on a transparent substrate [15].

First, we will determine the refractive index of the glass substrate. The transmittance coefficient of a glass plate T_0 located in air with refractive indices $n_1 = n_4 = 1$ (Fig. 1) disregarding absorption and with the condition that the thickness of the glass substrate $h_3 \gg \lambda$ is determined by the formula [14]:

$$T_0 = \frac{T_{34}^2}{1 - R_{34}^2} = \frac{2n_3}{n_3^2 + 1}, \tag{1}$$

where T_{34}, R_{34} are the transmittance and reflection coefficients on one surface of the glass substrate. These coefficients are determined by the formulas [14]:

$$T_{34} = \frac{4n_3}{(n_3 + 1)^2}, \quad R_{34} = \left(\frac{n_3 - 1}{n_3 + 1}\right). \tag{2}$$

The formula for determining the refractive index of a glass substrate in an air environment follows from the expression (1)

$$n_3 = \frac{1}{T_0} + \sqrt{\frac{1}{T_0^2} - 1}.$$

We obtain that the refractive index of the glass substrate is $n_3 = 1.517$ substituting the value $T_0 = 0.919$ into this formula.

Now let's consider a glass substrate system with a thin film of titanium dioxide applied to the surface. The transmittance coefficient of a glass substrate with a thin applied weakly absorbing layer, taking into account interference and at normal incidence of a light beam, is determined by the formula [14]:

$$T = \frac{T_{31}T_{34}}{1 - R_{31}R_{34}} = \frac{T_{34}}{1/T_{31} - R_{34}R_{31}/T_{31}}, \tag{3}$$

where T_{31} and R_{31} are the transmittance and reflection coefficients of the layer when light falls on the substrate layer. These coefficients are determined by the following formulas [14]:

$$\frac{1}{T_{31}} = \frac{1}{16n_2^2n_3}[(n_3 + n_2)^2(n_2 + 1)^2 \exp(\gamma_2) + (n_3 - n_2)^2 \times (n_2 - 1)^2 \exp(-\gamma_2) + 2(n_3^2 - n_2^2)(n_2^2 - 1)\cos\varphi_2 + 4k_2(n_3 + 1)(n_2^2 - n_3)\sin\varphi_2], \tag{4}$$

$$\frac{R_{31}}{T_{31}} = \frac{1}{16n_2^2n_3}[(n_3 - n_2)^2(n_2 + 1)^2 \exp(\gamma_2) + (n_3 + n_2)^2 \times (n_2 - 1)^2 \exp(-\gamma_2) + 2(n_3^2 - n_2^2)(n_2^2 - 1)\cos\varphi_2 - 4k_2(n_3 - 1)(n_2^2 + n_3)\sin\varphi_2]. \tag{5}$$

Here

$$\varphi_2 = \frac{4\pi n_2 h_2}{\lambda}, \quad \gamma_2 = \frac{4\pi k_2 h_2}{\lambda} = \alpha_2 h_2, \tag{6}$$

where h_2 is thickness of the applied film on the glass substrate; k_2 is film absorption index; α_2 is film absorption coefficient.

An expression for the transmittance coefficient extremes is obtained in [14] from the condition that

$$\frac{d}{d\varphi_2}\left(\frac{1}{T_{31}} - \frac{R_{31}}{T_{31}}R_{34}\right) = 0,$$

$$T_{ext} = \frac{A}{B\exp(\gamma_2) - (-1)^m C + D\exp(-\gamma_2)}, \tag{7}$$

where

$$A = 16n_2^2n_3, \quad B = (n_2 + 1)^3(n_2 + n_3^2),$$

$$C = 2(n_2^2 - 1)(n_2^2 - n_3^2), \; D = (n_2 - 1)^3(n_2 - n_3^2).$$

Expression (7) is obtained from the condition that $\sin\varphi_2 = 0$, and $\cos\varphi_2 = (-1)^m$, where m is the interference order. Therefore, for even m we have $T_{ext} = T_M$, and for odd m the transmittance coefficient is $T_{ext} = T_m$. Here T_m is the minimum bandwidth; T_M is the maximum bandwidth.

Let's first estimate the thickness of the five-layer film of TiO_2 using the expression

$$\frac{4\pi n_2(\lambda_m)h_2}{\lambda_m} = m\pi. \tag{8}$$

Hence we have

$$h_2 = \frac{m\lambda_m}{4n_2(\lambda_m)} = \frac{(m + 1)\lambda_{m+1}}{4n_2(\lambda_{m+1})}. \tag{9}$$

Let's assume that the refractive index dispersion is insignificant at large wavelengths [16], i.e. $n_2(\lambda_m) \approx n_2(\lambda_{m+1})$. With this in mind, the expression (9) has the form

$$m\lambda_m \approx (m+1)\lambda_{m+1}. \quad (10)$$

Hence, the interference order will be determined by the expression

$$m = \frac{\lambda_{m+1}}{\lambda_m - \lambda_{m+1}}. \quad (11)$$

We obtain $m = 3.012$ by substituting experimental data from Fig. 4 into (11): $\lambda_m = 810.5$ nm (minimum) and $\lambda_{m+1} = 608.5$ nm (maximum). As can be seen, the order of interference is practically the same as that of an integer. Therefore, the interference order is 3 for $\lambda_m = 810.5$ nm, and the interference order is $m = 4$ for $\lambda_{m+1} = 608.5$ nm.

In the case of weak light absorption, assuming $\gamma_2 \approx 0$, the formulas for T_M and $T_m(7)$ are simplified, and, according to (7), has the form [14]:

$$T_M = T_0 = \frac{2n_3}{n_3^2+1}, \quad T_m = \frac{4n_2^3 n_3}{(n_2^2+1)(n_3^2+n_2^2)}. \quad (12)$$

A comparison of $T_0 = 0.919$ and $T_M = 0.9182$ shows that these values are almost equal. This allows obtaining an expression from formula (12) for T_m to estimate the refractive index for the anatase film at $\lambda_3 = 810.5$ nm:

$$n_2 = \sqrt{\left[\frac{2n_3}{T_m} - \frac{1+n_3^2}{2}\right] + \sqrt{\left[\frac{2n_3}{T_m} - \frac{1+n_3^2}{2}\right]^2 - n_3^2}}. \quad (13)$$

The minimum transmittance is $T_m = 0.71278$ in the region of weak absorption at the light wavelength of $\lambda = 810.5$ nm according to experimental data. Substituting the numerical values for $T_m = 0.71278$ and $n_3 = 1.517$ into formula (13), we obtain that the refractive index of the titanium dioxide film is $n_2 = 2.174$ for $\lambda = 810.5$ nm. This allows calculating the thickness of a five-layer titanium dioxide film from the expression (9):

$$h_2 = \frac{m\lambda}{4n_2} = \frac{3 \cdot 810.5}{4 \cdot 2.172} \approx 279.87\,\text{nm}.$$

Since the film thickness is now known, we obtain an expression for calculating the refractive index at extreme points using the formula (8)

$$n_2(\lambda_m) = \frac{m\pi\lambda_m}{4\pi h_2}. \quad (14)$$

A single-oscillator model was used in [10] to approximate the dependence of the refractive index on the wavelength. The dependence of the refractive index on the wavelength has a monotonous decline according to this model. The titanium dioxide film in Ref. [10] was obtained by magnetron sputtering of a titanium cathode. The titanium dioxide film is obtained by a chemical method in our case. Although the numerical dependences of the refractive indices in our work and in Ref. [10] are similar, however, the qualitative dependences $n_2(\lambda)$ differ. As already noted, the optical properties of titanium dioxide films depend on the method of their production.

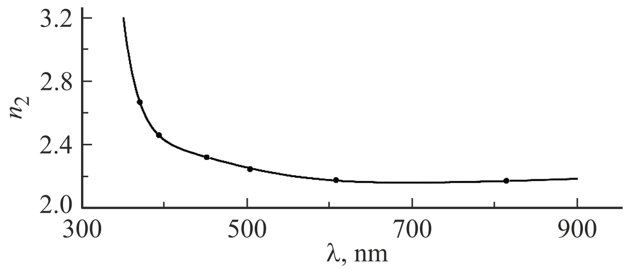

Figure 5. Spectral dependence of the refractive index of a titanium dioxide film on wavelength.

We obtain an empirical formula for approximating the dependence of the refractive index on the wavelength, using the Cauchy series [17]:

$$n_2 = a_0 + \frac{a_1}{\lambda^2} + \frac{a_2}{\lambda^4} + \frac{a_3}{\lambda^6} + \frac{a_4}{\lambda^8} + \frac{a_5}{\lambda^{10}}. \quad (15)$$

The number of coefficients of the Cauchy series was determined from the number of refractive indices at extreme points. We wrote down a system of six linear equations for the coefficients a_0 and a_i $(i = 1 \ldots 5)$ based on equation (15), which was solved by the Gauss method [18]. As a result, the following coefficient values were obtained: $a_0 = 2.2554$, $a_1 = -0.2208 \cdot 10^{-1}\,\mu\text{m}^2$, $a_2 = -0.6601 \cdot 10^{-1}\,\mu\text{m}^4$, $a_3 = 0.3963 \cdot 10^{-1}\,\mu\text{m}^6$, $a_4 = -0.7174 \cdot 10^{-2}\,\mu\text{m}^8$, $a_5 = 0.4363 \cdot 10^{-3}\,\mu\text{m}^{10}$.

The dependence of the refractive index of a five-layer titanium dioxide film on the wavelength is calculated using the formula (15) (Fig. 5). As can be seen from Fig. 5, the refractive index does not depend on the wavelength at $\lambda \geq 600$ nm. The wavelength-independent refractive index is observed, for example, for ionic crystals CsF, CsBr, KI, and KBr [16]. Knowing the spectral dependence of the refractive index of the titanium dioxide film, it was calculated using the formula (3) spectral dependence of the transmittance coefficient on the wavelength for a five-layer film at $\gamma_2 = 0$ (Fig. 4, curve *2*). Fig. 4 shows the coordinates λ_m for the experimental and calculated values T_M and T_m coincided, and the smaller the ratio h_2/λ, the better the agreement of the calculated values of the transmittance coefficient (curve *2*) with the experiment. Thus, using experimental data on the interference of the transmittance coefficient, the thickness of the titanium dioxide film and the dispersion of the refractive index were determined.

The absorption coefficient α_2 of the titanium dioxide film can be calculated at extreme points from the transformed expressions (7) for maxima T_M and minima T_m of transmittance coefficients:

$$\alpha_2 = \frac{1}{h_2} \ln\left[\frac{1}{2B}\left(\frac{A}{T_M} + C\right) + \sqrt{\frac{1}{4B^2}\left(\frac{A}{T_M} + C\right)^2 - \frac{D}{B}}\right], \quad (16)$$

$$\alpha_2 = \frac{1}{h_2} \ln\left[\frac{1}{2B}\left(\frac{A}{T_m} - C\right) + \sqrt{\frac{1}{4B^2}\left(\frac{A}{T_m} - C\right)^2 - \frac{D}{B}}\right], \quad (17)$$

The results of calculations of the absorption coefficient α_2 are shown in Fig. 6 and 7.

Fig. 6 shows the results of calculating the absorption coefficient of TiO_2 film according to the formula (16) in the range of maxima of transmittance coefficient T_M from the energy of the light quanta. As can be seen from Fig. 6, the experimental points fit well into the exponential dependence in the energy range of 2.0–3.5 eV. In addition, this figure shows points taken on an envelope drawn through points T_M using a scale at λ equal to 393.0, 504.0 and 810.5 nm. As can be seen from Fig. 6, these points lie quite close to the straight line drawn through the experimental maxima points of transmittance coefficient T_M.

The spectral dependence α_2 is determined by the expression

$$\alpha_2 = A_0 \exp(h\nu/\Delta_0), \quad (18)$$

where $A_0 = 2.955\,\mathrm{m}^{-1}$, $\Delta_0 = 0.2865\,\mathrm{eV}$. A similar dependence on energy for α_2 is observed in Ref. [19] for

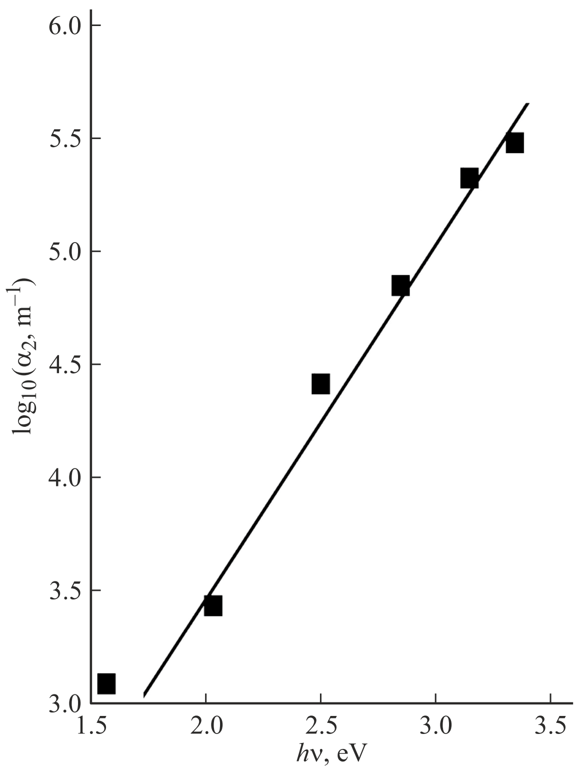

Figure 6. Spectral dependence of the absorption coefficient of a titanium dioxide film in the range of maxima of the spectral transmittance curve.

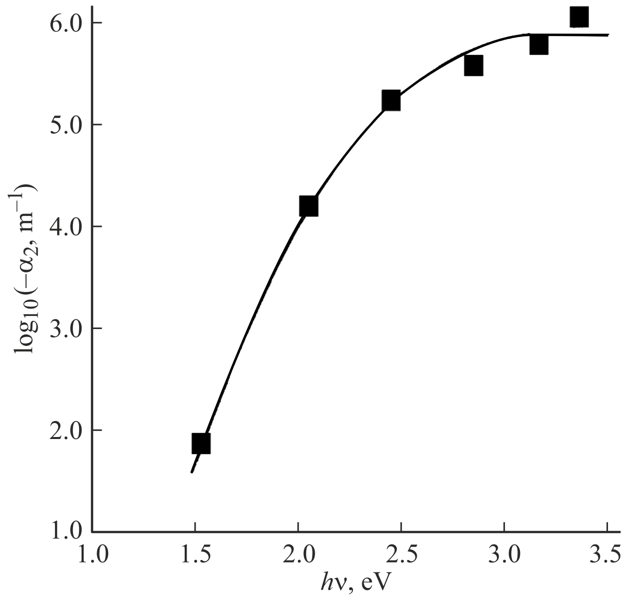

Figure 7. Spectral dependence of the „absorption coefficient" of the titanium dioxide film in the region of the minima of the spectral transmittance curve.

0.5–0.7 mm thick CdTe plate with an admixture of chlorine. The parameter Δ_0 characterizes the length of the tails of the density of states inside the band gap. The parameter Δ_0 is considered in Ref. [20] as a measure of semiconductor disordering. For example, $\Delta_0 \approx 0.067\,\mathrm{eV}$ for GeTe, which is significantly less than for TiO_2 film. The expression (18) can also be represented as

$$\alpha_2 = \alpha_0 \exp\left(-\frac{E_g - h\nu}{\Delta_0}\right),$$

where $\alpha_0 = A_0 \exp(E_g/\Delta_0)$, E_g is the band gap of titanium dioxide.

When calculating the spectral dependence of the transmittance coefficient, it was assumed that $\gamma_2 = 0$. According to [13], We obtain $\alpha_2 \approx 0.32 \cdot 10^6\,\mathrm{m}^{-1}$ $\gamma_2 = \alpha_2 h_2$. for $\lambda = 371\,\mathrm{nm}$. From here

$$\gamma_2 = \alpha_2 h_2 \approx 0.32 \cdot 10^6 \cdot 279.0 \cdot 10^{-9} \approx 9.0 \cdot 10^{-2} \ll 1.$$

Thus, the calculations for $\gamma_2 = 0$ and $\sim 9.0 \cdot 10^{-2}$ will differ by about 10%. The effect of absorption on T_M can be ignored in the region of longer wavelengths.

Fig. 7 shows the results of calculations of the absorption coefficient of TiO_2 film according to the formula (17) in the region of the minima of transmittance coefficient T_m depending on the energy of the light quanta. Calculations have shown that in the region of minima, the absorption coefficient is $\alpha_2 < 0$, i.e., the effect of „brightening" of a five-layer titanium dioxide film is observed. In addition, this figure shows the points taken on the envelope drawn through the experimental points T_m at λ equal to 371.0, 437.0 and 608.5 nm. The envelope drawn through the experimental points T_m is practically a straight line. The calculated curve $T(\lambda)$ in the minimum region, even without taking into account light absorption, passes below the experimental curve, and, as already noted, the longer the wavelength of light, the better the calculated curve agrees with the experiment.

What could be the reason for the „clarification" of five-layer TiO_2 film? If we assume that the surface of the titanium dioxide film has a roughness, this will lead to a change of the angular distribution of the luminous flux, i.e. to the scattering of light rays at the film–air interface and refraction of scattered rays in the film itself. When light rays are reflected from the film–glass substrate interface, the light rays are partially reflected from the film–air interface. If the angle of incidence of the light rays on the film–air interface inside the film is greater than the critical angle of total reflection β_C, then the light rays will be completely reflected from this surface and pass through the glass substrate, which will lead to the effect of illumination. The critical angle of total reflection from the interface is determined from the expression [17]:

$$\sin\beta_C = n_1/n_2,$$

which means that $\beta_C \sim 22.0^\circ - 27.0^\circ$. Thus, the angle of total internal reflection in the film depends on the dispersion of the refractive index. Since the angle β_C is not too big, the clarification effect is quite possible.

Let's estimate the thickness of a three-layer titanium dioxide film. According to Fig. 3, the first minimum of the transmittance coefficient is at $\lambda = 488.0$ nm. The second maximum of the transmittance coefficient is at $\lambda = 729.0$ nm. The refractive index changes insignificantly in this region of the spectrum, according to Fig. 5. Therefore, using the expression (11), we get that $m = 2.025$. Therefore, the interference index is $m = 2$ for $\lambda = 729.0$ the nm, while the interference index is 3 for $\lambda = 488.0$ nm. Further, using the formula (9) and the refractive index obtained for a five-layer film at $\lambda = 729.0$ nm, we obtain that the thickness of the three-layer film is $h_2 \approx 168.0$ nm.

The first maximum of the transmittance coefficient is at $\lambda = 370.5$ nm for a single-layer film, and the first minimum is at $\lambda = 563.0$ nm. The refractive index changes by about one and a half times in this region of the spectrum, according to Fig. 5. Therefore, the interference index can be estimated from the formula (9):

$$\frac{m\lambda}{4n_2(\lambda_m)} = \frac{(m+1)\lambda_{m+1}}{4n_2(\lambda_{m+1})},$$

where the refractive indices are calculated by the formula (9) for a five-layer TiO_2 film. Since the refractive indices for these wavelengths are known, we obtain after rounding $m = 1$, and the thickness of the single-layer film $h_2 \approx 70.0$ nm.

As can be seen from Fig. 2 and 3, the calculated and experimental values of the first maxima of the transmittance coefficients are shifted relative to each other. This is attributable to the fact that the refractive index depends on the thickness of TiO_2 film [10], and the lower the ratio h_2/λ, the better the agreement of the calculated values of the transmittance coefficient (curves *2*) with the experiment. Using the example of a three-layer TiO_2 film, we will show the dependence of the refractive index on the thickness. According to calculations using the formula (9), at the extreme points of the transmittance coefficient at $\lambda = 397.5$, 488.0 and 727.0 nm, the refractive indices of the film are respectively equal to $n_2 = 2.44$, 2.27 and 2.16. As can be seen from Fig. 3, the calculated and experimental values of the transmittance coefficient maxima coincide satisfactorily at $\lambda = 720.0$ nm. Therefore, let's assume that the experimental value is $n_2 = 2.16$. Next, using the formula (9) written as

$$n_2(\lambda_{m+1}) = \frac{(m+1)\lambda_{m+1}}{m\lambda_m} n_2(\lambda_m)$$

, we calculate the refractive index at $\lambda = 397.5$ and 488.0 nm. As a result, we obtain $n_2 = 2.36$ and 2.17, respectively. As can be seen from the calculation results, a decrease of the thickness of TiO_2 film results in a decrease of the refractive index, which is consistent with Ref. [7].

According to Figures 2 and 3, no clarification effect is observed in single- and three-layer titanium dioxide films. This may be attributable to the fact that the roughness of these films is less than the roughness of a five-layer film. In addition, a decrease of the refractive index n_2 leads to an increase of the angle of total internal reflection β_C.

Conclusion

Spectral dependences of transmittance coefficients for one-, three-, and five-layer titanium dioxide films on a transparent substrate are measured. The spectral dependences of the transmittance coefficients on the wavelength of the light of the spectrophotometer have pronounced alternating maxima and minima due to interference. The application of A.S.Valeev's work to the analysis of transmittance spectra made it possible to estimate the thickness of titanium dioxide films and calculate the transmittance spectrum. It is shown that the lower the ratio of the thickness of TiO_2 film to the wavelength, the better the calculated transmittance spectrum agrees with the experimental one.

An empirical formula based on the Cauchy series for the refractive index was obtained for a titanium dioxide film with a thickness of 279.87 nm. The refractive index of TiO_2 film in the range of $550.0 < \lambda < 900.0$ nm does not depend on wavelength. It is shown that as the thickness of the titanium dioxide film decreases, the refractive index decreases.

The spectral dependence of the light absorption coefficient of TiO_2 film is calculated from the maxima of the transmittance coefficient T_M. The light absorption coefficient exponentially depends on the energy of the light quanta. A clarification effect was detected in the region of minima

T_m. It is assumed that this effect is attributable to the total internal reflection of light rays in the film from its outer surface due to angular light scattering.

Funding

The work was performed using the instruments of „CUC" of scientific equipment of „Kemerovo State University" and „CUC of Kemerovo Scientific Center of the Siberian Branch of the RAS".

Conflict of interest

The author declares that he has no conflict of interest.

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Translated by A.Akhtayamov