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Hybrid modes in the antiferromagnet|ferromagnet heterostructure

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The phenomenon of mode hybridization in the exchange-coupled two-layer antiferromagnet|ferromagnet structure has been theoretically investigated. Expressions describing the dependence of the resonant frequencies of magnetization oscillations on the external magnetic field have been obtained by the method of Hamiltonian formalism. The effect of the coupling strength between the ferro- and antiferromagnetic layers on the width of the anti-crossing gap and on the magnitude of the external magnetic field at which hybridization is observed has been investigated.

Keywords: spintronics, ferromagnetic resonance, antiferromagnetic resonance, films, Hamiltonian formalism.

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1. Introduction

The possibility of using the transfer of spin (or magnetic moment) of electrons instead of transferring their charge (spintronics and magnonics) is currently studied for creation of a new class of devices for receiving, processing and transmitting information [1–3]. The active element in the prototypes of such devices is a layer of magnetically ordered material such as ferromagnet (FM) or antiferromagnet (AFM). Two-layer heterostructures of the antiferromagnet|ferromagnet type are also actively studied along with structures containing only one layer of magnetically ordered material. An exchange interaction occurs between the magnetic moments in the AFM and the magnetic moments in the FM in the interface layer in such heterostructures. It leads to the appearance of unidirectional anisotropy in FM [4–7], which is manifested in the so-called exchange bias. The presence of an exchange bias leads to a change of the width, shape and position of the magnetization curve $M(H)$ of the FM layer of the heterostructure [8,9], that is, the AFM layer acts as an element modifying the properties of the FM layer.

However, the interaction between the magnetic moments in AFM and FM also leads to hybridization of the homogeneous resonance modes in AFM and FM in addition to the exchange bias. The mode hybridization phenomenon constitutes a change of the nature of resonant oscillations of magnetization in two connected magnetic layers relative to these oscillations in unrelated layers due to mutual influence. This phenomenon has been observed for magnetics, for example, in case of the exchange interaction in synthetic AFM. A synthetic AFM consisting of two antiparallel magnetized ferromagnetic CoFeB layers separated by a thin metal layer was experimentally studied in Ref. [10]. It has been shown that the size of the gap between the acoustic and optical branches of ferromagnetic resonance can be controlled by changing the direction of the applied magnetic field. A similar pattern is observed in Ref. [11], where the AFM $CrCl_3$ is experimentally studied, whose sublattices can be considered as ferromagnetic layers under certain conditions. The excited acoustic and optical resonance branches hybridize with each other as a result of the exchange coupling between the layers. The hybridization was observed in Refs. [12,13] when studying the magnon-photon interaction of magnons in FM with ultrahigh-frequency photons. It is shown in Ref. [14] that magnetization oscillations in FM can interact with magnetization oscillations in AFM indirectly through coupling with the electromagnetic mode of the resonator. At the same time, hybridization of the resonator mode and the modes of ferro- and antiferromagnetic resonance is also observed. AFM|FM heterostructure (Mn_2Au|Py) was theoretically and experimentally studied in Ref. [15], and it was shown that the frequency of the spin-wave resonance mode in the FM layer changes due to the exchange interaction between AFM and FM.

In order to observe the hybridization of resonant modes in magnetic heterostructures, it is necessary to have an exchange interaction between the layers, which is achieved by breaking rotational symmetry relative to the direction of the external magnetic field [11]. Such symmetry breaking can be introduced into the structure in various ways, for example, using ferromagnets with different saturation magnetizations [16], with different thicknesses [17], or applying an additional magnetic field outside the plane [11]. The purpose of this work is a theoretical study of mode hybridization in a two-layer AFM|FM heterostructure.

2. Mathematical model

We assume that in the considered heterostructure, the magnetic moments of the FM involved in the exchange coupling with the magnetic moments of the AFM of both sublattices. In our case, the breaking of rotational symmetry is caused by the difference in saturation magnetization of FM and AFM. The considered heterostructure is a thin layer of antiferromagnet with easy-plane anisotropy, on which a thin layer of ferromagnet is applied. The entire heterostructure is placed in an external dc magnetic field, as shown in Figure 1.

To solve this problem, it is sufficient to consider a conservative system, so that Gilbert damping can be neglected. The dynamics of the magnetization vectors of the sublattices of AFM $\mathbf{M}_{1,2}$ and the magnetization vector of FM $\mathbf{M}_3$ are described by the Landau-Lifshitz equations

$$\frac{\partial \mathbf{M}_j}{\partial t} = \gamma \mathbf{M}_j \times \frac{\partial W}{\partial \mathbf{M}_j}, \tag{1}$$

where $j = 1, 2, 3$, γ is gyromagnetic ratio modulus, W is the total energy of the AFM|FM heterostructure:

$$\begin{aligned} W = &-\mu_0 \mathbf{H}(\mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3) - \frac{\mu_0 H_\mathrm{e}}{2M_\mathrm{s}}\big((\mathbf{M}_1\mathbf{y})^2 + (\mathbf{M}_2\mathbf{y})^2\big) \\ &+ \frac{\mu_0 H_\mathrm{h}}{2M_\mathrm{s}}\big((\mathbf{M}_1\mathbf{z})^2 + (\mathbf{M}_2\mathbf{z})^2\big) + \frac{\mu_0 H_\mathrm{ex}}{M_\mathrm{s}}\mathbf{M}_1\mathbf{M}_2 \\ &+ \frac{\mu_0}{2}(\mathbf{M}_3\mathbf{z})^2 + \frac{\mu_0 H_\mathrm{c}}{M_\mathrm{s}}(\mathbf{M}_1 + \mathbf{M}_2)\mathbf{M}_3, \end{aligned} \tag{2}$$

where μ_0 is the vacuum magnetic permeability, M_s is the saturation magnetization of AFM, $\mathbf{H} = H\mathbf{y}$ is the external magnetic field vector, $H_\mathrm{e,h}$ is the effective fields of magnetocrystalline anisotropy along the easy and hard axes, respectively, H_ex is the effective field of exchange interaction between AFM sublattices, H_c is the effective field of exchange interaction between AFM and FM. The demagnetizing field in the AFM is neglected due to its smallness. The Cartesian coordinate system is chosen in such a way that the equilibrium state of magnetization is directed along the **y**-axis. Let us represent the magnetization vector as the sum of vectors defining the equilibrium state and small oscillations around it as $\mathbf{M}_j = M_{jy}\mathbf{y} + m_{jx}\mathbf{x} + m_{jz}\mathbf{z}$. In this case, the normalization condition $M_{jy}^2 + m_{jx}^2 + m_{jz}^2 = 1$ is satisfied.

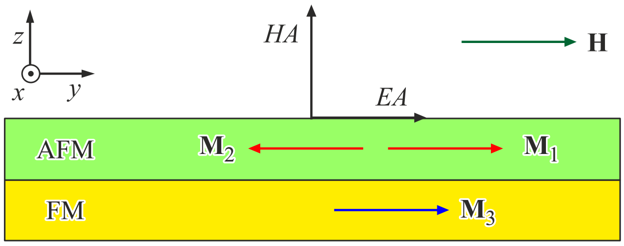

Figure 1. Diagram of the studied heterostructure. $\mathbf{M}_{1,2}$ is the magnetization vectors of the first and second AFM sublattices, $\mathbf{M}_3$ is the magnetization vector of FM, *EA* and *HA* are axes of easy and hard magnetization of AFM, **H** is the external magnetic field vector aligned with the easy axis.

3. Analytical calculation

It is convenient to analyze the considered system using the Hamiltonian formalism [18,19], which is well suited for analyzing both homogeneous and inhomogeneous magnetization oscillations in FM and AFM [14,20–22]. This requires a transition from the Landau-Lifshitz equations to the Hamilton equations. Let us introduce new complex variables $a_j = a_j(\mathbf{M}_j)$ for this transition which are related to the amplitude of magnetization oscillations, proposed by Holstein and Primakoff [23].

$$a_j = \frac{m_{jz} \pm i m_{jx}}{\sqrt{M_\mathrm{s} \pm M_{jy}}}, \tag{3}$$

where $j = 1, 2$, and the projections of the magnetization vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ obtained by the inverse transformation have the form

$$M_{jy} = \pm M_\mathrm{s} \mp |a_j|^2, \tag{4}$$

$$m_{jx} = \mp i\,\frac{1}{2}\sqrt{2M_\mathrm{s} - |a_j|^2}\,(a_j - a_j^*), \tag{5}$$

$$m_{jz} = \frac{1}{2}\sqrt{2M_\mathrm{s} - |a_j|^2}\,(a_j + a_j^*). \tag{6}$$

Expressions for the vector $\mathbf{M}_3$ are obtained from (3)–(6) by replacing the index 1 with 3 and the constant M_s with M_sF, where M_sF is the saturation magnetization of FM.

Substituting (4)–(6) into (2), we can write the Hamiltonian of the system as a series in powers of variables a_j. Since the problem of finding the frequencies of a homogeneous resonance is solved without taking into account nonlinearity, we will keep only the quadratic terms [18,19]. Then the Hamiltonian has the form

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3, \tag{7}$$

$$\begin{aligned} \mathcal{H}_1 = &A_1|a_1|^2 + A_2|a_2|^2 + B_1(a_1a_2 + a_1^*a_2^*) \\ &+ \frac{1}{2}C_1(a_1^2 + a_1^{*2} + a_2^2 + a_2^{*2}), \end{aligned} \tag{8}$$

$$\mathcal{H}_2 = A_3|a_3|^2 + \frac{1}{2}C_2(a_3^2 + a_3^{*2}), \tag{9}$$

$$\mathscr{H}_3 = -\mu_0 H_c \frac{M_{sF}}{M_s}(|a_1|^2 - |a_2|^2) + B_2(a_1 a_3^* + a_1^* a_3 + a_2 a_3 + a_2^* a_3^*), \quad (10)$$

where the Hamiltonian coefficients are

$$A_1 = \mu_0\Big(H_e + \frac{H_h}{2} + H_{ex} + H\Big), \quad (11)$$

$$A_2 = \mu_0\Big(H_e + \frac{H_h}{2} + H_{ex} - H\Big), \quad (12)$$

$$A_3 = \mu_0\Big(H + \frac{M_{sF}}{2}\Big), \quad (13)$$

$$B_1 = \mu_0 H_{ex}, \quad (14)$$

$$B_2 = \mu_0 H_c \sqrt{\frac{M_{sF}}{M_s}}, \quad (15)$$

$$C_1 = \frac{\mu_0 H_h}{2}, \quad (16)$$

$$C_2 = \frac{\mu_0 M_{sF}}{2}. \quad (17)$$

The terms $\mathscr{H}_1$ and $\mathscr{H}_2$ in (7) describe a homogeneous resonance in the layers of AFM and FM, respectively, and the term $\mathscr{H}_3$ is the interaction between AFM and FM. The next step is to find expressions for the natural oscillations of magnetization in the AFM|FM system, which would have the form of equations for an autonomous conservative system of two coupled linear oscillators. $\mathscr{H}_1$ and $\mathscr{H}_2$ can be represented in matrix form and then diagonalized to find resonant frequencies. Let us use the method proposed in Ref. [24] for diagonalization, which is a generalization of the Bogolyubov transformations [25]. For the antiferromagnetic layer

$$\mathscr{H}_1 = \frac{1}{2}\hat{X}_1^* \begin{pmatrix} \hat{H}_1 & \hat{H}_2 \\ \hat{H}_2 & \hat{H}_1 \end{pmatrix} \hat{X}_1 = \frac{1}{2}\hat{Y}_1^* \begin{pmatrix} \hat{\omega} & \hat{0} \\ \hat{0} & \hat{\omega} \end{pmatrix} \hat{Y}_1, \quad (18)$$

where $\hat{X}_1 = \begin{pmatrix} a_1 \\ a_2 \\ a_1^* \\ a_2^* \end{pmatrix}$, $\hat{Y}_1 = \begin{pmatrix} b_1 \\ b_2 \\ b_1^* \\ b_2^* \end{pmatrix}$,

$$\hat{H}_1 = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}, \ \hat{H}_2 = \begin{pmatrix} C_1 & B_1 \\ B_1 & C_1 \end{pmatrix}, \ \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}.$$

For the ferromagnetic layer

$$\mathscr{H}_2 = \frac{1}{2}\hat{X}_2^* \begin{pmatrix} A_3 & C_2 \\ C_2 & A_3 \end{pmatrix} \hat{X}_2 = \frac{1}{2}\hat{Y}_2^* \begin{pmatrix} \omega_3 & 0 \\ 0 & \omega_3 \end{pmatrix} \hat{Y}_2, \quad (19)$$

where $\hat{X}_2 = \begin{pmatrix} a_3 \\ a_3^* \end{pmatrix}$, $\hat{Y}_2 = \begin{pmatrix} b_3 \\ b_3^* \end{pmatrix}$,

Next, let us perform a linear transformation of the old complex variables a_j into the new ones b_j to diagonalize the coefficient matrix for complex variables

$$\hat{X}_1 = \begin{pmatrix} \hat{S}_1 & \hat{S}_2 \\ \hat{S}_2 & \hat{S}_1 \end{pmatrix} \hat{Y}_1, \quad (20)$$

$$\hat{X}_2 = \begin{pmatrix} u & -v \\ -v & u \end{pmatrix} \hat{Y}_2, \quad (21)$$

where

$$\hat{S}_1 = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}, \ \hat{S}_2 = \begin{pmatrix} s_{13} & s_{14} \\ s_{23} & s_{24} \end{pmatrix}.$$

Orthonormal relations for transition matrices can be obtained based on the commutation relations $[a_j, a_j^*] = 1$, $[a_j^*, a_j] = -1$, $[a_j, a_j] = 0$ [19]

$$\begin{pmatrix} \hat{S}_1 & \hat{S}_2 \\ \hat{S}_2 & \hat{S}_1 \end{pmatrix}^{-1} = \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & -\hat{1} \end{pmatrix} \begin{pmatrix} \hat{S}_1 & \hat{S}_2 \\ \hat{S}_2 & \hat{S}_1 \end{pmatrix} \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & -\hat{1} \end{pmatrix}, \quad (22)$$

$$\begin{pmatrix} u & -v \\ -v & u \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u & -v \\ -v & u \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (23)$$

where $\hat{1}$ is the identity matrix. Let us find the natural frequencies by using the relations that can be obtained by substituting (20) and (21) into (18) and (19), and also considering (22)–(23) [19]:

$$\begin{pmatrix} \hat{H}_1 & \hat{H}_2 \\ \hat{H}_2 & \hat{H}_1 \end{pmatrix} \begin{pmatrix} \hat{S}_1 & \hat{S}_2 \\ \hat{S}_2 & \hat{S}_1 \end{pmatrix} = \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & -\hat{1} \end{pmatrix} \begin{pmatrix} \hat{S}_1 & \hat{S}_2 \\ \hat{S}_2 & \hat{S}_1 \end{pmatrix} \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & -\hat{1} \end{pmatrix} \begin{pmatrix} \hat{\omega} & \hat{0} \\ \hat{0} & \hat{\omega} \end{pmatrix}, \quad (24)$$

$$\begin{pmatrix} A_3 & C_2 \\ C_2 & A_3 \end{pmatrix} \begin{pmatrix} u & -v \\ -v & u \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u & -v \\ -v & u \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \omega_3 & 0 \\ 0 & \omega_3 \end{pmatrix}. \quad (25)$$

The solution of the system of equations (24)–(25) allows obtaining the following expressions for the lower and upper modes of antiferromagnetic resonance $\omega_{1,2}$ and for the ferromagnetic resonance mode ω_3:

$$\left(\frac{\omega_{1,2}}{\gamma}\right)^2 = \frac{1}{2}\Big(A_1^2 + A_2^2 - 2(B_1^2 + C_1^2) \mp \big((A_1^2 - A_2^2)^2 - 4(A_1^2 + A_2^2)^2 B_1^2 + 16 B_1^2 C_1^2\big)^{1/2}\Big), \quad (26)$$

$$\left(\frac{\omega_3}{\gamma}\right)^2 = A_3^2 - C_2^2. \quad (27)$$

Substituting (20) and (21) in $\mathscr{H}_3$, we also obtain an expression describing the interaction of AFM and FM in variables b_1, b_2 and b_3. As will be seen later, the ferromagnetic resonance mode does not hybridize with the upper antiferromagnetic mode, so we can discard terms that depend on b_2. Then the Hamiltonian in variables b_1 and b_3 will have the form

$$\gamma\mathscr{H} = (\omega_1 + c_1)|b_1|^2 + \omega_3|b_3|^2 + c_2(b_1^2 + b_1^{*2}) + c_3(b_1 b_3 + b_1^* b_3^*) + c_4(b_1 b_3^* + b_1^* b_3), \quad (28)$$

where the coefficients responsible for the coupling:

$$c_1 = -\gamma\mu_0 H_c \frac{M_{sF}}{M_s}(s_{11}^2 + s_{13}^2 - s_{21}^2 - s_{23}^2), \quad (29)$$

$$c_2 = -\gamma\mu_0 H_c \frac{M_{sF}}{M_s}(s_{11}s_{13} - s_{21}s_{23}), \quad (30)$$

$$c_3 = \gamma B_2\Big((s_{13} + s_{21})u - (s_{11} + s_{23})v\Big), \quad (31)$$

$$c_4 = \gamma B_2\Big((s_{11} + s_{23})u - (s_{13} + s_{21})v\Big). \quad (32)$$

Let us now write down the equations describing the dynamics of magnetization in the Hamiltonian formalism, to which we have moved from the Landau-Lifshitz equations. They will look as follows in variables b_1 and b_3

$$\frac{\partial b_1}{\partial t} = -i\gamma\frac{\partial\mathscr{H}}{\partial b_1^*} = -i(\omega_1 + c_1)b_1 - i(2c_2 b_1^* + c_3 b_3^* + c_4 b_3), \quad (33)$$

$$\frac{\partial b_3}{\partial t} = -i\gamma\frac{\partial\mathscr{H}}{\partial b_3^*} = -i\omega_3 b_3 - i(c_3 b_1^* + c_4 b_1). \quad (34)$$

Equations (33)–(34) are called the form of coupled oscillations [26]. Their solution gives the natural frequencies of a system of coupled oscillators. In this case, ω_1 and ω_3 are the partial frequencies of such a system, that is, the frequencies of individual oscillators (in our case, the AFM and FM layers) without regard to coupling. Let us look for a solution of (33)–(34) in the form of $b_j = b_j(0)e^{-i\omega t}$, $b_j^* = b_j^*(0)e^{-i\omega t}$ $(j = 1, 3)$. Let us find the determinant of the resulting system of algebraic equations. The characteristic equation is obtained by equating the determinant to zero

$$\begin{vmatrix} \omega_1 + c_1 - \omega & c_4 & 2c_2 & c_3 \\ c_4 & \omega_3 - \omega & c_3 & 0 \\ 2c_2 & c_3 & \omega_1 + c_1 - \omega & c_4 \\ c_3 & 0 & c_4 & \omega_3 - \omega \end{vmatrix} = 0. \quad (35)$$

Assuming that the interaction between AFM and FM is weak [26], we can neglect the coefficients c_2 and c_3 linking b to b^*. Then the characteristic equation takes the form

$$\begin{vmatrix} \omega_1 + c_1 - \omega & c_4 \\ c_4 & \omega_3 - \omega \end{vmatrix} = 0. \quad (36)$$

Finally, from (36) we can obtain expressions for the natural oscillation frequencies in the system of exchange-coupled AFM and FM

$$\omega_{a,b} = \frac{1}{2}\Big(\omega_1 + c_1 + \omega_3 \mp \big((\omega_1 + c_1 - \omega_3)^2 + 4c_4^2\big)^{1/2}\Big). \quad (37)$$

4. Results

Figure 2 shows the dependences of the natural oscillation frequencies $\omega_{a,b}$ on the external magnetic field

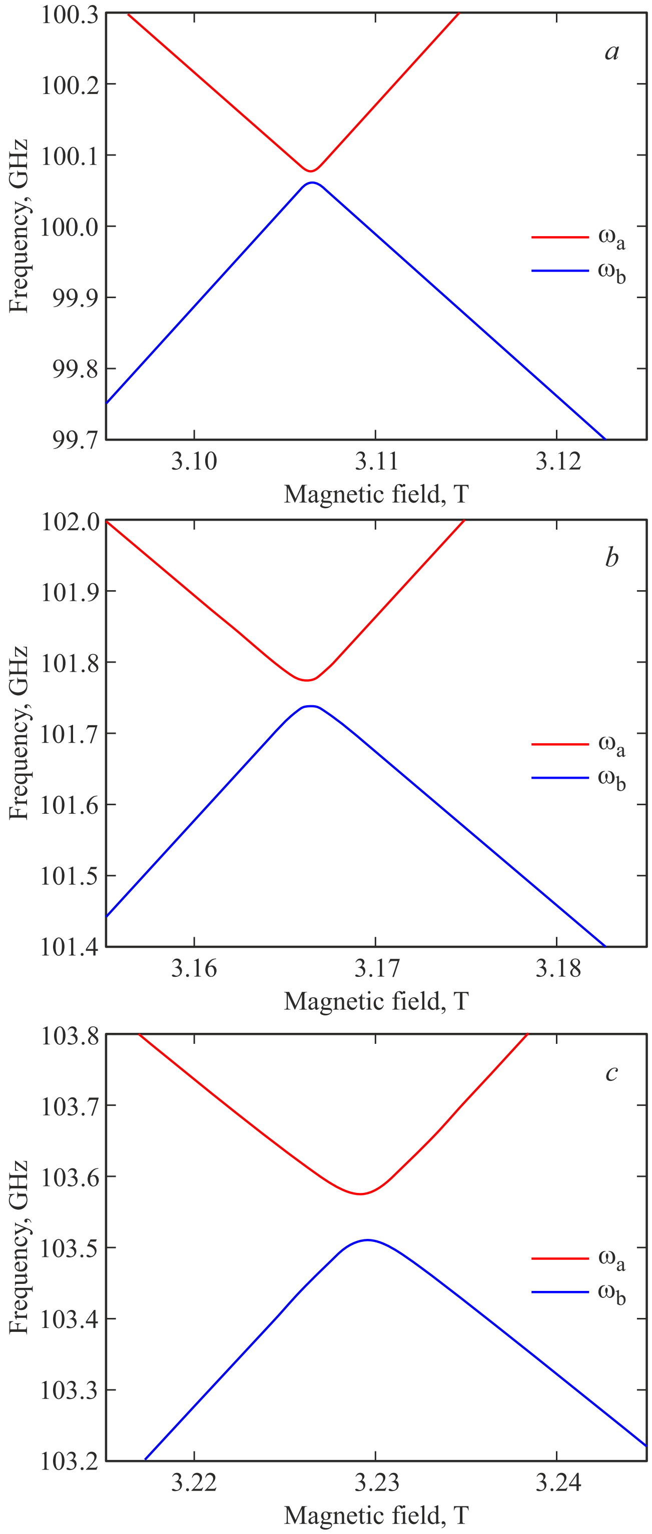

Figure 2. Dependences of the resonant frequency on the magnetic field for various effective coupling fields: *a* — $\mu_0 H_c = 0.1$ T; *b* — $\mu_0 H_c = 0.15$ T; *c* — $\mu_0 H_c = 0.2$ T.

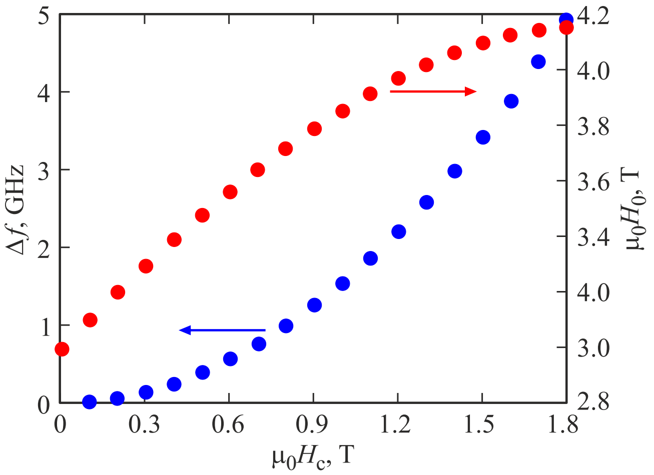

Figure 3. The dependence of the gap width and the external field at which hybridization is observed on the effective coupling field.

H at different values of the effective coupling field. These dependencies were built using the values of constants for nickel oxide NiO (AFM) and permalloy $Ni_{80}Fe_{20}$ (FM) $\gamma/2\pi = 28$ GHz/T, $M_{sF} = 800$ kA/m [27], $M_s = 351$ kA/m [28], $\mu_0 H_e = 0.011$ T [29], $\mu_0 H_h = 0.635$ T [29], $\mu_0 H_{ex} = 968.4$ T [29]. It is shown in [26] that the introduction of coupling into an autonomous conservative system of two oscillators should increase the interval between the natural frequencies of the linear system. Figure 2 shows that when the resonant frequencies of the two magnetic subsystems (antiferromagnetic and ferromagnetic) approach each other, a gap in the frequency spectrum is formed at the place of their intended intersection. It should be noted that the expression (37) is valid for a limited range of values H_c. Figure 3 shows the dependence of the width of the gap Δf and the external field H_0, at which hybridization is observed, on the effective coupling field H_c. With an increase in the effective coupling field, an expansion of the interval between modes is observed, as well as a strong shift of the hybridization region towards large magnetic fields. The values Δf and H_0 increase non-linearly with the increase in H_c.

5. Conclusion

A two-layer AFM|FM heterostructure is considered. It is shown that the exchange interaction between the magnetic moments of AFM and FM leads to hybridization of resonant modes. This phenomenon has been studied by the method of Hamiltonian formalism, and expressions have been obtained for the natural frequencies of the AFM|FM structure as a system of two coupled linear oscillators. The dependences of the gap width and the external field at which hybridization is observed on the effective coupling field are analyzed. The results are demonstrated using the nickel oxide|permalloy structure as an example.

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Conflict of interest

The authors declare that they have no conflict of interest.

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