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The Holtsmark model for a spin glass

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Impurity atoms with a magnetic moment are distributed rarely and randomly among the non-magnetic material in a spin glass. We suggest the Holtsmark model for description of spatial distribution of these atoms. Within the framework of the mean self-consistent field, the dependence of the spontaneous magnetic field on the temperature, as well as the critical temperature where this magnetic field disappears, have been found analytically.

Keywords: Holtsmark distribution, critical temperature, spontaneous magnetic field.

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1. Introduction

Spin glasses are distinguished by the possibility of a phase transition associated with the occurrence of a static spontaneous magnetic field below a certain critical temperature T_c [1]. This randomly directed field is created by chaotically directed magnetic moments of impurities in a non-magnetic material. The interaction between impurity centers can be neglected in case of their low concentration and the analysis is simplified. The system becomes uniformly ordered at temperature $T < T_c$. An accurate consideration of the problem is possible only in the (unrealistic) case of magnetic moments parallel to each other [2].

Let us use H_i to denote the magnetic field strength at the point where the ith impurity atom with spin 1/2 is located. Let us use μ to denote the magnetic moment of this atom. The energy of an atom in a magnetic field is split into two values: $\pm\mu H$. The statistical sum associated with the magnetic field has the form: $Z_i = \exp(-\mu H_i/T) + \exp(\mu H_i/T)$. Let us find the corresponding free energy:

$$F_i = -T \ln\left\{2\cosh\left(\frac{\mu H_i}{T}\right)\right\}, \tag{1}$$

Let us determine the average magnetic moment of a given ith impurity atom:

$$M_i = -\frac{\partial F_i}{\partial H_i} = \mu \tanh\left(\frac{\mu H_i}{T}\right). \tag{2}$$

This magnetic moment creates a magnetic field at the point j where some other impurity atom is located:

$$\mathbf{H}_j = \frac{3\mathbf{n}(\boldsymbol{\mu}\mathbf{n}) - \boldsymbol{\mu}}{R^3} \tanh\left(\frac{\mu H_i}{T}\right). \tag{3}$$

Here R is the distance between the atoms, $\mathbf{n}$ is the unit vector along the direction connecting the atoms. The magnetic field $\mathbf{H}_j$ decreases very rapidly as this distance increases. Therefore, it is possible to assume that R — this is actually the distance to the nearest neighbors.

The magnetic field (3) becomes zero in case of averaging over the corners of the vector $\boldsymbol{\mu}$. But it turns out that a large number of such magnetic moments, randomly located in space and having a random direction, create a well-defined magnetic field other than zero, although also randomly directed (nonergodic system).

Let's turn to the nonzero square of the magnetic field strength:

$$H_j^2 = \frac{\mu^2 + 3(\boldsymbol{\mu}\mathbf{n})^2}{R^6} \tanh^2\left(\frac{\mu H_i}{T}\right). \tag{4}$$

We obtain the following from (4) averaging it over the chaotically directed angles of the vector $\boldsymbol{\mu}$:

$$H_j^2 = \frac{2\mu^2}{R^6} \tanh^2\left(\frac{\mu H_i}{T}\right). \tag{5}$$

2. Nearest neighbor approximation

The magnetic field H_j is actually created not by a single atom i, but by a large number of such atoms in its vicinity. Let us determine the probability normalized by one $w(R)dR$ that the impurity atom closest to the ith atom is located at a distance of $[R, R + dR]$ from it. This probability is equal to the product of the probability that there are no atoms inside R and the probability of detecting one atom inside a thin spherical layer $[R, R + dR]$. So, we get the equation:

$$w(R)dR = \left\{1 - \int_0^R w(R')dR'\right\} 4\pi R^2 n dR. \tag{6}$$

Here n is the number of impurity atoms per unit volume. The differential equation follows from (6) in case of differentiation

$$\frac{d}{dR}\left(\frac{w(R)}{R^2}\right) = -4\pi n w(R). \tag{7}$$

The boundary condition to equation (7) follows from (6):

$$w(R \to 0) \to 4\pi R^2 n. \tag{8}$$

The solution of the equation (7) with the boundary condition (8) has the simple form:

$$w(R) = 4\pi R^2 n \exp\left(-\frac{4\pi R^3 n}{3}\right). \tag{9}$$

Determining the average value of the distance$R$:

$$\bar{R} = \int_0^\infty 4\pi R^2 n \exp\left(-\frac{4\pi R^3 n}{3}\right) dR$$

$$= \frac{1}{(4\pi n)^{1/3}} \int_0^\infty x^3 \exp\left(-\frac{x^3}{3}\right) dx = \frac{1.288}{(4\pi n)^{1/3}}; \quad n = \frac{0.17}{R^3}. \tag{10}$$

Substituting (10) into (5), we replace $R \to \bar{R}$ and obtain an implicit equation for determining a randomly directed static spontaneous magnetic field.:

$$H = 8.3\mu n \tanh\left(\frac{\mu H}{T}\right). \tag{11}$$

Let us reduce it to a dimensionless form by substitutions: $j = H/8.3\mu$n; $t = T/8.3\mu^2$n. We obtain the implicit equation:

$$h = \tanh\left(\frac{h}{t}\right). \tag{12}$$

Figure 1 shows the dependence of the magnetic field on temperature according to (12). The spontaneous magnetic field disappears at temperatures $t > 1$, $T > 8.3\mu^2$n $= \mu^2\sqrt{2}/\bar{R}^3$. We have $h = 1$, $H = H_0 = 8.3\mu$n $= \mu\sqrt{2}/\bar{R}^3$ at temperature $T = 0$.

The nearest neighbor approximation is close to, but not reducible to, the Holtsmark model. Both approximations coincide in the limit of small distances R, when the exponent in (9) can be replaced by one.

3. Gaussian approximation

Next, let us will consider another model: a Gaussian distribution for the random arrangement of rare impurities in space. It is usually used when describing spin glass [3–5].

Due to the random location of the impurity atoms, its preset value is determined by a normal (Gaussian) distribution. More precisely, according to the central limit theorem, the sum of identically distributed independent random variables has a normal distribution. A magnetic field, as the sum of magnetic fields from randomly located magnetic moments, can be described by a normal distribution or not, depending, for example, on the average distance between them. The correspondence of the distribution to the normal is simply postulated in the theory of spin glasses. This is done below.

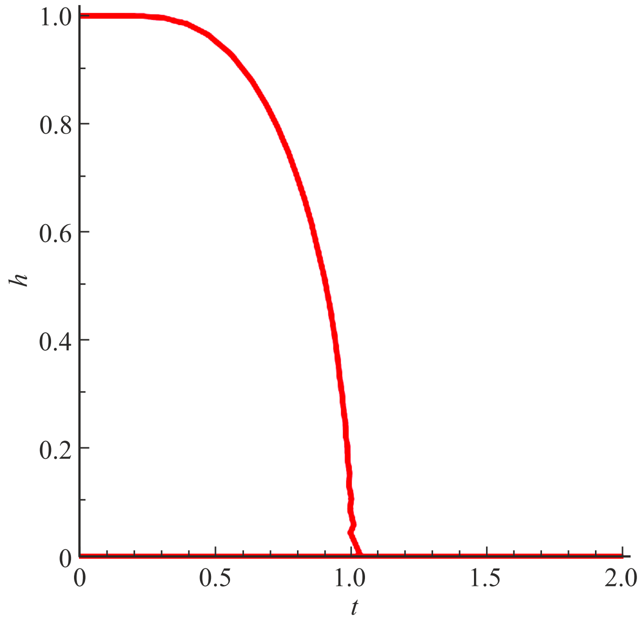

Figure 1. Spontaneous magnetic field as a function of temperature in the nearest neighbor model.

So, it is possible to write the following using (5):

$$\langle H^2 \rangle = H_0^2 \int_{-\infty}^{\infty} \frac{dH}{\sqrt{2\pi\langle H^2 \rangle}} \exp\left(-\frac{H^2}{2\langle H^2 \rangle}\right) \tanh^2\left(\frac{\mu H}{T}\right);$$

$$H_0 = \frac{\mu\sqrt{2}}{\bar{R}^3}. \tag{13}$$

Next, let us introduce dimensionless quantities: the mean-square dimensionless magnetic field $h = \sqrt{\langle H^2 \rangle / h_0^2}$, the dimensionless temperature $t = \frac{T}{\mu H_0}$, and the dimensionless integration variable $x = \frac{H}{\sqrt{\langle H^2 \rangle}}$. Let us obtain an integral equation for dimensionless quantities from (13):

$$h^2 = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tanh^2\left(\frac{hx}{t}\right). \tag{14}$$

This equation defines the implicit mean-square spontaneous magnetic field as a function of temperature. Both zero and non-zero solutions occur at $t < 1$ (Figure 2). A nonzero solution is implemented experimentally, since it corresponds to a lower energy of the spin system. We obtain $h = 1$, $\sqrt{\langle H^2 \rangle} = \mu\sqrt{2}/\bar{R}^3$ for $t = 0$. There is only a zero solution to equation (14) for $t > 1$. This can be seen from Figure 2. This can also be seen analytically if, for small h and $t = 1 + \delta$, $0 < \delta \ll 1$, the hyperbolic tangent is decomposed into a Taylor's series. We obtain the equation $h^2 = h^2/t^2$ from (14), which has only a zero solution. If $\delta < 0$, then a small nonzero solution appears due to the next term of the decomposition of the hyperbolic tangent into a

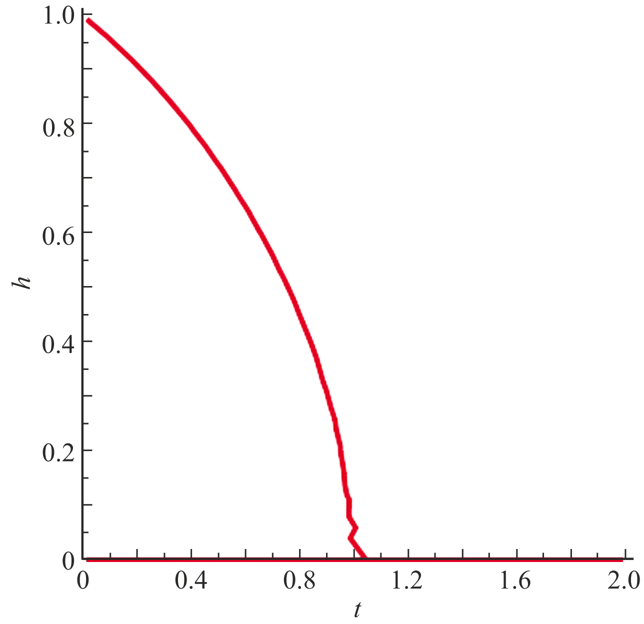

Figure 2. Spontaneous magnetic field as a function of temperature in the Gauss model.

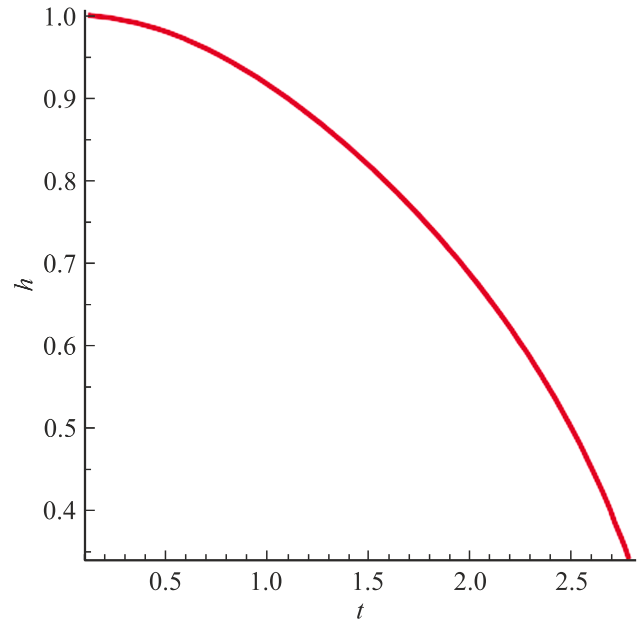

Figure 3. Spontaneous magnetic field as a function of temperature in the Holtsmark model.

Taylor's series. The magnetic field disappears at temperature $T = \mu H_0 = \mu^2\sqrt{2}/\bar{R}^3$.

We see that both the critical temperature of the disappearance of the magnetic field and the value of the field at zero temperature in both models coincide. Figure 2 shows the numerical solution of the implicit equation (14). It can be seen that the curves in Figure 1 and Figure 2 are very different from each other, although they coincide at both endpoints.

4. Holtsmark distribution

The Holtsmark distribution determines, in particular, the stationary distribution for the force acting on the Sun due to gravity from surrounding randomly distributed stars [6] The Holtsmark distribution is used when considering the broadening of lines of hydrogen ideal plasma [7]. Protons randomly distributed in space create a static electric field. This field produces a linear Stark splitting of the levels of hydrogen atoms. The Holtsmark distribution is used to calculate the electric field distribution in compensated semiconductors in the paper in Ref. [8].

Let us insert the Holtsmark distribution, normalized by one in equation (14), instead of the Gaussian distribution:

$$H(x) = \frac{2}{\pi x}\int_0^\infty \exp\left\{-(z/x)^{3/2}\right\}\sin(z)z\,dz. \quad (15)$$

We obtain a universal implicit equation for determining the spontaneous magnetic field:

$$h^2 = \int_0^\infty H(x)\tanh^2\left(\frac{hx}{t}\right)dx. \quad (16)$$

Its numerical solution is shown in Figure 3. The critical temperature $T = 2.7\mu^2\sqrt{2}/\bar{R}^3$ turns out to be almost three times higher than in the case of the Gaussian distribution (Figure 2). And the spontaneous magnetic field at zero temperature coincides with the field for the Gaussian distribution.

5. Conclusion

The dipole interaction of magnetic moments in a spin glass is directly taken into account in this study, whereas the majority of papers usually use some abstract random exchange field with a given distribution function to describe the interaction. If the self-consistent field approximation is not used, then the spontaneous magnetic field should, of course, be calculated differently. It is necessary to calculate the average square of the magnetic field when using the Gaussian distribution. To do this, it is necessary to multiply the square of the magnetic field defined by equation (5) by the probability of finding the magnetic moment at a distance of R (it is equal to $4\pi nR^2dR$) and integrate the resulting expression from zero to infinity. It is easy to see that this integral diverges. The same problem arises for the Holtsmark distribution. It is well known (see, for example, [6]) that the second moment of the Holtsmark distribution is equal to infinity. A way to circumvent these divergences has been found in this paper.

A fairly complete review of the experimental data on spin glasses is contained in a recent publication [9]. Typical temperatures for the occurrence of a spontaneous magnetic field in various spin glasses are 15–30 K. It can be concluded from the above dependences of the spontaneous magnetic field on temperature, as well as from the dependences given in Ref. [3] that the Holtsmark distribution should be given greater preference than the usually considered Gaussian distribution for describing the random distribution of rare magnetic impurity centers in a non-magnetic material.

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Conflict of interest

The author declares that he has no conflict of interest.

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