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Shaping of optical vortices in elliptic Gaussian beams with uniaxial crystal

© N.V. Shostka, B.V. Sokolenko, Yu.A. Egorov

Vernadskii Crimean Federal University, Simferopol, Russia
E-mail: nataliya_shostka@mail.ru

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A simple method for practical realization of beams carrying optical vortices with a single topological charge, as well as their chains, based on the inclined propagation of an elliptic Gaussian beam to the perpendicular to the optical axis of a uniaxial crystal is proposed. By changing the beam tilt angle and rotating the uniaxial crystal around the beam axis, the shape of the vortex beam and the number of singularities in the visible region are controlled.

Keywords: Optical vortex, elliptical Gaussian beam, uniaxial crystal.

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Laser beams carrying phase singularity and orbital angular momentum are widely used in optical traps and microparticle rotators [1], fiber acousto-optics and microcavity systems [2,3], as well as in measuring devices and microscopes [4]. The object of research is an optical vortex — observed area of the minimum of the light field intensity, in which the energy flux takes a spiral form and the phase changes from 0 to 2π as it traverses a closed loop around a singularity. Taking into account the long history of singular optics, which dates back to the 1970s [5], and the development of the main approaches to the generation of vortex beams, we can conclude that the issues of improving the methods of singularity generation in laser beams are still relevant at this time [6]. Laser resonators and optical fibers [7], spiral phase plates [8], diffraction elements and fork holograms formed with spatial light modulators [9,10] are considered to be the most common tools for generating vortex beams. A separate class of polarization elements, such as q-plates, liquid and birefringent crystals [11,12], whose operation is based on controlling the geometric phase that gives rise to polarization singularities [13]. Their major advantages — the simplicity of manipulating the polarization of the light beam, applicability to polychromatic radiation and high-power laser beams [14,15].

In this paper, we propose a method for generating single optical vortices and their chains based on the propagation of an elliptic Gaussian beam at a small angle to the perpendicular to the optical axis of a [16] uniaxial crystal. This model is shown schematically in fig. 1.

Let us an inclined circularly polarized Gaussian beam propagating in a uniaxial crystal at a small angle α ($\alpha \approx \sin\alpha \ll 1$) to the $z$axis. The optical axis of the crystal C is oriented perpendicular z, and the tensor of the dielectric permittivity has a diagonal form $\hat{\varepsilon} = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_1)$. The paraxial wave equation in this case will be satisfied by two transverse beam components: ordinary and extraordinary, having different wave numbers $k_1 = 2\pi\sqrt{\varepsilon_1}/\lambda$ and $k_2 = 2\pi\sqrt{\varepsilon_2}/\lambda$, where λ — the wavelength of the incident radiation. Let us write expressions for the complex amplitudes of the ordinary E_o and extraordinary E_e components in the linear polarization basis, taking into account the ellipticity of the Gaussian beam incident on the crystal in the coordinate system associated with the crystal [16,17]:

$$E_o(x, y, z) = \frac{1}{\sqrt{\sigma_{ox}\sigma_{oy}}} \exp\left[-\frac{x_o^2}{\omega_x^2\sigma_{ox}} - \frac{y_o^2}{\omega_y^2\sigma_{oy}} - f_o\right] \times \exp(-ik_1 z), \quad (1)$$

$$E_e(x, y, z) = \frac{i\chi}{\sqrt{\sigma_{ex}\sigma_{ey}}} \exp\left[-\frac{x_e^2}{\omega_x^2\sigma_{ex}} - \frac{y_e^2}{\omega_y^2\sigma_{ey}} - f_e\right] \times \exp(-ik_2 z), \quad (2)$$

where $\sigma_{ox} = 1 - iz/z_{ox}$, $\sigma_{oy} = 1 - iz/z_{oy}$, $\sigma_x = 1 - iz/z_{ex}$, $\sigma_{ey} = 1 - iz/z_{ey}$, $\chi = \pm 1$. The choice of the sign χ in expression (2) depends on the direction of the circular polarization: right ($\chi = -1$) and left ($\chi = +1$). The Rayleigh

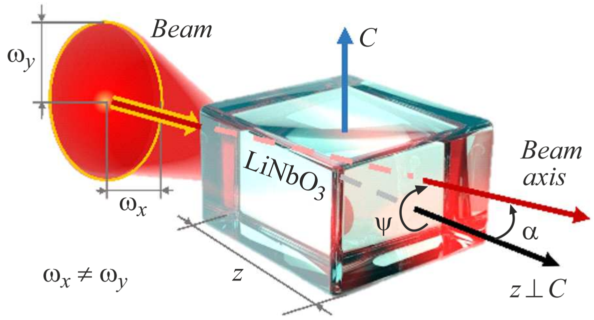

Figure 1. A scheme of propagation of an inclined beam at a small angle to z axis in single-axis crystal.

lengths in this case are different for each of the field components $z_{ox} = k_1\omega_x^2/2$, $z_{oy} = k_1\omega_y^2/2$, $z_{ex} = k_2\omega_x^2/2$, $z_{ey} = k_2\omega_y^2 n_1^2/2n_2^2$, where ω_x, ω_y — waists of the original elliptic beam at the input facet of the $(z = 0)$crystal. The beam inclination and the rotation of the crystal by an angle ψ around the z axis are accounted for using a coordinate transformation of the form

$$x_o \to x + i\alpha z_{ox}\cos\psi,$$

$$y_o \to y + i\alpha z_{oy}\sin\psi,$$

$$x_e \to x + i\alpha(n_1/n_2)z_{ex}\cos\psi,$$

$$y_e \to y + i\alpha(n_2/n_1)z_{ey}\sin\psi;$$

$$f_e = \left(\alpha^2(n_1/n_2)^2 k_2 z_x\cos^2\psi + \alpha^2 k_1(n_2/n_1)z_y\sin^2\psi\right)/2,$$

$$f_o = (\alpha^2 k_1 z_{ox} + \alpha^2 k_1 z_{oy})/2.$$

The superposition of ordinary and unusual beams will be observed at the output facet of the crystal, the amplitude of which can be found from the expression

$$E(x, y, z) = E_x(x, y, z) + E_y(x, y, z). \qquad (3)$$

Thus, the spatial structure of the light field is described by an appropriate set of parameters: the original beam waist (ω_x, ω_y), the inclination angle α and the angle of rotation of the crystallographic axes ψ. Fig. 2 shows the results of computational modeling of the transverse distribution of the resulting complex amplitude modulus after uniaxial lithium niobate crystal. Due to a rather large difference in refractive indices: $n_o = 2.286$, $n_e = 2.203$ $(\lambda = 632.8\,\text{nm})$, a change in the spatial polarization of the field is observed along with the ellipticity of the cross section of the ordinary beam. This is due to the difference in the phase velocities of the ordinary and extraordinary beams, which have different divergence and initial nonzero inclination angle. As follows from Fig. 2, *a*, an asymmetric conoscopic pattern in the form of a family of isoclines is observed when beams with large divergence propagate through the crystal. As ω increases, only a part of the isoclines is illuminated, while a local amplitude minimum is isolated in the overlap region of the ordinary and extraordinary beams. Note that the beam inclination angle remains so small that no spatial splitting of the ordinary and extraordinary beams occurs. In their overlap area, a wavefront singularity with a characteristic spiral phase portrait in the vicinity of the amplitude modulus minimum is formed (Fig. 2, *a, c*).

Increasing the inclination angle α of the beam incident on the crystal (Fig. 2, *b*) with other constant parameters leads to a gradual splitting of the ordinary and extraordinary beams and an increase in the region of their overlap, in which an increasing number of isocline branches are localized (the maximum density of which falls on the angle ψ, a multiple of $\pi/4$). As a consequence, the number of local minima with phase singularity also increases, degenerating into a chain of vortices with unit topological charge. The distance between local minima in the chain is inversely proportional to the inclination angle α, so that more vortices are localized in the beam aperture as α grows.

Rotation of the crystal by the angle ψ allows us to control the shape of the local minima of the amplitude modulus, the ellipticity of which greatly increases when ψ values multiple of $\pi/2$ are reached. An important consequence of crystal rotation is a variable change in the sign of the topological charge of the optical vortex. As shown in Fig. 2, *c*, as a consequence of increasing the angle ψ to the value of $\sim 3/4\pi$ the phase in the region of the minimum of the amplitude modulus has the form of a spiral whose direction is inverted relative to the case shown in Fig. 2, *a*, at $\psi = 5/4\pi$. The sign change of the topological charge occurs with a periodicity of $\pi/2$ rad. Further increase of the angle $\psi \to \pi$ leads to the transformation of local minima into dark horizontal bands with edge dislocation of the wavefront. Note that the above phenomena do not occur if the initial Gaussian beam has circular symmetry $(\omega_x = \omega_y)$.

The possibility of forming singular beams as well as topological dipoles in white light when propagating along the optical axis of a crystal has been shown in the works of the A.V. Volar [12,14] group. Provided that the beam and the optical axis of the crystal lie in mutually orthogonal planes, the energy losses can be reduced due to a smaller number of necessary polarization elements, which is relevant when working with high-power beams. In Figure 3, *a, b* the possibility of obtaining a „white“ vortex beam by adding beams with three basic wavelengths (633, 550, and 450 nm) is shown. The refractive indices for the ordinary and extraordinary beams in the lithium niobate crystal were calculated using the Sellmeier [18] equations. The insignificant shift of the amplitude minimum relative to the beam center is due to the deviation of the optical difference of the ordinary and extraordinary beams from the value $m\lambda/2$, where $m \in \mathbb{N}$. The $k_{o,e}(\lambda)$ dependence is reflected by the unequal size of the cross section of the partial beams, due to which the resulting „white“ vortex beam has an orange halo, and short-wavelength components are observed in the minimum area. The distribution of field polarization in the vicinity of the amplitude minimum is shown in Fig. 3, *c*. The force lines plotted along the major axis of the polarization ellipses indicate the formation of a spatial polarization singularity „star“ in accordance with the Nye [19] classification with a characteristic region of circular polarization (*C*-points) at the vortex center and linear polarization (*L*–lines) at the periphery of the beam.

The obtained results suggest the possibility of using the phenomenon of superposition of ordinary and extraordinary beams spreading after a uniaxial crystal to generate single optical vortices as well as their chains. The control of the number of vortices, their shape and location is provided by selecting such parameters

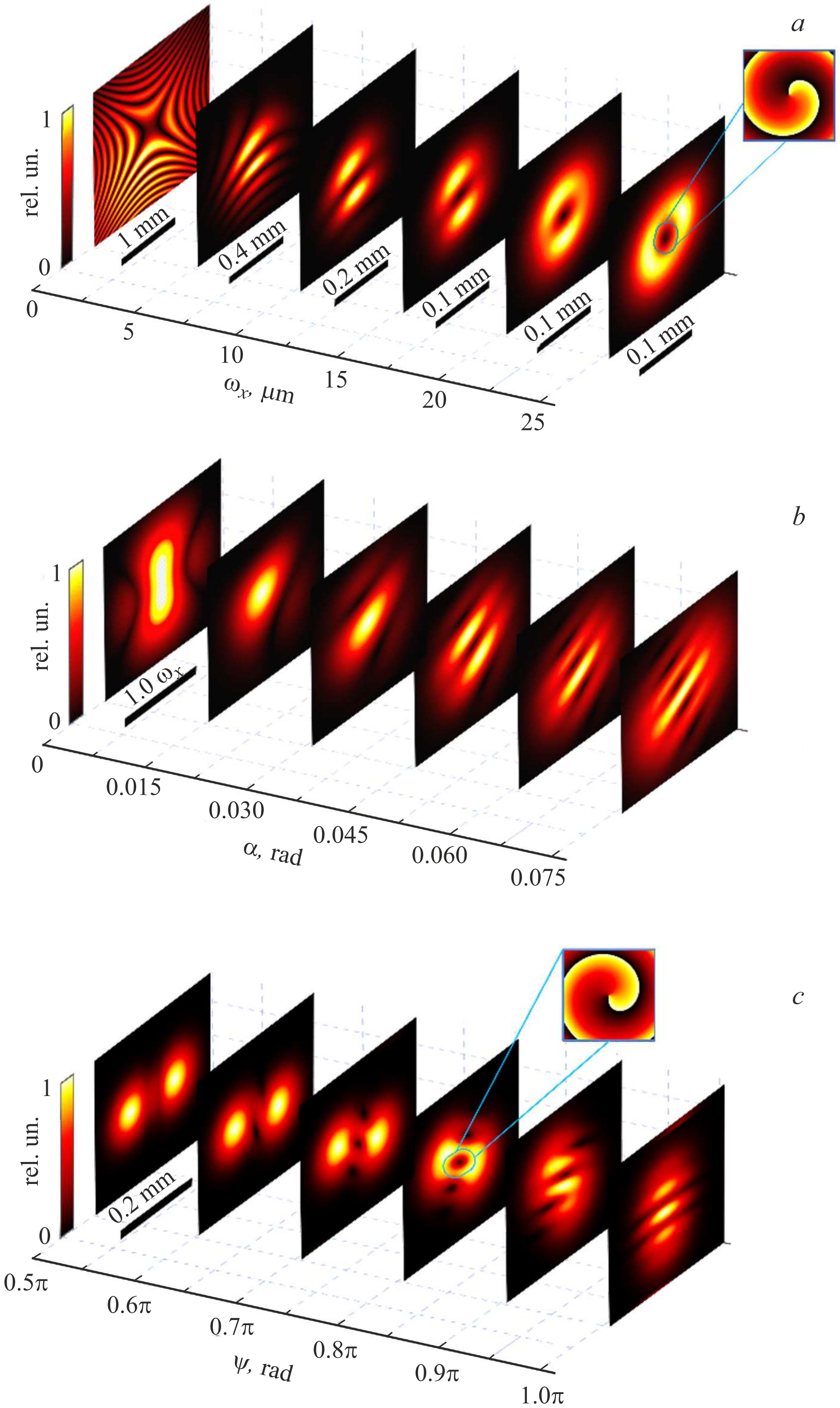

Figure 2. Numerical calculation of the transverse distribution of the amplitude modulus at the output facet of the crystal $LiNbO_3$ for the case of different radii of the original beam waist ω (a), inclination angle α (b) and crystal rotation angle ψ (c). Simulation parameters: source and resultant beams in the left circular polarization projection $(\chi = +1)$, $z = 20$ mm, a — $\psi = -3\pi/4$ rad, $\alpha = 0.031$ rad; b — $\psi = \pi/4$ rad, $1.2\omega_x = \omega_y = 12\,\mu$m; c — $1.2\omega_x = \omega_y = 18\,\mu$m, $\alpha = 0.08$ rad.

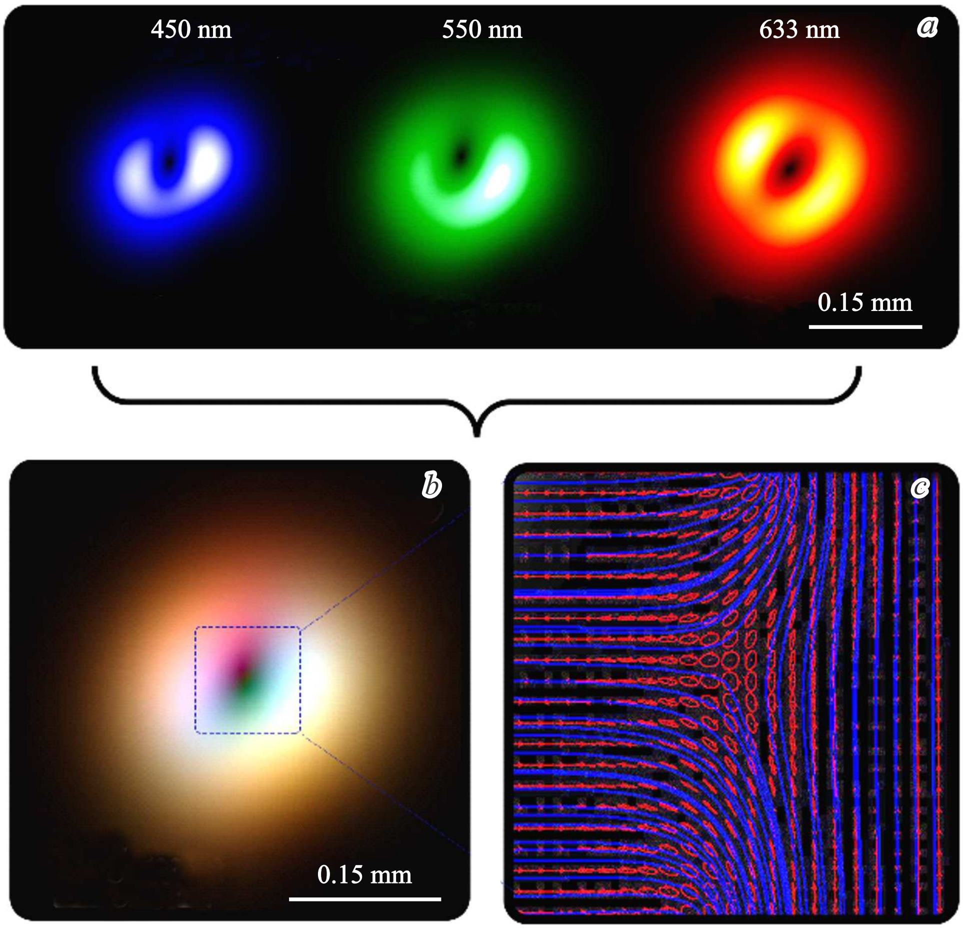

Figure 3. Numerical calculation of the transverse distribution of the amplitude modulus at the output facet of the $LiNbO_3$ crystal for the case of superposition of beams with wavelengths 633, 550, 450 nm (a), the result of their addition (b), and the distribution of the field polarization in the vicinity of the amplitude minimum (c). Simulation parameters: initial and resulting beams in the projection of the left circular polarization $(\chi = +1)$, $\psi = -3\pi/4$ rad, $\alpha = 0.031$ rad, $z = 20$ mm, $1.2\omega_x = \omega_y = 24\,\mu$m.

as the waist of the incident beam, the angle of inclination relative to the perpendicular to the optical axis of the crystal, and the rotation of the crystallographic axes. This approach has the advantage of being implemented without a spatial light modulator, spiral phase wafer, or artificial beam matching using an interferometer [20].

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Conflict of interest

The authors declare that they have no conflict of interest.

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