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Effect of the thermal boundary condition on the flat channel wall and axial thermal conductivity of liquid on heat transfer in a laminar pulsating flow in a quasi-stationary regime

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The possibility of increasing the efficiency of heat transfer devices by imposing high-amplitude flowrate pulsations on the laminar flow has been investigated. The greatest heat transfer increase in the pulsating flow compared to that in the stationary flow occurs in the region of relatively low flow pulsation frequencies, i. e. in the quasi-stationary regime. The average Nusselt number over the channel length and time may increase by 50 % as compared to that in the stationary flow and by several times in the presence of influence of the liquid's axial thermal conductivity at low Peclet numbers and relatively short pipes.

Keywords: heat transfer, pulsating quasi-stationary flow, axial thermal conductivity.

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The paper considers heat transfer in the case of a laminar pulsating flow in a flat channel. Velocity $\langle u\rangle$ averaged over cross-section varies with time according to harmonic law

$$\langle U\rangle = 1 + A\cos(\omega t),$$

where $\langle U\rangle = \langle u\rangle/\langle\overline{u}\rangle$, $\langle\overline{u}\rangle$ are the average velocity values over the cross-section and time, A is the pulsation amplitude, ω is the circular frequency.

The non-stationary energy equation for a developed flow has the following form:

$$\mathrm{Wo}_T^2\frac{\partial\vartheta}{\partial t_\omega} + U\frac{\partial\vartheta}{\partial X} = \frac{\partial^2\vartheta}{\partial Y^2} + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2\vartheta}{\partial X^2}. \qquad (1)$$

Here $t_\omega = t\omega$ is the dimensionless time; $X = x/(h\mathrm{Pe})$, $Y = y/h$ are the dimensionless longitudinal and transverse coordinates; $\vartheta = \lambda(T - T_0)/(hq_w)$ is the dimensionless temperature at the given heat flux density on the wall $q_w = \mathrm{const}$; $U = u/\langle\overline{u}\rangle$ is the dimensionless longitudinal velocity; $\mathrm{Pe} = \mathrm{RePr}$ is the Peclet number; $\mathrm{Re} = \langle\overline{u}\rangle h/\nu$ is the Reynolds number; ν is the kinematic viscosity coefficient; $\mathrm{Wo}_T = 2\mathrm{Wo}\sqrt{\mathrm{Pr}}$, $\mathrm{Wo} = \frac{h}{2}\sqrt{\frac{\omega}{\nu}}$ is the Womersley number; h is the channel width; T_0 is the channel inlet temperature.

One of the methods for intensifying heat exchange in various heat transfer devices may be imposing flowrate pulsations on the laminar flow. Paper [1] presents classification of pulsating laminar flow regimes. Based on the Womersley numbers, the following regimes are distinguished: quasi-stationary (Wo, $\mathrm{Wo}_T < 1$), high-frequency (Wo, $\mathrm{Wo}_T > 10$) and intermediate ones. The paper has shown that an increase in the pulsating flow heat transfer relative to that in the stationary flow is possible only at high pulsation amplitudes $A > 1$. It has been established that the greatest heat transfer enhancement occurs in the range of relatively low flowrate pulsation frequencies, i. e. in the quasi-stationary regime. In this case, when $\mathrm{Wo}_T \ll 1$, it is possible to omit the first term in (1) and solve the stationary energy equation. If heat transfer is considered for a developing flow, the longitudinal and transverse velocity components will be found by solving a set of stationary equations of motion and continuity. Paper [2,3] has proposed a method for utilizing data on the stationary-flow hydrodynamic and thermal characteristics to obtain these characteristics for the quasi-stationary pulsating flow:

$$U(X, Y, \omega t) = U_s(X/\langle U\rangle, Y)\langle U\rangle,$$
$$\vartheta(X, Y, \omega t) = \vartheta_s(X/\langle U\rangle, Y),$$
$$\Delta\vartheta(X, \omega t) = \Delta\vartheta_s(X/\langle U\rangle). \qquad (2)$$

Here U_s, ϑ_s, $\Delta\vartheta_s$ are the velocity, temperature and temperature head (difference between the wall temperature and average liquid bulk temperature) obtained by solving stationary equations.

For the analysis of the effect of flowrate pulsations on hydrodynamics and heat transfer, work [3] proposed to divide the pulsation period into two parts; in the first one, the liquid moves from the channel inlet to outlet (forward flow), while in the second one the flow direction is opposite (reverse flow). This separation allows the obtained data to be employed in designing various heat transfer devices, e. g. heat exchangers used to intensify the heat transfer. In practice, heated channel section length L_q is a finite value. For the reverse flow, distance from the heating onset X should be replaced in relations (2) by $X - L_q$.

Since 1960s, research on heat transfer in the presence of fluid's axial thermal conductivity has been conducted.

Studies [4–8] were performed for a developed flow and infinite lengths of the heated section and preceding adiabatic section: $L_q \to \infty$, $L_0 \to -\infty$. At the adiabatic section inlet, a uniform temperature profile was set. In some studies, temperature distribution along the transverse coordinate at the heated section outlet was found by solving the energy equation with ignoring the axial thermal conductivity of the liquid. In other works it was assumed that outlet temperatures change in the longitudinal direction according to the laws corresponding to the case of absence of the influence of axial thermal conductivity. Studies [5–8] considered a flow in a round pipe, while papers [7,8] considered in addition a flow in a flat channel. Calculations with the first-type boundary condition on the heated section wall were carried out in [4,6,8], those with the second-type boundary condition were performed in [5–8]. An approximate analytical method for solving the problem (expansion in eigenfunctions) was applied in [4,5,7]. In this case, at the interface between the adiabatic and heated sections there were set conditions of equality of temperatures and their derivatives along the section length (exact fulfillment of the last condition is not mandatory, since it does not follow from the energy equation). In [6,8], the problem was solved by the finite difference method.

In heat transfer devices, heat-carrier agents are typically fed into the channel at a constant temperature. The adiabatic section is absent, its length is zero. The heated section has finite length L_q. At the heated section outlet, the liquid temperature does not change in the longitudinal direction. The problem formulated as above has been solved for the first time.

Here the stationary energy equation for a developed flow is being solved:

$$U\frac{\partial\vartheta}{\partial X} = \frac{\partial^2\vartheta}{\partial Y^2} + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2\vartheta}{\partial X^2}. \tag{3}$$

The equation (3) boundary conditions are as follows: $\vartheta = 0$ at $X = 0$, $\partial\vartheta/\partial X = 0$ at $X = L_q$, $\partial\vartheta/\partial Y = 0$ at $Y = 0$ $\partial\vartheta/\partial Y = 1$ at $Y = 1/2$.

For the developed flow in a flat channel, $U = 1.5(1 - 4Y^2)$.

The solution depends on the Peclet number Pe and heated section length L_q. The Nusselt number is calculated via relation $\mathrm{Nu}_s = 1/\Delta\vartheta_s$.

To solve (3), a new unconditionally stable finite-difference algorithm has been developed as a combination of two known schemes: iterative methods of Gauss–Seidel and longitudinal-transverse sweep. This scheme accounts for the transition from solving an elliptic-type equation to solving that of the parabolic type in the absence of the axial thermal conductivity effect. Besides, the proposed algorithm allows one to minimize the number of iterations.

The developed numerical-simulation method has been verified. In the absence of effect of the liquid's axial thermal conductivity ($\mathrm{Pe} \to \infty$), a good agreement between the $\mathrm{Nu}_s(X)$ calculations and data given in [9] was achieved.

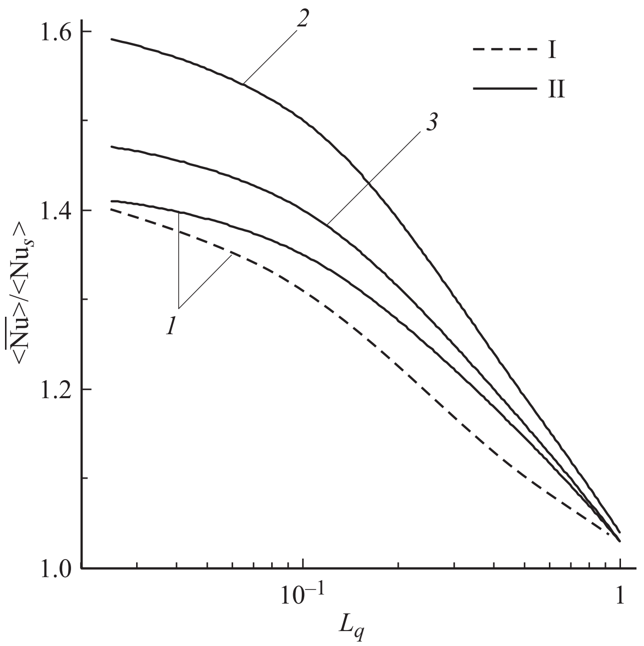

Figure 1. Average Nusselt number over time and channel length versus the heated section length under the first-type boundary condition. *1* — $\langle\overline{\mathrm{Nu}}\rangle/\langle\mathrm{Nu}_s\rangle$, *2* — $\langle\overline{\mathrm{Nu}^d}\rangle/\langle\mathrm{Nu}_s\rangle$, *3* — $\langle\overline{\mathrm{Nu}^r}\rangle/\langle\mathrm{Nu}_s\rangle$. I — $\langle\overline{\mathrm{Nu}_1}\rangle/\langle\mathrm{Nu}_s\rangle$, II — $\langle\overline{\mathrm{Nu}_2}\rangle/\langle\mathrm{Nu}_s\rangle$.

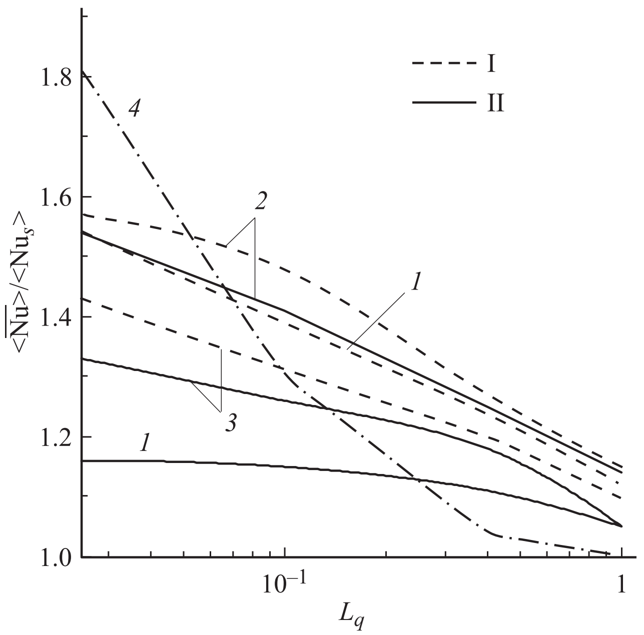

Figure 2. Average Nusselt number over time and channel length versus the heated section length under the second-type boundary condition. *1* — $\langle\overline{\mathrm{Nu}}\rangle/\langle\mathrm{Nu}_s\rangle$, *2* — $\langle\overline{\mathrm{Nu}^d}\rangle/\langle\mathrm{Nu}\rangle_s$, *3* — $\langle\overline{\mathrm{Nu}^r}\rangle/\langle\mathrm{Nu}_s\rangle$, *4* — $\langle\mathrm{Nu}_s\rangle/\mathrm{Nu}_\infty$. I — $\langle\overline{\mathrm{Nu}_1}\rangle/\langle\mathrm{Nu}_s\rangle$, II — $\langle\overline{\mathrm{Nu}_2}\rangle/\langle\mathrm{Nu}_s\rangle$.

The difference is about 10 % at $X = 10^{-5}$ and about a few units of percent at $X = 0.1$. Taking into account the axial thermal conductivity, the calculated variations in the Nusselt

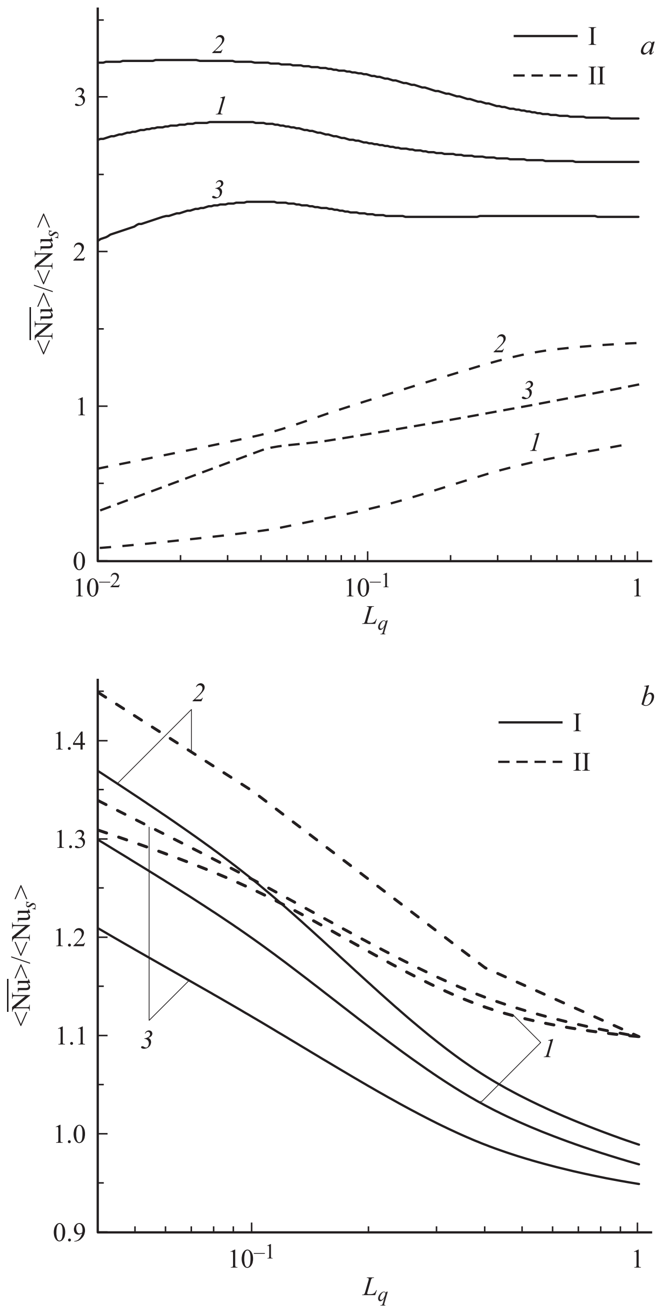

Figure 3. Average Nusselt number over time and channel length versus the heated section length for $\mathrm{Pe} = 1$ (a) and 1000 (b). *1* — $\langle\overline{\mathrm{Nu}}\rangle/\langle\mathrm{Nu}_s\rangle$, *2* — $\langle\overline{\mathrm{Nu}^d}\rangle/\langle\mathrm{Nu}_s\rangle$, *3* — $\langle\overline{\mathrm{Nu}^r}\rangle/\langle\mathrm{Nu}_s\rangle$. I — $\langle\overline{\mathrm{Nu}_1}\rangle/\langle\mathrm{Nu}_s\rangle$, II — $\langle\overline{\mathrm{Nu}_2}\rangle/\langle\mathrm{Nu}_s\rangle$.

number along the channel at low Peclet numbers and $L_q \to \infty$, $L_0 \to -\infty$ coincide qualitatively and quantitatively with the results of the above-mentioned previous studies. The maximum difference, as well as difference between data acquired in the above-mentioned works, is observed at $\mathrm{Pe} = 1$ at the heated section inlet and ranges within 10−15 %.

The Nusselt number calculations for the stationary flow with the finite heated section length were used to obtain data for the quasi-stationary pulsating flow by the above-described method.

The method for designing various heat transfer devices uses data on the Nusselt number averaged over the channel length and time. Time averaging may be performed in two ways: $\mathrm{Nu}_1 \propto \overline{1/\Delta\vartheta}$, $\mathrm{Nu}_2 \propto 1/\overline{\Delta\vartheta}$. For the forward flow, time averaging is performed in that part of the period where the flow velocity averaged over the cross-section is positive. Time averaging for the reverse flow is performed in the period part where the liquid flow is directed from the channel outlet to inlet.

Calculations at $A = 5$ are presented in Figs. 1−3. In addition to average Nusselt numbers for the forward (Nu^d) and reverse (Nu^r) flows, the figures demonstrate average Nusselt numbers $\overline{\mathrm{Nu}}$ over the entire oscillation period. Values of $\overline{\mathrm{Nu}_1}$ and $\langle\overline{\mathrm{Nu}_1}\rangle$ are between $\overline{\mathrm{N}_1^d}$, $\langle\overline{\mathrm{N}_1^d}\rangle$ and $\overline{\mathrm{N}_1^r}$, $\langle\overline{\mathrm{N}_1^r}\rangle$. Ratios between Nu_2 and Nu_1, as well as between $\overline{\mathrm{Nu}_2}$, $\langle\overline{\mathrm{Nu}_2}\rangle$ and $\overline{\mathrm{Nu}_2^d}$, $\langle\overline{\mathrm{Nu}_2^d}\rangle$, $\overline{\mathrm{Nu}_2^r}$, $\langle\overline{\mathrm{Nu}_2^r}\rangle$, may be different. They are determined by the temperature head dependence on time and distance from the heating onset for the forward and reverse flows.

Calculations whose results are presented in Figs. 1, 2, were performed earlier for the developing flow at $\mathrm{Pr} = 0.7$ without taking into account the liquid's axial thermal conductivity. In the case of the second-type boundary condition on the channel wall ($q_w = \mathrm{const}$, Fig. 2), the heat transfer enhancement due to imposing flowrate pulsations is somewhat higher than that for the first-type boundary condition ($T_w = \mathrm{const}$, Fig. 1). This difference is not observed at $\mathrm{Pr} = 7$. Fig. 2 also shows variations in the Nusselt number averaged along the channel for the stationary developing flow with respect to its stabilized value $\mathrm{Nu}_\infty = 4.12$. Note that at $\mathrm{Pr} \geqslant 7$ the effect of the flow development on the ratio between Nusselt numbers for quasi-stationary and stationary flows becomes insignificant.

Via the developed numerical-simulation technique, there were performed calculations of the developing and developed stationary flows in the flat channel with accounting for the liquid's axial thermal conductivity at finite lengths of the adiabatic and heated sections. The data obtained were used to calculate the quasi-stationary flow Nusselt numbers which depend on the longitudinal coordinate and time. Calculations have shown that effect of the flow development on the ratio between Nusselt numbers for quasi-stationary and stationary flows is insignificant at $\mathrm{Pr} > 7$.

Calculations whose results are given in Fig. 3 were performed for the developed flow with accounting for the liquid's axial thermal conductivity under the second-type boundary condition. In this case, the Nusselt number depends not on the Prandtl number, but on the Peclet number. When Peclet numbers are as low as about a few units (Fig. 3, a) and pipes are relatively short, heat transfer increases several times with respect to that for the stationary flow. At high Peclet numbers ($\mathrm{Pe} > 100$) (Fig. 3, b), when the axial thermal conductivity effect is insignificant, heat transfer increases to a somewhat lesser extent than in the case of the developing fluid flow with Prandtl number Pr $= 0.7$ (Fig. 2). This difference increases with increasing Prandtl number. Calculations performed for the first-type boundary condition showed that the type of boundary condition slightly affects the Nusselt number ratio to its stationary-flow value.

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Conflict of interests

The author declares that she has no conflict of interests.

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