

Scattering of electrons by the anode mesh in virtual cathode oscillators

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Received October 8, 2024

Revised December 10, 2025

Accepted March 4, 2025

Electron interaction with matter which it passes through, significantly changes its velocity both in magnitude and direction due to multiple scattering and energy losses. Thus, a beam of electrons incident on a target forms flows of charged particles reflected from and passing through matter. The intensities of these flows depend significantly on the matter properties, the initial energy of beam particles and the angle of electron incidence on a target. In devices with an oscillating virtual cathode, energy losses and multiple scattering of relativistic electrons in the anode mesh lead to formation of an electron cloud near the anode. The cloud particles possess large spread in velocities. The electrons captured by the cloud do not participate in the oscillations of the virtual cathode and partially block a vircator. As a result, the amplitude of the electric field oscillations is reduced. In order to increase the oscillation amplitude, the thickness of the anode mesh should be approximately equal to the mean free path of electrons in the mesh material.

Keywords: Multiple scattering, ionization energy losses, vircator, reflex triode.

DOI: 10.61011/TP.2025.07.61463.317-24

Introduction

Interaction of charged particles with matter, along with interaction with fields that originate with propagation of powerful ion and electron beams in complex electrodynamic structures, defines efficiency of operation of the high-current electronic devices [1]. Without quantitative analysis of transmittance of the charged particles through matter, which experience multiple scattering as well as ionization and radiation energy losses, it is impossible to correctly design high-current devices and describe their operation.

Actually, with unavoidably hitting the structural elements (the anode mesh, the collector, the drift tube, etc.), a part of the high-current beam particles reflects therefrom due to multiple scattering, returns back to the system and continues to interact with the electromagnetic field inside it. Thus, when the electrons normally hit a steel structural element, approximately a quarter of the particles is reflected. If they hit at an oblique angle, the portion of the reflected particles is much more. The spectrum of their kinetic energy ranges from zero to the energy of incident particles. Since the number of the incident electrons is comparable with the number of the reflected ones, the latter can noticeably affect operation of the high-current devices.

Consequent quantitative description of interaction of the relativistic high-current beams with electrodynamic structures with taking into account multiple scattering may be obtained by means of computational modeling by the Monte Carlo method [2–4] that is based on the strictest Goudsmit-Sanderson theory of multiple scattering [5]. It is also necessary to take into account the ionization and radiation energy losses [6–8].

The present paper describes an approach to numerical modeling of interaction of the high-current electron beams with electrodynamic structures in the high-current electronic devices. Its correctness has been checked on the exemplified calculation of transmission and reflection coefficients of the relativistic electrons reflected from the plates made of different materials, whose results were compared with the data published in the literature [9–12]. After verification, the said method of calculation was integrated into a one-dimensional computer code for simulating devices with an oscillating virtual cathode (VC). We note that the interest to this type of the devices is not accidental. The devices with the oscillating virtual cathode have long attracted researchers and are widely used, in particular, as sources of powerful electromagnetic radiation and to generate fast particles (see the papers [13–19] and the references contained therein).

The present paper has demonstrated that scattering of the relativistic electrons by the anode mesh resulted in formation of a cloud of charged particles with large energy spread. This cloud results in substantial reduction of an amplitude of field oscillations in vircators. It is shown that the use of the anode meshes with a thickness approximately equal to the mean free path of electrons in the anode material contributes to reduction of the charge in the cloud and increase of the amplitude of the oscillations.

1. Electron interaction with matter

1.1. Simulation by the Monte Carlo method

The computational modeling of transmittance of electrons through matter is based on the Monte Carlo method, whose

most popular variant can be described as follows [2,3]: a trajectory of each electron in matter is divided into many short segments and within each portion the particle energy is assumed to be constant. When passing from one portion to another, the energy of the particles varies in accordance with the theory of ionization and radiation losses [6,7]. Possible angular deviations of the particle within each of the segments are described by the Goudsmit-Sanderson distribution [2]. It is calculated until the particle leaves the substance or loses a substantial part of its energy (in practice, the calculations stop when the electrons reach the energy of 10 keV).

One significant drawback of the standard Monte Carlo method is its computation complexity. For calculation of the Goudsmit-Sanderson distribution function, it is necessary to summarize a large series of the terms, wherein each of them has an integral of the rapidly oscillating function. This significantly complicates the use of the Monte Carlo method in the high-current electronics due to a huge number of particles used when simulating the high-current devices.

The way out of the situation was found in the study [4], which presents a function of distribution of angle deviations by means of a simple formula that is obtained based on strict theoretical analysis [20] of average values calculated by means of the Goudsmit-Sanderson distribution. Based on the example of calculation of the angular distribution of electrons after transmittance through golden foils the authors of the paper [4] have demonstrated that in use of simplified expressions for the Goudsmit-Sanderson distribution the computation complexity of the Monte Carlo method is substantially reduced, while the results of simulation of transmittance of the particles through matter do not change.

The method proposed by the authors of the paper [4] was selected by us for integration into the one-dimensional computer code for simulating the high-current devices with the oscillating virtual cathode.

1.2. Verification of the method: reflectance and transmission coefficient

In the first step for evaluating suitability of the above-described approach to simulation of interaction of electrons with matter, we have calculated the reflection coefficients R_N of electrons reflected from the foils (plates) made of various substances (C, Al, Fe, Ag, Au, U and Be). At the same time, the foil thicknesses significantly exceeded the mean free paths of electrons in the respective materials. The energy of the particles normally incident to the target varied within the range from 0.1 to 2 MeV. The results of simulations were compared to the experimental data [11,12] and the comparison results are shown on Fig. 1. Good compliance was obtained for the most materials except for beryllium (Fig. 1, *b*).

Apparently, the significant difference of the simulation results from the experimental data for beryllium is attributed to the fact that the calculations do not take into account electron-electron collisions that increase an angle of multiple

scattering approximately in $1 + 1/Z$ times. In case of the light elements, the nucleus charge of the denominator provides more substantial impact of this factor on all the magnitudes that depend on the angle of multiple scattering. Besides, it should be noted that shells of the valence electrons of the atom are modified in the solid body, thereby also contributing to deviation of the calculated values from the experimental data.

It also included calculation of the transmission coefficient T_N and the reflectance R_N of the electrons with the energy from 0.1 to 1 MeV for the aluminum and beryllium foils of the different thickness. The figures 2, 3 show comparison of the results of simulation of the transmission coefficient T_N and the reflectance R_N with the calculated values of the study [10] and the experimental data [12]. The results obtained by us well comply with the published calculations [10]. At the same time, there is some discrepancy of the results of simulation and the experiment in the reflectance, which, as noted above, is most likely attributed to the neglect of the electron-electron collisions both in our simulation and in the calculations [10].

Fig. 4 show the dependences of the reflection coefficients on the kinetic energies of the particle incident to the aluminum plate. The different curves correspond to the different incident angles. The results obtained in the present study excellently comply with the results of the study [9] up to ~ 2 MeV. With the energy of ~ 2 MeV, the reflection coefficients of [9], unlike our calculations, demonstrate noticeable reduction, including under normal incidence. Let us note that the experimental dependence corresponding to the normal incidence ($\theta = 0^\circ$) (Fig. 1) has no such reduction of the reflectance.

Thus, the results of simulation obtained in the present study demonstrate good compliance both with the experimental data and the numerical calculations published in the literature. Some discrepancy with the experiment ($\sim 20\%$) is observed in the reflectance the electrons with the energy of ~ 0.1 MeV reflected from the beryllium foils (Fig. 3), which is apparently attributed, as noted above, to the neglect of electron-electron scattering.

2. Oscillations of the virtual cathode

The algorithm described in the study [4] and used by us in the previous paragraph for calculating the reflectance and the transmission coefficient of electrons was integrated into the one-dimensional computer code for simulating the high-current devices with the oscillating virtual cathode in a particle-in-cell methods [17,19,21,22]. This computer code is designed to calculate motion of the relativistic electrons in a longitudinal self-consistent electric field. The electric field is found by numerically solving the Poisson equation for the potential. It should be noted here that although the one-dimensional approach to investigating operation of the devices with the oscillating cathode takes into account only the longitudinal electric field and in a simplified form describes a geometry of the device, it still qualitatively

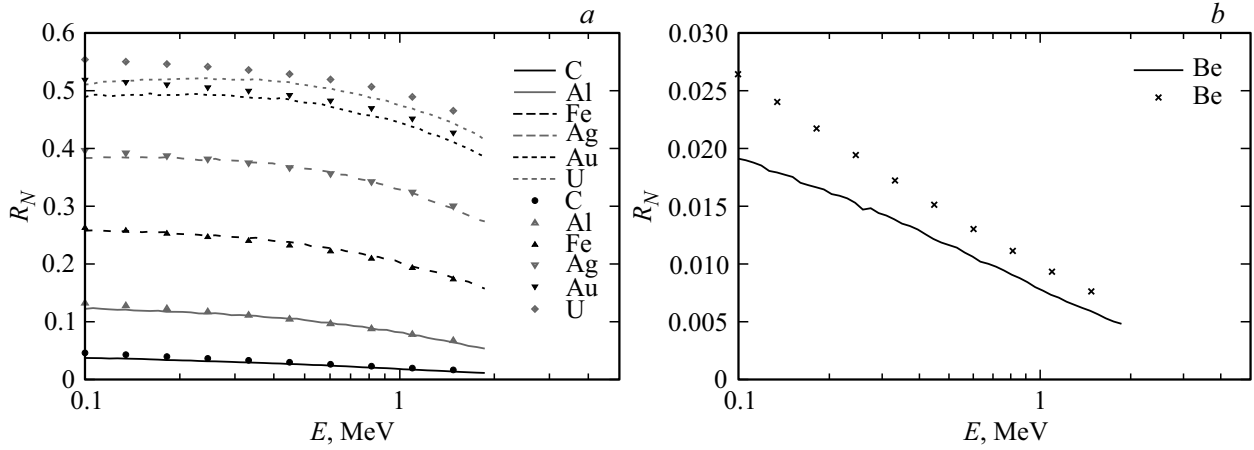


Figure 1. Reflection coefficients of electrons reflected from thick plates made of various substances (C, Al, Fe, Ag, Au, U and Be). The solid and dashed lines mark the simulation results, while the dots correspond to the experimental data [11,12]. Due to the scale difference, the reflection coefficients for beryllium are shown on (b).

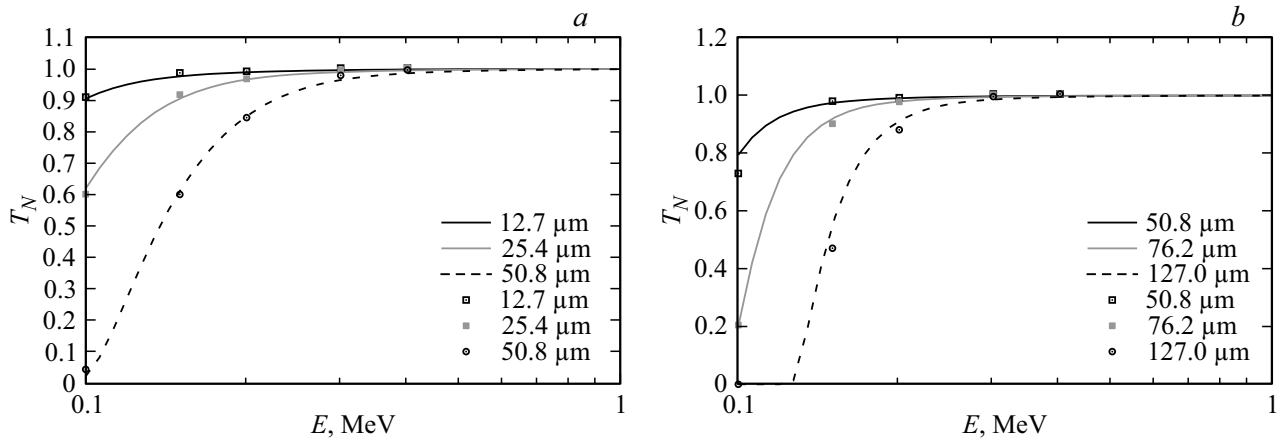


Figure 2. Transmission coefficients of electrons transmitted through aluminum (a) and beryllium (b) plates of the different thickness. The solid and dashed lines mark the simulation results, while the dots correspond to the calculation results [10].

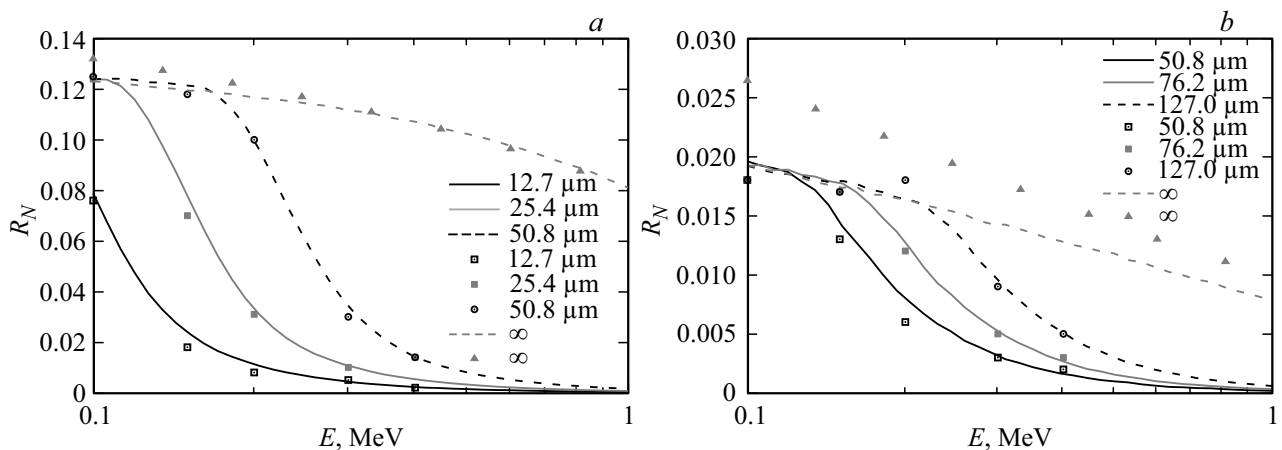


Figure 3. Reflection coefficients of the electrons reflected from aluminum (a) and beryllium (b) plates of the different thickness. The solid and dashed lines mark the simulation results, while the dots correspond to the calculation results for the plates of a finite thickness [10] and the experimental data for the thick plates [12], whose thickness significantly exceeds the mean free path of particles in the respective material. The dependences that correspond to the thick plates are marked on the graphs with the symbol ∞ .

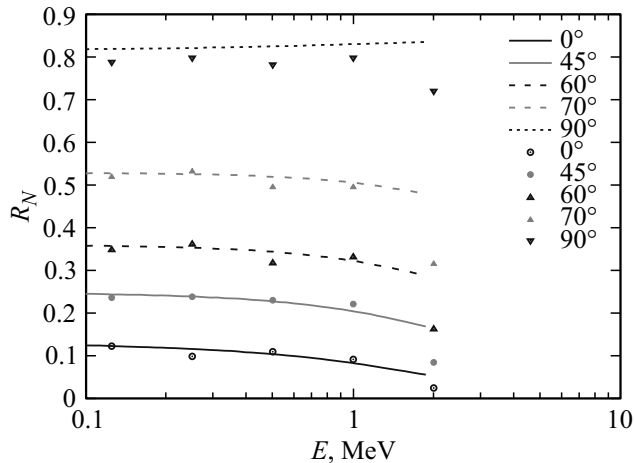


Figure 4. Dependences of the reflectance on the kinetic energy at the various incident angles of electrons incident to the thick aluminum plate. The lines correspond to the simulations results, so do the dots to the calculations of the study [9].

reproduces a process of formation of the oscillating virtual cathode and a spectrum of its oscillations [16].

The calculation region consists of two parts (Fig. 5): a cathode-anode gaps and a drift space. The potential of the right wall of the drift space may be equal both to the anode potential (the vircator case) and to the cathode potential (the case of a reflex triode). The super-particles, each of which contains a huge number of the elementary charge carriers are injected into the system in conditions of unlimited emissivity of the cathode that assumes a zero electric field on its surface.

This condition the most accurately corresponds to a mode of explosive electron emission, which is typical for the high-current accelerators. In the one-dimensional approximation, it is assumed that the electron beam is uniform within the plane perpendicular to the axis z , along which the virtual cathode oscillates. Each particle has three components of a velocity.

Before taking into account scattering, the anode mesh was supposed to be semi-transparent and characterized by a single parameter — geometric transparency η , which varies from zero to unity. With transmittance of the superparticle through the anode, its charge was multiplied to a transparency coefficient, which corresponds to absorption of a part of the particles by the anode mesh, so the initial model was called the „absorption model“. Neither particle reflection, nor deceleration of the particles in the anode material, nor origination of velocity spread due to multiple scattering was not taken into account in the computer code. Fig. 6 shows oscillations of the electric field E and a phase portrait of the beam in the vircator in the absorption model for the anode mesh with the geometrical transparency $\eta = 0.7$. The accelerating potential applied to the cathode-anode gap is 0.5 MV. On the curves, the velocity component v_z , the electric field E , the time t , the frequency f and the coordinate z are respectively rated to

the magnitudes typical for the considered problem: the light speed c , the field $\frac{mc^2}{eh_{ac}}$, the time interval h_{ac}/c , the frequency interval c/h_{ac} and the cathode-anode gap h_{ac} (e — the elementary charge, m — the mass of the electron). The electric field corresponds to a point in the drift space located from the anode at the distance of the interelectrode gap h_{ac} , which approximately corresponds to the position of the virtual cathode — an area in which the velocity component v_z changes its sign and the electrons are partially reflected (Fig. 6, b).

Addition of the algorithm described in the study [4] and used by us for simulation of reflection and transmittance of electrons through matter made it possible to describe their scattering by the anode mesh — the respective approach is called the „scattering model“. For description of the anode mesh, the improved version of the computer code requires to use three parameters: the geometric transparency of the mesh, the anode material and its thickness d . The geometric transparency η determines probability $(1 - \eta)$ of falling of the particle into the anode mesh, which is a perforated metal foil of the thickness d . The ratio of the sum of hole areas to the total foil area is equal to the transparency coefficient η .

Thus, the particle transmitted through the anode is scattered with probability $1 - \eta$, or it is not scattered with probability η . In case of scattering, motion of the particle in the anode material is calculated in the same way as in the flat foil. (In doing so, the possible complex structure of the anode mesh is not taken into account. For example, the mesh can be made of circular wires, and it is impossible to describe it within the framework of the one-dimensional model.) The scattered (unabsorbed) particle either enters into the cathode-anode gap or the drift space with the longitudinal velocity that is reduced due to the ionization energy losses and multiple scattering. At this, we neglect the time of stay of the particle inside the anode material, as the typical thickness d is approximately by one or two orders less than the typical value of the cathode-anode gap.

The figures 7 and 8 show the oscillations of the electric field, their spectrum and the phase portrait of the beam in case of the steel anode meshes with the geometric transparency $\eta = 0.7$ that have a different thickness (the

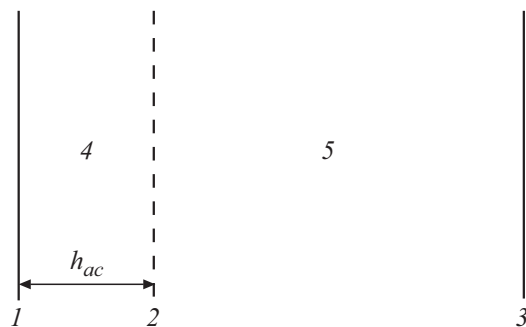


Figure 5. One-dimensional model of the device with the oscillating virtual cathode (1 — the cathode, 2 — the anode mesh, 3 — the collector, 4 — the cathode-anode gap, 5 — the drift space).

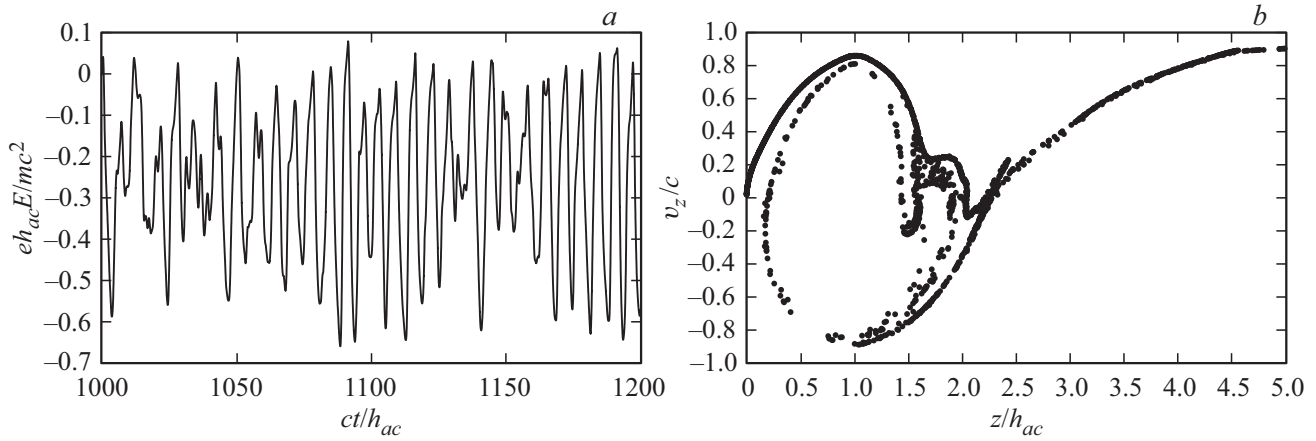


Figure 6. Electric field in the virtual cathode area (a) and the phase portrait of the electron beam (b) in the absorption model. Along the abscissa axis of the graph a, there is a longitudinal coordinate in units of the cathode-anode gap h_{ac} . Along the ordinate axis — the longitudinal component of the particle velocity in units of the light speed.

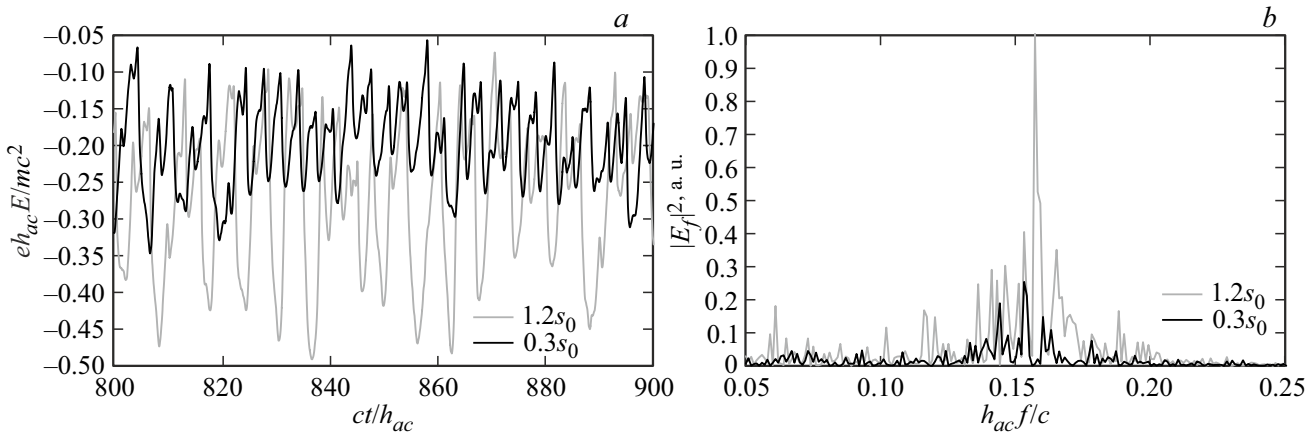


Figure 7. Electric field within virtual electrode area and its spectrum in the scattering model. The anode is made of steel.

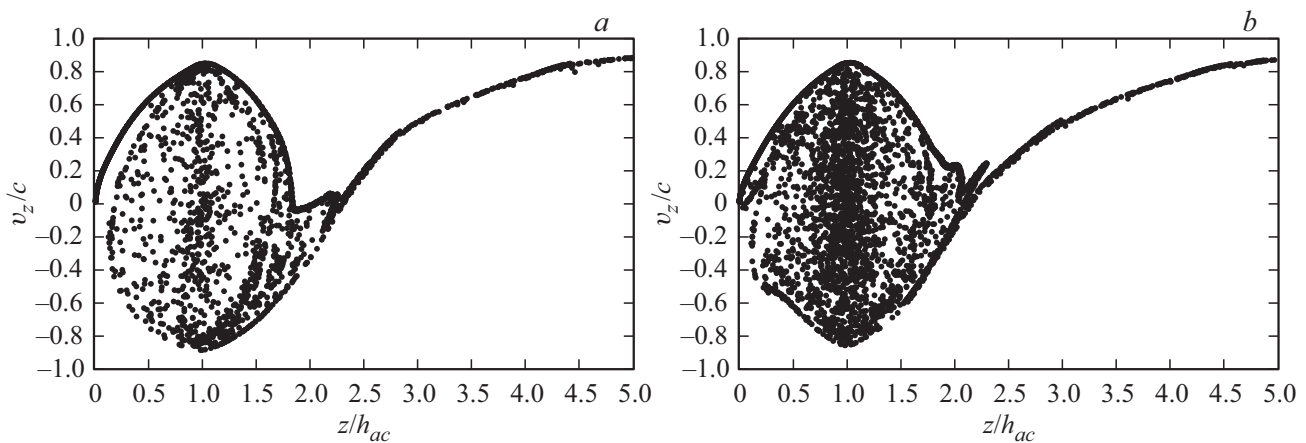


Figure 8. Phase portraits of the electron beam in the scattering model with the thicknesses of the steel anode $1.2s_0$ (a) and $0.3s_0$ (b).

mesh thickness of the graph is specified in units of the mean free path s_0). The accelerating potential is 0.5 MV. The simulation of the vircators has shown that scattering of

the electrons by the anode results in generation of a cloud of electrons near the anode, which have a large velocity spread. This cloud is generated as a result of loss of the

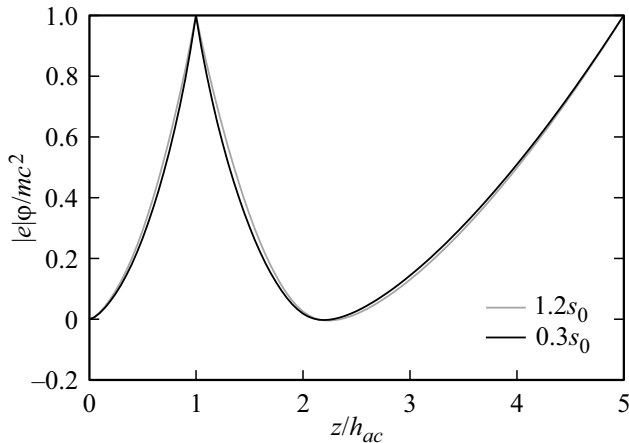


Figure 9. Distribution of the potential averaged along many oscillation periods and rated to the value $mc^2/|e| \approx 0.5$ MV in the device with the oscillating virtual cathode.

longitudinal momentum by the electrons being scattered by the anode mesh, causing lack of energy of the electrons for hitting the cathode or the virtual cathode. As a result, they oscillate in a potential well between the cathode and the virtual cathode until being fully absorbed. Scattering by the anode results in random change of a phase of oscillations of the electrons, thereby preventing them from participating in generation of coherent oscillations. Besides, the hanging cloud of the scattered electrons partially screens the anode and blocks the relativistic vacuum diode. As a consequence, the electrons injected from the cathode surface that could have participated in collective oscillations enter the diode in smaller quantities. As a result, the amplitude of the oscillations in the vircators decreases.

As seen on the figures 7 and 8, with the thickness of the anode mesh close to the mean free path of electrons in the anode material, the number of the electrons in the cloud is less than in case of a thinner mesh. Thus, the cloud affecting decrease of the amplitudes of the oscillations will have less particles blocking the vircators. It is indicated by the curves of Fig. 9 for the potential ϕ averaged along many oscillation periods, for the thin and thick anode meshes. It can be noted that the curve corresponding to the thinner anode mesh has a steeper slope near the anode, thereby indicating greater concentration of electrons in the said region, as the second derivative of the potential by the spatial coordinate is proportional to the charge density.

Conclusion

Scattering of the relativistic electron beams on the electrodynamic structures, which is due to multiple scattering and accompanied by the ionization and radiation energy losses, substantially affects operation of the high-current vircators. In particular, scattering of the beams by the anode mesh results in generation of the cloud of electrons near the anode, which have the large velocity spread. The scattered

electrons that entered this cloud fail to participate in the oscillations of the virtual cathode and partially block the vacuum diode, resulting in reduction of the amplitude of oscillation of the fields in the system and, therefore, in radiation power drop. It has been shown that in order to increase the amplitude of the oscillations, it is necessary to use the anode meshes with a thickness that is approximately equal to the mean free path of electrons in the respective material. These meshes are capable of minimizing the number of the scattered electrons transmitted through the anode.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by M. Shevelev