06

Statistical method for determining the range limit of non-destructive load and the range limit of non-destructive deformation ranges of synthetic yarns

© L.F. Vyunenko, E.S. Tsobkallo, T.B. Koltsova

St. Petersburg State University of Industrial Technologies and Design, St. Petersburg, Russia
E-mail: Viunenko.LF@suitd.ru

Received April 9, 2025
Revised May 13, 2025
Accepted May 15, 2025

The paper proposes a new method for estimating the ultimate non-destructive load and deformation values during stretching of elementary synthetic yarns, based not on critical breaking values, but on a statistical analysis of the breaking load and deformation values determined experimentally. The described method for estimating „safe" ranges of loads and deformations under mechanical impacts, using the three-parameter Weibull distribution, can be extended to a wide range of materials.

Keywords: synthetic yarns, non-destructive loads, non-destructive deformations, Weibull–Gnedenko distribution, three-parameter Weibull distribution.

DOI: 10.61011/TPL.2025.08.61540.20340

Polymer materials (synthetic fibers and yarns included) are among the most important structural materials, since they form the basis for the production of composite materials and other marketable technical products in which yarns and fibers act as load-bearing elements. Approaches based on the fulfillment of strength and rigidity conditions seem to be well established in calculations of the reliability of products and feature the most important characteristics of mechanical properties: the permissible stress and deformation magnitudes. These values, which depend on the properties of materials, are normally determined experimentally for various structural materials and represent the critical values of stress (for the strength condition) and deformation (for the rigidity condition) at failure that are reduced by a certain factor. They are provided as reference data for traditional construction materials. However, the determination of permissible values for polymer structural materials, including fibers and yarns, requires a special approach, since their mechanical properties have certain peculiar features (pronounced relaxation properties, nonlinearity of the tension curve, etc.). The solution to the problem of correct determination of permissible magnitudes of stress and deformation is especially relevant for thin elementary synthetic yarns (components of complex yarns) due to the large relative error of determination of their cross-section areas.

The Weibull distribution, which has various forms and notations, is often used as a probabilistic model to describe and model the values of breaking characteristics of materials. The most common is its two-parameter form (Weibull–Gnedenko distribution) with probability density function [1–3]

$$f(x;a,k)=\frac{a}{k}\left(\frac{x}{k}\right)^{a-1}\exp\left(-\left(\frac{x}{k}\right)^{a}\right),\qquad x\geqslant 0,\quad (1)$$

where a is the shape parameter and k is the scale parameter.

It was demonstrated in [1] that the distribution of strength parameters is characterized adequately within the Weibull model, and the features of statistical distributions of experimental values of breaking forces and deformations of elementary yarns of polyamide-6, which are potentially associated with different localization of the fracture process, were revealed in [1,2] using model (1). The two-parameter Weibull distribution was used in [3] to analyze the tensile and bending strength of polymers and composite materials produced from polymer mixtures. When applied to synthetic materials, such a model may be used to characterize the statistical distributions of breaking stress values in oriented monofilament and polyfilament polymer fibers [4]. Model (1) provides a fairly accurate description of the center of the distribution, but is not suitable for correct estimation of the minimum values of a random variable. We propose to use the three-parameter form of the Weibull distribution for this purpose. It contains shift parameter x_0:

$$f(x;x_0,a,k)=\begin{cases}\frac{a}{k}\left(\frac{x-x_0}{k}\right)^{a-1}\exp\left(-\left(\frac{x-x_0}{k}\right)^{a}\right), & x\geqslant x_0>0,\\ 0, & x<x_0.\end{cases}\quad (2)$$

In physical terms, x_0 is the threshold (lowest) value of a random variable. This model is used to assess the probability of failure-free operation of technical equipment [5] and

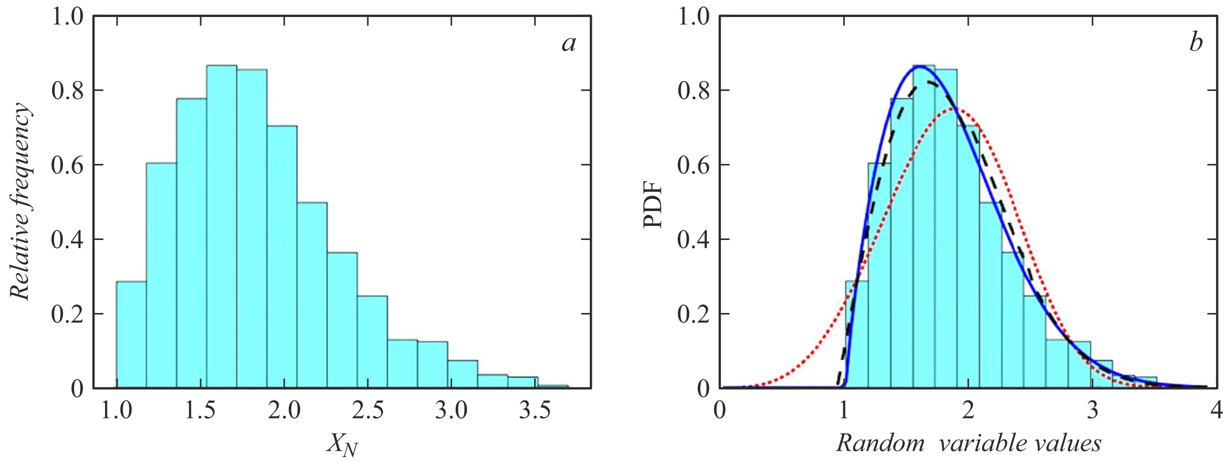


Figure 1. Results of statistical processing of a sample from the three-parameter Weibull distribution W ($x_0 = 1$, $k = 0.95$, $a = 1.8$). *a* — Relative frequency histogram; *b* — probability density functions (PDFs) of two-parameter (dotted curve), semi-empirical three-parameter (dashed curve), and theoretical three-parameter (solid curve) distributions.

operational risks [6] and in certain applied economic and medical problems [7]. A method for predicting the fracture of steel plates containing surface cracks based on model (2) was proposed in [8]. This statistical approach has not yet been used to evaluate the characteristics (including mechanical ones) of synthetic polymer materials. The advantage of model (2) in studying the mechanical behavior of synthetic oriented polymer materials (fibers, yarns) is that it provides an opportunity to estimate the limit load or deformation that does not induce fracture. General guidelines for calculating the confidence limits of parameters of the three-parameter Weibull distribution, which are set out in GOST 11.007–75 [9], do not allow one to construct an estimate of the shift parameter needed in the case under consideration. It was proposed in [9] that an estimate of this parameter should be the smaller of two values: the minimum of those observed in the experiment and the one obtained in the recommended calculation. This approach is inadequate for evaluating the boundaries of the range of non-destructive loads and deformations. In addition, modern numerical methods make it possible to obtain a more accurate solution to the corresponding computational problem. The aim of the present study is to develop a probabilistic method for estimating the range of safe non-destructive loads and deformations of synthetic materials using the three-parameter Weibull distribution. Elementary yarns of polyamide-6, which make up complex industrial-use yarn, with a diameter of $9 \pm 0.5\,\mu$m were chosen as the object of study. The process of yarn fracture was studied in the mode of active tension with an Instron 1122 instrument at a stretching rate of 20 mm/min in accordance with GOST 6611.2–73 [10]. The values of breaking force (P_p, N) and relative elongation at break (ε_p, %) were determined experimentally. A total of 200 samples were tested.

The proposed method for determining the boundaries of the ranges of non-destructive loads and deformations, which is laid out below, is applicable if there is good reason to assume that the experimental values follow the Weibull distribution law. Regarding experimental data as sample values of a random variable with probability density function (2), we construct an estimate of its minimum possible value. Three probabilistic models are used for this purpose in statistical processing of experimental data. The first one is two-parameter model (1) with its parameters determined by the maximum likelihood method (MLM) [11]. The second one is three-parameter model (2). Shift parameter x_0 in it is determined using the semi-empirical formula from [12], and the remaining two parameters (a and k) are determined by MLM. In what follows, it is referred to as the semi-empirical three-parameter model. The third one is three-parameter model (2) with all its parameters determined simultaneously by MLM. It is referred to as the theoretical three-parameter model. The second and third models allow one to calculate estimates of the minimum value of the random variable being analyzed. The smaller value should then be chosen as the safe value boundary. Let us clarify the proposed method using the following example. We model sample X_N with size $N = 1000$ from the three-parameter Weibull distribution with probability density function (2) and the following parameter values: $x_0 = 1$, $k = 0.95$, and $a = 1.8$. The values of the random variable are considered to be dimensionless, since the proposed method for estimating the shift parameter is applicable to any data that follows the Weibull distribution and does not depend on units of measurement. The simulation result is presented in Fig. 1, *a* in the form of a relative frequency histogram. In Fig. 1, *b*, it is shown together with the plots of probability density functions corresponding to the three models listed above. The parameters of models were obtained numerically by searching for the maximum of the logarithmic likelihood function of sample X_N of a given probability model. The following parameter values were determined in calculations: $x_0 = 0$, $k = 2.0189$, and $a = 3.9491$ for the two-parameter model; $x_0 = 0.9182$, $k = 1.0360$, and $a = 2.0248$ for the

Table 1. Distribution center characteristics and minimum value for probabilistic models

	Statistical expectation	Median	Mode	Minimum value
Theoretical values	1.8448	1.7750	1.6054	1
Sampled values	1.8344	1.7694	1.6048	1.0164
Two-parameter model	1.8286	1.8400	1.8750	0
Three-parameter model (semi-empirical)	1.8337	1.7647	1.5972	0.9182
Three-parameter model (theoretical)	1.8338	1.7654	1.5995	0.9978

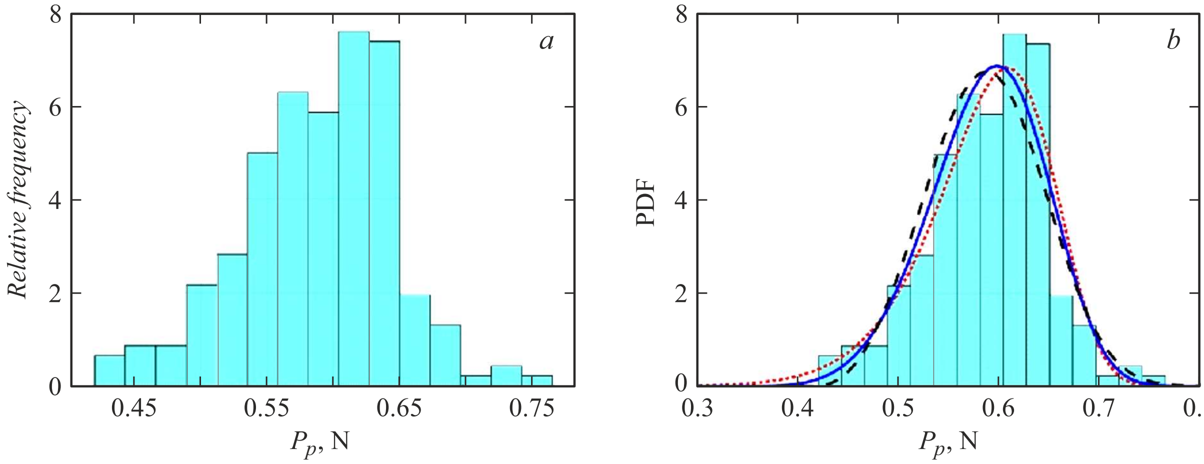


Figure 2. Results of statistical processing of experimental breaking loads of elementary polyamide-6 yarns. *a* — Relative frequency histogram; *b* — probability density functions of two-parameter (dotted curve), semi-empirical three-parameter (dashed curve), and theoretical three-parameter (solid curve) distributions.

semi-empirical three-parameter model; and $x_0 = 0.9978$, $k = 0.9403$, and $a = 1.8064$ for the theoretical three-parameter model. The maximum value of the logarithmic likelihood function corresponded to the theoretical three-parameter model.

When processing the results of modeling for each model, we fixed the minimum value and calculated statistical expectation MX, median $\mathrm{Med}\,X$, and mode $\mathrm{Mod}\,X$, which are the characteristics of the distribution center [13]:

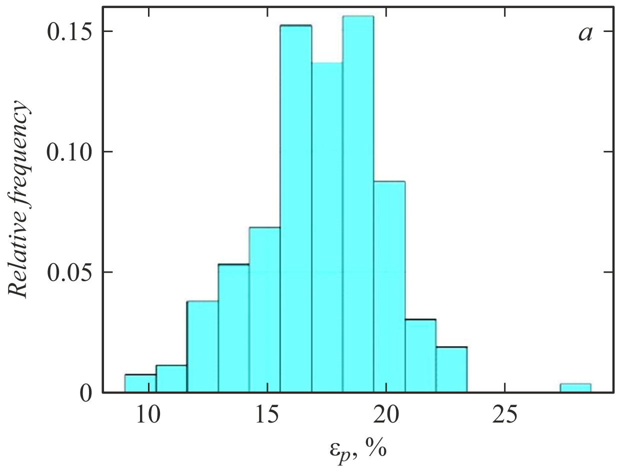
$$MX = k\Gamma\Big(1 + \frac{1}{a}\Big) + x_0, \quad \Gamma(z) = \int_0^{+\infty} t^{z-1} e^t dt$$

— gamma function,

$$\mathrm{Med}\,X = k(\ln 2)^{1/a} + x_0,$$

$$\mathrm{Mod}\,X = k\frac{(a-1)^{1/a}}{a^{1/a}} + x_0, \quad a > 1.$$

The calculation results are listed in Table 1. It follows from the obtained results that the theoretical three-parameter model provides the closest agreement with the simulation results and the most accurate estimate of the minimum value for simulated data. The semi-empirical three-parameter model yields a smaller value of the shift parameter; therefore, it may be treated as a conservative estimate. Note that the two-parameter model overestimates $\mathrm{Med}\,X$ and $\mathrm{Mod}\,X$ and is essentially unsuitable for estimating the smallest possible value of a random variable. Let us apply the proposed method in estimation of the range of non-destructive loads for elementary synthetic yarns of polyamide-6. Figure 2, *a* shows the relative frequency histogram constructed based on the data of breaking force P_p measurements. The probability density functions of the two-parameter, semi-empirical three-parameter, and theoretical three-parameter distributions are plotted in Fig. 2, *b*. The minimum values of P_p and the characteristics of the distribution center obtained with the use of three probabilistic models are listed in Table 2. The minimum value of P_p obtained with the theoretical three-parameter model turned out to be the smaller of two such values. According to the proposed probabilistic method for assessing the boundary of the range



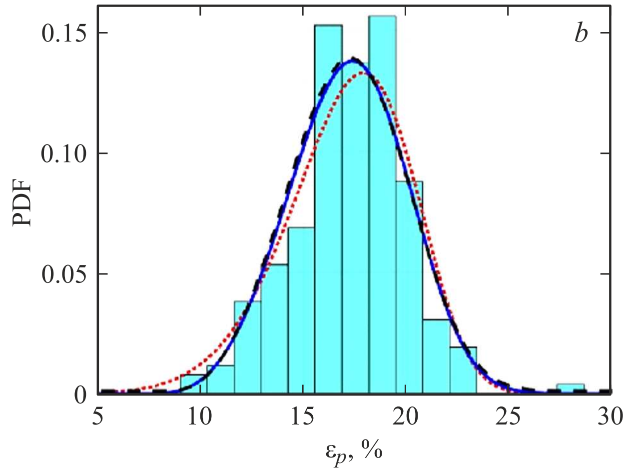


Figure 3. Results of statistical processing of experimental breaking deformations of elementary polyamide-6 yarns. *a* — Relative frequency histogram; *b* — probability density functions of two-parameter (dotted curve), semi-empirical three-parameter (dashed curve), and theoretical three-parameter (solid curve) distributions.

Table 2. Values of distribution center characteristics and minimum value for probabilistic models of breaking forces P_p and breaking deformations ε_p of polyamide-6 synthetic yarns

	Average value		Median		Mode		Minimum value	
	P_p, N	ε_p, %	P_p, N	ε_p, %	P_p, N	ε_p, %	P_p, N	ε_p, %
Sampled values	0.59	17.20	0.60	17.40	0.63	17.52	0.43	9.60
Two-parameter model	0.59	17.10	0.59	17.35	0.61	17.89	0	0
Three-parameter model (semi-empirical)	0.59	17.17	0.59	17.18	0.59	17.27	0.42	8.29
Three-parameter model (theoretical)	0.59	17.17	0.59	17.22	0.60	17.37	0.34	7.62

of safe non-destructive loads for the material under study, the value of $P_p = 0.34$ N may be used as this boundary.

In addition, correct calculations of the mechanical reliability of structures in many practical cases require the determination of boundaries of the range of safe deformations. In the present study, the proposed approach was also used to determine the boundary of the range of non-destructive deformations. Figure 3, *a* shows the relative frequency histogram constructed based on the results of measurements of deformations at failure ε_p. The probability density functions of the two-parameter, semi-empirical three-parameter, and theoretical three-parameter distributions are plotted in Fig. 3, *b*. The minimum values of ε_p and the characteristics of the distribution center obtained with the use of three probabilistic models are listed in Table 2. According to the proposed method for assessing the boundary of the range of safe non-destructive reformations for the material under study, the value of $\varepsilon_p = 7.62$ % may be set as this boundary.

In critically important areas, one may introduce strength and rigidity assurance coefficients to ensure greater reliability. Thus, a method for determining safe values of stresses and deformations for materials under mechanical loads was proposed. The method is based on a statistical approach to determination of the range of non-destructive loads and deformations rather than on critical breaking values. It was demonstrated that statistical evaluation of the boundaries of safe ranges of load and deformation of materials requires the use of the three-parameter Weibull model. It was proposed to calculate the parameters of the three-parameter model in two ways (semi-empirical and theoretical), which lead to different estimates of the shift parameter, and to determine the boundaries of safe loads and deformations based on the smallest of the two estimates of this parameter. The obtained results may be used in certification of products made from synthetic materials, to develop the methods for their production and determination of operating modes, and to develop new standards and technical specifications establishing scientifically validated performance standards.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Yu.M. Boiko, V.A. Marikhin, O.A. Moskalyuk, L.P. Myasnikova, E.S. Tsobkallo, Tech. Phys. Lett., **45**, 404 (2019). DOI: 10.1134/S1063785019040229.

[2] T.B. Kol'tsova, E.S. Tsobkallo, O.A. Moskalyuk, Izv. Vyssh. Uchebn. Zaved. Tekhnol. Legk. Prom-sti., №. 2, 9 (2021) (in Russian). DOI: 10.46418/0021-3489_2021_52_02_02

[3] R.K. Sundaram, K.S. Senthil, S.P. Edwin, R. Pandiyarajan, J. Chin. Inst. Eng., **45** (7), 588 (2022). DOI: 10.1080/02533839.2022.2101538

[4] Yu.M. Boiko, V.A. Marikhin, L.P. Myasnikova, J. Surf. Investig., **16** (3), 321 (2022). DOI: 10.1134/S1027451022030247.

[5] A. Shangguan, N. Feng, R. Fei, X. Hei, Y. Jin, L. Mu, Eksploatacja Niezawodność — Maintenance Reliability, **27** (3) (2025). DOI: 10.17531/ein/199426

[6] M.Yu. Prus, Tekhnol. Tekhnos. Bezop., **96** (2), 161 (2022) (in Russian). DOI: 10.25257/TTS.2022.2.96.161-179

[7] N.T. Thomopoulos, *Statistical distributions: applications and parameter estimates* (Springer, Cham, 2017). DOI: 10.1007/978-3-319-65112-5

[8] X. Gao, R.H. Dodds, Jr., R.L. Tregoning, J.A. Joyce, R.E. Link, Fatigue Fracture Eng. Mater. Struct., **22** (6), 481 (1999). DOI: 10.1046/j.1460-2695.1999.00202.x

[9] *Applied Statistics. Point and Interval Estimators for Parameters of Weibull Distribution*, GOST 11.007–75 (in Russian). https://www.russiangost.com/p-224011-gost-11007-75.aspx

[10] *Textile Threads. Methods for Determination of Breaking Load and Elongation at Rupture*, GOST 6611.2–73 (in Russian). https://meganorm.ru/Data2/1/4294823/4294823030.pdf

[11] S.A. Aivazyan, V.S. Mkhitaryan, *Prikladnaya statistika i osnovy ekonometriki* (YuNITI, M., 1998) (in Russian).

[12] L.F. V'yunenko, Obozr. Prikl. Prom. Mat., **8** (2), 561 (2001).

[13] R.N. Vadzinskii, *Spravochnik po veroyatnostnym raspredeleniyam* (Nauka, SPb., 2001), p. 159 (in Russian).

Translated by D.Safin