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New scenario of low-threshold decay of ordinary microwave in a blob

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Received March 3, 2025
Revised June 3, 2025
Accepted June 3, 2025

A new scenario of parametric instability of a beam of ordinary waves in a blob at the plasma edge is considered, resulting in the excitation of a 2D localized upper-hybrid wave and forced heavily damped oscillations.

Keywords: blob, microwave, upper hybrid wave, decay.

DOI: 10.61011/TPL.2025.08.61549.20303

Controlled thermonuclear fusion in a tokamak reactor requires an improved plasma confinement regime achieved at steep density gradients at the plasma edge [1]. Steep gradients in any physical system are a source of free energy, which is normally released in the form of instabilities. In the present case, these are edge localized modes [2]. Density perturbations with a filamentary structure extended along the magnetic field lines are observed at the nonlinear stage of development and saturation of this instability. These perturbations (blobs, filaments) are coherent structures formed and supported by turbulence.

Additional electron cyclotron (EC) heating is considered to be a reliable local method and is needed to achieve thermonuclear temperatures in a discharge and non-inductive current drive. According to the latest plans developed at the 34th meeting of the ITER Council in 2024, it is to be used to introduce up to 70 MW of microwave power into plasma at the ITER tokamak reactor [3]. However, the idea of a linear and perfectly predictable pattern of propagation and absorption of EC waves by electrons is confronted with contradictory results of recent experiments on electron cyclotron resonance heating (ECRH) at various toroidal plasma confinement facilities. Specifically, it was established [4,5] that the passage of a microwave beam through any local maximum of the density profile (including a blob at the plasma edge) makes it unstable with respect to parametric decays accompanied by the excitation of daughter waves and the emergence of further nonlinear phenomena. These data served as an experimental confirmation of predictions of the theory of low-threshold parametric decay instabilities of microwaves [6]. A scenario of low-threshold decay of an ordinary microwave has recently been investigated theoretically in the context of upcoming experiments at the ITER tokamak reactor. According to it, two electron Bernstein waves localized two-dimensionally in a blob at the plasma edge are excited [7]. This scenario is not dominant under the expected conditions of ECRH experiments at the ITER facility, where beams of ordinary waves with a frequency corresponding to the fundamental EC harmonic are planned to be used.

In the present study, we consider a new scenario of decay of an ordinary microwave that could be implemented in a blob at the plasma edge and involves nonlinear excitation of a 2D-localized upper hybrid wave and forced heavily damped oscillations.

Let us introduce local cylindrical and Cartesian coordinate systems (r, θ, z) and (x, y, z) with a common origin at the blob center and coordinate z parallel to the magnetic field. Coordinate x is associated with the magnetic surface label, and y is the coordinate perpendicular to the magnetic field line on the magnetic surface. We present the density as a sum of background density $n_0(r)$ and blob density $\delta n(r) = \delta n_0 \exp(-r^2/r_b^2)$, where $r_b \propto (5-10) r_s$ is the blob size [8], r_s is the ion gyroradius with electron temperature, and δn_0 is the blob amplitude that may reach 50 % of the local n_0 value [9]. Since the characteristic scales of background plasma are much larger than r_b, only the spatial dependence of density in the blob $\delta n(r)$ is taken into account.

Let us consider a quasi-transverse beam of ordinary microwaves incident onto a blob. The typical transverse size of the beam along y is significantly larger than blob dimensions r_b. The distribution of electric field in the beam along z may be modeled by a piecewise continuous function with the following spectrum:

$$a_0(y, k_z) = a_0(y, 0)\frac{2\sin(k_z w_z/2)}{k_z}. \tag{1}$$

The electric field of a pump wave with power P_0 may be presented in the following form [10]:

$$\mathbf{E}_0 = \sqrt{\frac{2\pi P_0}{w_y w_z c}} \sum_{s=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk_z w_z}{\sqrt{2\pi}} \mathbf{e}_0(r) a_0(0, k_z) A_s(r)$$
$$\times \exp(is\theta + ik_z z - i\omega_0 t) + \text{c.c.}, \tag{2}$$

where

$$A_s(\mathbf{r}) = \big(k_r(s, r) r\big)^{-1/2} \exp\bigg(-\frac{c^2 s^2}{2\omega_0^2 w^2}$$
$$- i\int^r k_r(s, \xi) d\xi + is\pi/2\bigg)$$

is the amplitude, k_r is the wave number of an ordinary wave [10], $w_{y,z}$ are the beam dimensions, $\mathbf{e}_0 = \left(-n_z\mathbf{e}_x, in_z\mathbf{e}_y\omega_0/\omega_{ce}, \mathbf{e}_z\right)$ is the polarization vector of a wave in plasma [6], ω_{ce} is the EC frequency, and $n_z = ck_z/\omega_0 \ll 1$ is the longitudinal refraction index.

Next, we analyze the behavior of upper hybrid (UH) waves in a blob. In weakly inhomogeneous plasma, the integral equation characterizing a UH wave has the form

$$\hat{D}_E\phi_E = \frac{1}{(2\pi)^4}\int\limits_{-\infty}^{\infty}\phi_E(\mathbf{r}', t')\left(\int\limits_{-\infty}^{\infty} D_E\left(\mathbf{q}, \frac{\mathbf{r}+\mathbf{r}'}{2}, \omega\right)\right.$$
$$\left.\times \exp\left(i\mathbf{q}(\mathbf{r}-\mathbf{r}') - i\omega(t-t')\right)d\mathbf{q}d\omega\right)d\mathbf{r}'dt' = 0, \tag{3}$$

where the kernel of the integral transform set equal to zero $(D_E(\mathbf{q}, \omega) = 0)$ is the dispersion relation of quasi-longitudinal UH waves [11]. The solution of Eq. (3) takes the following form [6]:

$$\phi_E = B_{n,m}\varphi_{n,m}(r)\exp\left(im\theta + i\omega_{n,m}t\right) + \text{c.c.}, \tag{4}$$

where eigen function $\varphi_{n,m}(r)$ may be presented as [6]:

$$\varphi_{n,m}(r) = \left(\pi|\partial D_E/\partial q_r|_{q_{Er},r}\int\limits_{r_{m1}}^{r_{m2}}|\partial D_E/\partial q_r|^{-1}_{q_{Er},r'}r'dr'\right)^{-1/2}$$
$$\times\cos\left(\int\limits_{r_{m1}}^{r} q_{Er}\left(\xi, \omega_{n,m}\right)d\xi - \frac{\pi}{4}\right), \tag{5}$$

$q_{Er} = \sqrt{q_{E\perp}^2(r) - (m^2 - 1/4)/r^2}$, $q_{E\perp}$ — solution of dispersion relation $D_E(\mathbf{q}, \omega) = 0$, and $r_{m1,2}$ — points of mode m turning in the radial direction. Eigen frequency $\omega_{n,m}$ is determined from the condition $\int\limits_{r_{m1}}^{r_{m2}} q_{Er}\left(\xi, \omega_{n,m}\right)d\xi = \pi n$.

At $m \gg 1$, Eq. (4) characterizes the whispering gallery modes of a UH wave [12] localized in a plane perpendicular to the magnetic field and travelling predominantly along the azimuthal angle with constant amplitude $B_{n,m} = \text{const}$. This is illustrated in Fig. 1, which shows the trajectory of the whispering gallery mode $n = 1$, $m = 6$ of a UH wave that was calculated using the ray tracing procedure with the parameters of ECRH experiments at the ASDEX-Upgrade tokamak [4]. The beat of pump wave (2) with the whispering gallery mode of a UH wave (4), the initial amplitude of which is at the level of thermal noise, induces forced oscillations with imposed wave number $\mathbf{q}_I = (q_{Er}, m+s, k_z)$ at difference frequency $\omega_0 - \omega_{n,m}$. The potential of these oscillations with attenuation prevailing over convective losses is characterized by the Poisson

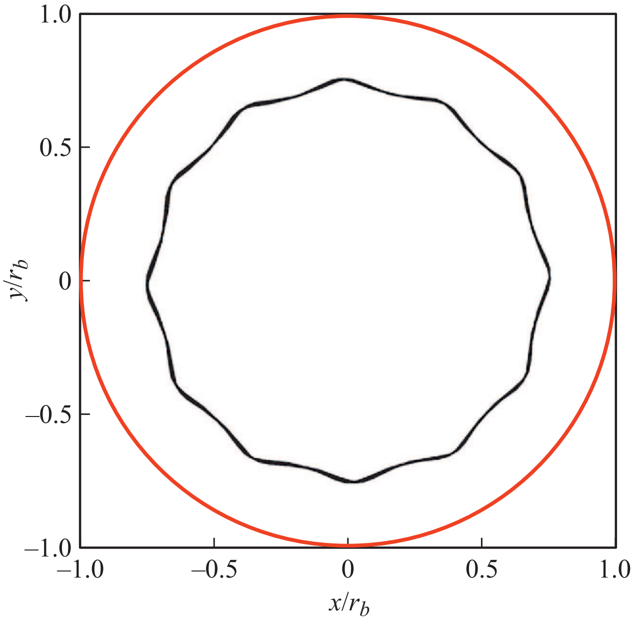

Figure 1. Trajectory of a UH wave corresponding to the whispering gallery mode ($n = 1$, $m = 6$) under the conditions of ECRH experiments at the ASDEX-Upgrade tokamak [4]. The outer solid line indicates the blob boundary.

equation and may be presented in the following form [6]:

$$\phi_I = \sum_{p=-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{dk_z w_z}{\sqrt{2\pi}}\frac{C_p(k_z)}{2}$$
$$\times\exp\left(i\int^r q_{Er}(m)d\xi + ip\theta + ik_z z - i(\omega_0 - \omega_{n,m})t\right) + \text{c.c}, \tag{6}$$

where the partial amplitude is written as

$$C_p(k_z) = \sqrt{\frac{8\pi P_0}{w_y w_z c}}$$
$$\times\sum_{m=-\infty}^{\infty}\left.\frac{\chi_e^{nl}(k_z)}{2(D_I' + iD_I'')}\right|_{\omega_0-\omega_{n,m}, q_{Er}, p, k_z}\frac{A_{p-m}}{\overline{B}}B_{n,m}, \tag{7}$$

where $\overline{B}$ is the local magnetic field in a blob, D_I' and D_I'' are the residual and the imaginary part of the dispersion relation of longitudinal oscillations [11], and $\chi_e^{nl} \propto k_z$ is the second-order plasma susceptibility [13].

The equation for the whispering gallery mode of a UH wave induced by a nonlinear excitation $\sim \chi_e^{nl^*}\phi_I\mathbf{E}_0^*\cdot\mathbf{e}_0^*/\overline{B}$ takes the following form [6]:

$$\hat{D}_E\phi_E = i\chi_e^{nl^*}\phi_I\mathbf{E}_0^*\cdot\mathbf{e}_0^*/\overline{B}. \tag{8}$$

Let us use the perturbation theory procedure. At the first step, we neglect nonlinear pumping. This reduces the equation to homogeneous Eq. (3), a particular solution of which is expression (4) with a constant amplitude. At the second step of the perturbation theory procedure, we take nonlinear pumping, which is characterized by the right-hand side of Eq. (8), into account. This makes the UH wave amplitude dependent on time and longitudinal coordinate. In the coordinate representation, the resulting equation for the amplitude of whispering gallery mode m is

$$\frac{\partial B_{n,m}}{\partial t} - i\Lambda_{n,m}\frac{\partial^2 B_{n,m}}{\partial z^2} + \nu_{ea}B_{n,m} = \gamma_0 w_z\left(\delta\left(z-\frac{w_z}{2}\right)+\delta\left(z+\frac{w_z}{2}\right)\right)B_{n,m}, \quad (9)$$

where $\Lambda_{n,m}$ — averaged coefficient of UH wave diffraction along the magnetic field, ν_{ea} — electron–atom collision rate,

$$\gamma_0 = \gamma_0' + i\gamma_0'' = \frac{i}{\langle|\partial D_E/\partial\omega|_{\omega_{n,m},q_{Er}}\rangle\omega_0 w_y w_z^2\overline{B}^2} \times \sum_{p=-\infty}^{\infty}\int_0^{\infty}\frac{|\chi_e^{nl}|^2 A_{m-p}^* A_{p-m}}{(D_I' + iD_I'')_{\omega_0-\omega_{n,m},q_{Er}+k_r,p,k_{z0}}}|\varphi_{n,m}(r)|^2 r dr. \quad (10)$$

We seek an unstable solution of Eq. (9) in the form $B_{n,m} = B_0\exp(i\delta\omega_{n,m}t + \gamma_{n,m}t)$, which reduces it to the one-dimensional Schrödinger equation for a molecular hydrogen ion H_2^+ [14]:

$$\partial^2 B_0/\partial z^2 + 2Q\big(\delta(z-w_z/2)+\delta(z+w_z/2)\big)B_0 = -2EB_0, \quad (11)$$

where

$$Q = -i\tilde{\gamma}_0 w_z/(2\Lambda_{n,m}),\ E = (\delta\omega_{n,m} + i\gamma_{n,m} + i\nu_{ea})/(2\Lambda_{n,m})$$

and $\delta(z)$ — delta function. According to [14], the solution of Eq. (11) may be presented as a linear combination of two functions

$$B_0 = B_0^{(1)} + B_0^{(2)} = D_1\exp\big(-\kappa|z-w_z/2|\big) + D_2\exp\big(-\kappa|z+w_z/2|\big), \quad \kappa = \sqrt{-2E}. \quad (12)$$

Function B_0 is continuous, but its derivative has discontinuities at points $z = \pm w_z/2$. Equation (11) also features integrable singularities associated with the contributions of delta functions δ. The discontinuities of the derivative of function (12), which are specified by κ, should be such that they are compensated by the contributions of the double delta potential. The behavior of $dB_0^{(k)}/dz$ $(k = 1, 2)$ in the neighborhood of $z = \pm w_z/2$ may be estimated by inserting (12) into (11) and integrating around these points within the range from $-\varepsilon \pm w_z/2$ to $\pm w_z/2 + \varepsilon$, where ε is an infinitesimal quantity. This procedure yields two conditions for variation of the derivative when travelling along coordinate z in the positive direction $(d\ln B_0^{(k)}/dz)_{+\varepsilon} - (d\ln B_0^{(k)}/dz)_{-\varepsilon} + 2QB_0|_{z=\mp w/2} = 0$, $k = 1, 2$. Thus, at the first and second points:

$$2\kappa D_1 = 2Q\big(D_1 + D_2\exp(-\kappa|w_z|)\big),$$

$$2\kappa D_2 = 2Q\big(D_1\exp(-\kappa|w_z|) + D_2\big).$$

The determinant of this system leads to the sought-for dispersion relation

$$\kappa_\pm = Q\big(1 \pm \exp(-\kappa_\pm w_z)\big), \quad (13)$$

where the upper sign corresponds to the symmetric state with $C_1/C_2 = 1$. Well above the instability threshold ($Qw_z \gg 1$), the approximate solution of Eq. (13) is $\kappa_+ \approx Q$. This allows us to determine instability increment IFx75Xe in explicit form and the correction to eigen frequency $\omega_{n,m}$

$$\delta\omega_{n,m} = \frac{\tilde{\gamma}_0''^2 - \tilde{\gamma}_0'^2}{4\Lambda_{n,m}}w_z^2. \quad (15)$$

Let us use Eq. (14) to analyze the possibility of nonlinear excitation of the whispering gallery mode of a UH wave during ECRH at the ASDEX-Upgrade tokamak [4]. Figure 2 shows the dependence of instability increment on power under these conditions. The threshold power is 0.352 MW, which is significantly lower than the power of the megawatt beams used [4].

Thus, a new scenario of instability of an ordinary microwave with a frequency exceeding the fundamental EC resonance frequency in a plasma blob with axial symmetry was analyzed. Within this scenario, the pump wave decay leads to the emergence of a whispering gallery mode of an upper hybrid wave, which is confined two-dimensionally in the plasma volume, and forced heavily damped oscillations. An expression for the increment of this nonlinear phenomenon was obtained using the proposed model. The obtained results may help interpret the anomalous scattering of microwaves at the plasma edge that was correlated with the excitation of edge localized modes [2] and observed at the ASDEX-Upgrade and TCV tokamaks [4,5]. They may also be used to analyze the strong anomalous absorption of microwaves in a plasma blob that was discovered in model experiments at a linear facility [15].

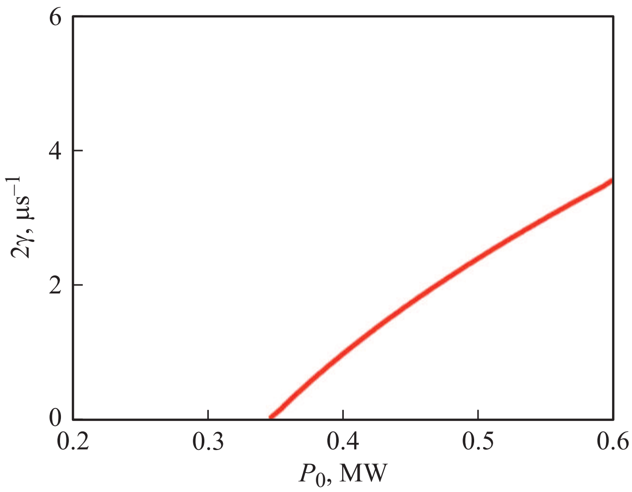

Figure 2. Dependence of instability increment on power under the conditions of ECRH experiments at the ASDEX-Upgrade tokamak [4]. The threshold power is 0.352 MW.

Funding

This study was supported by grant No. 23-72-00024 from the Russian Science Foundation.

Conflict of interest

The authors declare that they have no conflict of interest.

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Translated by D.Safin