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Surface plasma oscillations in a semiconductor nanolayer

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The problem of surface plasma oscillations propagating along a layered „dielectric-semiconductor-dielectric“ nanostructure is solved within the quantum theory of charge carrier transport phenomena. The case of symmetrical charge carrier distribution at the boundaries of a semiconductor nanolayer is considered. Expressions are derived for the wave propagation and attenuation coefficients as a function of the semiconductor layer thickness, surface wave frequency, chemical potential, permittivity of insulating layers, and roughness parameters of the „semiconductor-dielectric“ interfaces. Oscillations of the frequency dependences of the wave propagation length are detected, the period and amplitude of which correspondingly depend on the semiconductor nanolayer thickness and the surface roughness parameters.

Keywords: surface plasmons, semiconductor nanolayer, kinetic equation, Soffer model.

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Introduction

Recently, researchers are greatly interested in studying specific features of propagation of surface plasma oscillations in various nanostructures [1–5]. Active development of plasmonics is due to improvement of technologies that can create nanostructures of any complexity with a typical size of on the order of nanometers. It can be exemplified by metal nanolayers with an applied periodic lattice or with nanoholes used as plasmon waveguides [6–8]. Plasmonics development is facilitated by improvement of high-sensitivity optical technologies that make it possible to experimentally study characteristics of surface waves [9]. The above said indicates relevance of theoretical research in the field of plasmonics.

Today, we can highlight a number of theoretical studies that are dedicated to investigating special features of propagation of the surface plasma wave in conducting nanolayers. In the papers [2], the authors have taken into account spatial dispersion of polarizability of the HgTe/CdHgTe electron gas and shown that within a high-frequency area the plasmon dispersion law goes from the root one to the linear one. The studies [10,11] have obtained expressions for coefficients of propagation and decay of the surface plasmon taking into account surface charge carrier scattering for a symmetric and antisymmetric configuration. We note that studies [10,11] considered a case of a metal without taking into account quantization of the energy spectrum of the charge carriers.

The present study has obtained an analytical solution of the problem about surface plasma oscillations in the semiconductor nanolayer with taking into account the quantum theory of the transfer phenomena. Surface charge carrier

scattering is taken into account by the Soffer boundary conditions [12].

1. Problem formulation

Let us review a semiconductor nanolayer of the thickness a , which is located between two non-magnetic insulating layers with the same permittivities. We take the Cartesian system of coordinates with the axis Z that is oriented inwards the nanolayer and the axis X that is directed parallel to propagation of the surface wave (Fig. 1). We note that the surface wave can not propagate in case of TE polarization [1]. We will review only TM polarization, when the vectors of strengths of the electric and magnetic fields are as follows: $\mathbf{E} = \{E_x, 0, E_z\}$, $\mathbf{H} = \{0, H_y, 0\}$.

It is known that plasma oscillations with the symmetric and antisymmetric charge distribution at boundaries can propagate along thin layers [1]. The first case corresponds to a situation when the charge signs at the upper and lower boundaries of the layer coincide, while the second case correspond to a situation when they are different. The first case will be reviewed in the present study, which is related to the following considerations. The layer, along which the oscillations propagate, can be conditionally divided into areas where an excessive positive (negative) charge is concentrated. Fig. 1 shows that a center of these areas, where the charge is the greatest, will be dominated by the electric field component E_z (and, therefore, H_y). Between the areas, i.e. where the charge at the boundaries is the least, the component E_x will be predominant. In case of symmetrical charge distribution and the thin layer, the electric field strength will practically be without the z -component inside the layer. It follows from a superposition principle, i.e. power lines of the electric field strength vector,

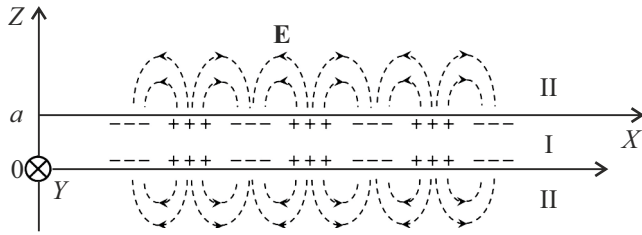


Figure 1. „Dielectric-semiconductor-dielectric“ nanostructure, along which the surface wave propagates: I — the semiconductor, II — the dielectric. The dashed lines mark power lines of the electric field strength vector.

which go from the upper and the lower boundaries of the layer, are oppositely directed to each other. Therefore, only the component E_x will be inside the layer. In case of antisymmetric charge distribution, the electric field's transverse component E_z will be predominant. Under effects of this field, the charge carriers move perpendicular to the layer surface and are accumulated at the boundaries, thereby modifying the field itself. This situation necessitates solving a self-consistency problem that is reduced to a system of integral equations of the function of the charge carrier distribution and the field strength E_z . It is a more complicated task as compared to the case when the charge sign at the nanolayer boundaries is the same. Therefore, we stop on reviewing the case of symmetrical charge distribution.

We note that the surface charges at the boundaries of the conducting layer affect a nature of distribution of the charge and the electric field along the thickness, which requires solution of the system of equation for determining the function of distribution of charge carriers and the electric field strength. This issue was reviewed in the studies [13–15], in which dependences of the charge carrier concentration and the electric field strength on a coordinate were found for a lanthanum-strontium cuprate and an indium oxide nanoparticle. The results showed that a width of the area of nonuniform charge distribution near the layer boundaries is estimated to be 5 nm. The surface charge generates a quite strong transverse electric field of the strength of 10^8 V/m. This field causes motion of the charge carriers in a direction perpendicular to the layer surfaces. As noted above, in case of symmetrical charge distribution the nature of the plasma oscillations is predominantly affected by a longitudinal electric field. Therefore, nonuniformity of the charge and heterogeneity of the transverse electric field will be omitted.

In the present study, permittivity of the insulating layers is assumed to be constant. This factor limits a material from which the insulating layers are made of. We will assume that these layers are made of a nonpolar dielectric. This case involves only electron polarization that does not noticeably contribute to a frequency dependence of permittivity up to ultraviolet frequencies [16]. It is possible

to use nonpolar dielectrics or wide-band semiconductors with an ion type of a bond between atoms (for example, the solid solutions $\text{Al}_{1-x}\text{Ga}_x\text{As}$, $\text{Al}_{1-x}\text{In}_x\text{Sb}$, etc.). At the same time, resonance absorption is possible at the IR-range frequencies, but away from a resonance the permittivity can be assumed to be constant.

It is assumed that a thickness of the semiconductor nanolayer can be comparable or less than the de Broglie wavelength of the charge carriers. In this situation the charge carrier systems shall be reviewed as a quantum one. The electron gas shall be reviewed as a quasi-two-dimensional gas included in a quantum well with infinitely high walls. The expression for the full energy of the electron (hole) for the l -th subband will be as follows:

$$\varepsilon_l = \frac{\hbar^2}{2m}(k_x^2 + k_y^2) + \varepsilon_l l^2, \quad (1)$$

$$l = \pm 1, \pm 2, \pm 3 \dots \pm N, \quad (2)$$

where m — the effective mass of the electron (hole), $\hbar$ — the Planck constant, $\varepsilon_1 = (\pi\hbar)^2/(2ma^2)$ — the eigenvalue of the charge carrier energy at the first energy level, N — the total number of the subbands.

In accordance with the quantum theory of the transfer phenomena, the charge carrier system is described by a density operator [17]:

$$\hat{\rho}(z, \mathbf{k}_{\parallel}, t) = \sum_l W_l \langle \psi_l(z, \mathbf{k}_{\parallel}, t) \rangle \langle \psi_l(z, \mathbf{k}_{\parallel}, t) \rangle, \quad (3)$$

which complies with the Liouville's quantum equation:

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}], \quad (4)$$

where ψ_l — the wave function of the charge carrier system at the l -th subband, W_l — the statistic weight, $\mathbf{k}_{\parallel}$ — the longitudinal component of the wave vector.

We note that in reality the nanolayer surface is not perfectly smooth. The mean square height of the surface relief may vary from tenths to several nanometers. With the thickness of the nanolayer of about 10 nm, surface irregularity at an atom level can significantly affect the transfer phenomena in the nanolayer. One of the factors affecting surface charge carrier scattering is nontrivial distribution of the field and the charge in dependence on surface curvature due to its roughness. The study [18] has investigated a distribution of the electric field and the charge in a metal tip that imitates an element of surface irregularity. It was shown that the most significant change of the field strength is observed near a cone apex within the area of the width of 0.5 nm. The value of the electric field near the cone apex can exceed the applied field in four times. Study of distribution of the electric field and the charge in dependence on the surface curvature is a separate difficult problem. It is possible to significantly simplify the calculations when assuming that the height of the surface relief is small as compared to the nanolayer thickness. In this

case, the quantum well with an irregular boundary can be replaced by a quantum well with smooth boundaries, while the surface irregularity can be interpreted as a disturbance of the system Hamiltonian. The full Hamiltonian of the carrier charge system can be presented as a sum of the Hamiltonian that describes the equilibrium state $\hat{H}_0$ and a correction that takes into account volume and surface scattering $\hat{V}$ [19]:

$$\hat{H} = \hat{H}_0 + \hat{V}. \quad (5)$$

We note that in the most semiconductors of the n type the Hamiltonian of the electron system in the conduction band depends on the wave number as per the quadratic law. Some semiconductors, whose crystal structure has no inversion center (for example, GaAs that has a zinc blende structure) have a deviation from the quadratic dependence of $H_0(\mathbf{k})$ due to spin-orbit interaction of the charge carriers [20] by several percent. In most cases, this deviation can be neglected. In a valence band, spin-orbit interaction of the charge carriers becomes significant [21]. Specific features of a law of dispersion of the valence band in the semiconductors have been analyzed to show that the dependence of the energy on the wave vector is not described by the quadratic law.

The present study assumes that the conducting nanolayer is a semiconductor of the n type of conductivity. We will think that the Hamiltonian of the charge carrier system depends on the wave number of the charge carrier by the quadratic law.

$$\hat{H}_0 = \frac{\hbar^2 k^2}{2m}. \quad (6)$$

It is shown in the papers [19,22] that in a situation when the charge carrier system slightly deviates from the equilibrium state, the equation (4) can be reduced to the following kinetic equation:

$$-v f_l^{(1)} + v_z \frac{\partial f_l^{(1)}}{\partial z} + \frac{e\mathbf{E}}{\hbar} \frac{\partial f_l^{(0)}}{\partial \mathbf{k}_{\parallel}} = 0. \quad (7)$$

Here, $\nu = \tau^{-1} - i\omega$ — the complex scattering frequency, τ — the relaxation time, $\hbar$ — the Planck constant, v_z — the projection of the vector of the charge carrier velocity on the axis Z, $\mathbf{k}_{\parallel}$ — the component of the charge carrier vector, which is parallel to the nanolayer plane, e — the charge of the electron (hole), f_l — the function of distribution of the charge carriers at the l -th subband, which performs as the l -th diagonal element of the density matrix. The function f_l can be presented in an expansion that is linear by the external electric field:

$$f_l(z, \mathbf{k}_{\parallel}, t) = f_l^{(0)} + f_l^{(1)}(z, \mathbf{k}_{\parallel}) \exp(-i\omega t), \quad (8)$$

$$f_l^{(0)} = \frac{1}{1 + \exp((\varepsilon_l - \mu)/k_B T)}, \quad (9)$$

where $f_l^{(0)}$ — the equilibrium distribution function, $f_l^{(1)}$ — the non-equilibrium correction, μ — the chemical potential, k_B — the Boltzmann constant, T — the temperature.

The surface charge carrier scattering is taken into account by the Soffer boundary conditions [12] applied to the equation (7):

$$\begin{cases} f_l^{(1)+} = q_1(g_1, \theta) f_l^{(1)-}, & z = 0, \\ f_l^{(1)-} = q_2(g_2, \theta) f_l^{(1)+}, & z = a, \end{cases} \quad (10)$$

$$q_{1,2}(g_{1,2}, \theta) = \exp\left(-4\pi g_{1,2} \cos \theta\right)^2, \quad (11)$$

$$g_{1,2} = \frac{g_{s1,2}}{\lambda_B}, \quad (12)$$

where $f_l^{(1)\pm}$ — the functions of distribution of electrons with a positive and negative projection of the wave vector to the axis Z, respectively, $g_{s1,2}$ — the mean square height of the surface relief of the upper and the lower surface, respectively, λ_B — the de Broglie wavelength of the electron (hole), θ — the incident angle of the charge carrier to the internal surface of the semiconductor nanolayer.

We note that the Soffer model (10)–(12) is obtained as a result of solving the problem about interaction of the quasi-monochromatic wave with the metal boundary with taking into account a far field approximation. A package of the flat waves was assumed to be quite big so that the wave could be considered to be quasi-monochromatic, but quite small so that the far field approximation is fulfilled for calculating an amplitude of the de Broglie wave scattered from the nanolayer surface. We note that the author of the study [12] expresses doubts about applicability of the model of boundary conditions for semimetals and semiconductors (the far field approximation is not fulfilled). We will show that in case of a semiconductor film the far field approximation and is applicable and the Soffer model can be used. In a metal, free charge carriers which ensure flow of electric current have a fixed energy that corresponds to the Fermi energy. Therefore, the de Broglie wavelength of electrons in the metal will be constant. The charge carriers experience volume and surface scattering, the electron gas can be presented as a set of spherical waves with the same wavelength, which are reflected from scattering sources. The wave package considered in the paper [12] is a result of interference of the spherical waves and its size is determined by a distance between the scattering sources (in case of the metal it is atoms of a crystal lattice). In the semiconductor, an additional factor that affects the size of the wave package is a spread of thermal velocities of the charge carriers. As a result of phonon and impurity scattering, the wave vector can change its amplitude, i.e. the de Broglie wavelength is changed. This factor reduces a size of the wave package, within which the wave can be assumed to be monochromatic. It makes it possible to apply the far field approximation in the semiconductors. We will assume that the mean square height of the surface relief is assumed to be small as compared to the film thickness, because the kinetic equation (7) was obtained within the disturbance theory. Therefore, the parameters g_{s1} and g_{s2} are less than the de Broglie wavelength of the charge carriers

λ_B in case when the nanolayer thickness is comparable with λ_B . It corresponds to the condition, at which the far field approximation is fulfilled [23]:

$$\frac{g_{s1,2}^2}{b\lambda_B} \ll 1, \quad (13)$$

where b — the typical scale of variation of the amplitude of the de Broglie wave.

We note that the Soffer model is applicable in case when there are effects of dimensional quantization of the charge carriers [12].

The non-equilibrium distribution function makes it possible to calculate the current density and the integral conductivity by the formulas [19,22]:

$$j = \frac{2ek_1}{(2\pi)^3} \sum_l \int \int v_x (f_l^{(1)+} + f_l^{(1)-}) dk_x dk_y, \quad (14)$$

$$\sigma_a = \int_0^a \frac{j}{E} dz, \quad (15)$$

where k_1 — the z -component of the wave vector of the charge carrier at the first subband.

2. Calculation of characteristics of the surface wave

By solving the equation (7) with taking into account the boundary conditions (10), successively substituting $f_l^{(1)+}$ and $f_l^{(1)-}$ into (14) and (15) and carrying out a series of mathematical calculations, we will obtain the following expression for the integral conductivity:

$$\sigma_a = \sigma_0 a \Sigma(x_0, x_\lambda, y_0, g_1, g_2, u_\mu), \quad (16)$$

$$\sigma_0 = \frac{n_v e^2 \tau_v}{m}, \quad (17)$$

$$\Sigma = \frac{\sqrt{u_{0v}}}{2x_0 I_{1/2} z_0} \sum_{l=1}^{\infty} \ln(\exp(u_\mu - u_1 l^2) + 1) \times \left(1 - \chi\left(\frac{2x_0^2 z_0}{l x_\lambda}\right)\right), \quad (18)$$

$$\chi(p) = \frac{1}{2p} (1 - e^{-p}) \frac{2 - q_1 - q_2 + (q_1 + q_2 - 2q_1 q_2) e^{-p}}{1 - q_1 q_2 e^{-2p}}, \quad (19)$$

$$q_{1,2}(g_1, 2, \theta) = \exp\left(-\left(2\pi g_{1,2} l / x_0\right)^2\right), \quad (20)$$

$$z_0 = v \tau_v = \frac{\tau_v}{\tau} - i y_0, \quad x_0 = \frac{a}{\lambda_{B0}}, \quad x_\lambda = \frac{\Lambda}{\lambda_{B0}}, \quad y_0 = \omega \tau_v, \quad (21)$$

$$I_s = \int_0^\infty \frac{(u^s du)}{\exp(u - u_\mu) + 1}, \quad (22)$$

$$u_0 = \frac{mv_0^2}{2k_B T}, \quad u_{0v} = \frac{mv_{0v}^2}{2k_B T}, \quad u_1 = \frac{mv_1^2}{2k_B T} = \frac{u_{0v}}{4x_0^2}, \quad u_\mu = \frac{\mu}{k_B T}. \quad (23)$$

Here, the following dimensionless parameters are introduced: z_0 — the complex scattering frequency, x_0 — the thickness of the semiconductor nanolayer, x_λ — the mean free path of the charge carriers, y_0 — the surface wave frequency. The parameters x_0 and x_λ are rated to the de Broglie wavelength of the charge carrier that moves with the typical velocity u_{0v} . z_0 and y_0 are rated to the relaxation time of the charge carriers τ_v without taking into account quantization of their energy spectrum. u_1 — the transverse velocity of the charge carriers for the first subband, u_0 and u_{0v} — the typical velocities of the charge carriers with and without taking into account quantization of the energy spectrum of the charge carriers, respectively, which are introduced as follows:

$$nv_0^2 = 4\left(\frac{m}{h}\right)^3 v_1 \frac{5}{3} \sum_{l=1}^{\infty} \int \int (v_\parallel^2 + v_1^2 l^2) f_l^{(0)} dv_x dv_y, \quad (24)$$

$$n_v v_0 v^2 = 2\left(\frac{m}{h}\right)^3 \frac{5}{3} \int \int \int v^2 f_0 d^3 v, \quad (25)$$

$v_\parallel$ — the longitudinal velocity of the charge carrier, n_v and n — the concentration of the charge carriers in the macroscopic sample and the nanolayer, respectively, which are determined by the expressions:

$$n = 4 \sum_{l=1}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_l^{(0)} k_1 \frac{dk_x dk_y}{(2\pi)^3} = 4\pi \left(\frac{m}{h}\right)^3 \left(\frac{2k_B T}{m}\right)^{3/2} \sqrt{u_1} P, \quad (26)$$

$$n_v = 2 \int \int \int f_0 \frac{d^3 k}{(2\pi)^3} = 4\pi \left(\frac{m}{h}\right)^3 \left(\frac{2k_B T}{m}\right)^{3/2} I_{1/2}. \quad (27)$$

Using (24)–(25), we obtain the following expressions for the parameters u_0 and u_{0v} :

$$u_0 = \frac{5}{3} \frac{K}{P}, \quad u_{0v} = \frac{5}{3} \frac{I_{3/2}}{I_{1/2}}, \quad (28)$$

$$K = \sum_{l=1}^{\infty} \int_{u_1 l^2}^{\infty} \frac{u du}{\exp(u - u_\mu) + 1}, \quad (29)$$

$$P = \sum_{l=1}^{\infty} \ln(\exp(u_\mu - u_1 l^2) + 1). \quad (30)$$

In case of a degenerate electron gas, u_0 and u_{0v} transform into the Fermi velocity u_F , and in case of a non-degenerate Fermi gas they are of the order of the average thermal velocity of the charge carriers.

$$v_0 v^2 \approx \frac{5k_B T}{m}, \quad (31)$$

$$v_0^2 \approx \frac{10k_B T}{3m} \left(1 + \frac{\Sigma u_1 l^2 \exp(-u_1 l^2)}{\Sigma \exp(-u_1 l^2)}\right). \quad (32)$$

The meaning of introduction of the magnitudes τ_v and u_{0u} is as follows. The change of the thickness of the nanolayer results in variation of a position of the energy level and, therefore, in the change of the mean square velocity of motion of the charge carriers u_0 (there is a dependence $u_0(a)$). The present study assumes that without surface scattering the mean free path of the charge carriers does not depend on the thickness and is determined as a product of the average velocity of motion of the charge carriers by the mean free time:

$$\Lambda = u_0 \tau = u_{0v} \tau_v. \quad (33)$$

Therefore, the availability of the dependence $u_0(a)$ results in that the parameter τ depends on the nanolayer thickness. The parameters τ_v and u_{0v} do not depend on the thickness, therefore, they have been used for rating the magnitudes v , a , Λ , ω .

Using the expression (33), we will express the ratio τ_v/τ included in the parameter z_0 via the typical velocities:

$$z_0 = \frac{\tau_v}{\tau} - iy_0 = \frac{\sqrt{u_0}}{\sqrt{u_{0v}}} - iy_0. \quad (34)$$

In order to find the characteristics of the surface wave, we will find surface impedances inside the semiconductor nanolayer and from the side of the insulating layers to be determined as a ratio of the x -component of the electric field strength E_x and the y -component of the magnetic field strength H_y .

The electric and magnetic components of the surface wave are described by the system of wave equations:

$$\begin{cases} \frac{\varepsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \Delta \mathbf{E} = 0, \\ \frac{\varepsilon}{c^2} \frac{\partial^2 \mathbf{H}}{\partial t^2} - \Delta \mathbf{H} = 0. \end{cases} \quad (35)$$

Outside the conducting layer, the dependence of $\mathbf{E}$ and $\mathbf{H}$ on the time and the coordinates x and z can be obtained as a solution of the system of equations (35), which is presented as the wave that is running along the axis X and evanescent when $z \rightarrow \pm\infty$:

$$\begin{cases} \mathbf{E} = \mathbf{E}_0 \exp(-i\omega t + \alpha z + ikx), & z < 0, \\ \mathbf{E} = \mathbf{E}_0 \exp(-i\omega t + \alpha(a - z) + ikx), & z > a, \end{cases} \quad (36)$$

$$\begin{cases} \mathbf{H} = \mathbf{H}_0 \exp(-i\omega t + \alpha z + ikx), & z < 0, \\ \mathbf{H} = \mathbf{H}_0 \exp(-i\omega t + \alpha(a - z) + ikx), & z > a. \end{cases} \quad (37)$$

Here, α — the transverse coefficient of decay that is determined via the parameters k , ω and ε by the relationship

$$\alpha = \sqrt{k^2 - \frac{\omega^2}{c^2} \varepsilon}. \quad (38)$$

From the side of the insulating layers the components E_x and H_y are related to each other by the Maxwell equation:

$$-\frac{\partial H_y}{\partial z} + i\varepsilon \frac{\omega}{c} E_x = 0. \quad (39)$$

Here, ω — the wave frequency, k — the wave number, c — the speed of light, ε — the permittivity.

Using (39) and taking into account (36) and (37), we obtain the surface impedance outside the conducting layer:

$$Z_1 = \frac{E_x}{H_y} \Big|_{z=0} = -\frac{iac}{\varepsilon\omega}. \quad (40)$$

Inside the conducting layer, the dependences $\mathbf{E}$ and $\mathbf{H}$ on the time and the coordinates x and z will be written so that amplitudes of the components E_{0x} and H_{0y} be functions of the coordinate z :

$$\begin{cases} E_x = E_{0x}(z) \exp(-i\omega t + ikx), \\ E_z = E_{0z}(z) \exp(-i\omega t + ikx), \\ H_y = H_{0y}(z) \exp(-i\omega t + ikx). \end{cases} \quad (41)$$

By replacing ε in the expression (39) with complex permittivity $\varepsilon_m = 1 + i4\pi\sigma/\omega$, where σ is conductivity, we will obtain a relation between E_x and H_y inside the conducting layer:

$$-\frac{\partial H_y}{\partial z} + i \frac{\omega}{c} E_x = \frac{4\pi}{c} \sigma E_x. \quad (42)$$

Symmetrical distribution of the charge carriers (Fig. 1) corresponds to a situation, when the components of the vectors E_x and H_y at the nanolayer boundaries are related to each other by the relationship:

$$\begin{cases} H_y(0) = -H_y(a), \\ E_x(0) = E_x(a). \end{cases} \quad (43)$$

It follows from the condition (43) that the component of the electric field strength E_x is virtually independent of the coordinate z . From the equation (42) we will obtain an expression for the surface impedance from the side of the conducting layer:

$$Z_2 = \frac{E_x}{H_y} \Big|_{z=a} = \frac{2ic}{(\omega a + 4\pi i \sigma_a)}. \quad (44)$$

By equating the surface impedances, we will obtain expressions for the transverse coefficient of decay of the surface wave:

$$\alpha = -\frac{2\varepsilon\omega}{(\omega a + 4\pi i \sigma_a)}. \quad (45)$$

By using the relationship (38), we will obtain an expression for the wave vector of the surface wave:

$$k = \sqrt{\frac{\omega^2}{c^2} \varepsilon + \frac{4\varepsilon^2 \omega^2}{(\omega a + 4\pi i \sigma_a)^2}}. \quad (46)$$

The expression (46) describes the dispersion law of the surface plasmon.

We rate the parameters k and α to the thickness of the semiconductor nanolayer and write the expressions (45) and (46) via dimensionless parameters:

$$\alpha_0 = -\frac{2\varepsilon y_0}{(y_0 + iy_p^2 \Sigma)}, \quad (47)$$

$$k_0 = \sqrt{y_0^2 \frac{x_0^2}{x_\lambda^2} \rho^2 \varepsilon + \frac{4y_0^2 \varepsilon^2}{(y_0 + iy_p^2 \Sigma)^2}}, \quad (48)$$

where the following notation is introduced: $\rho = v_{0v}/c$, $y_p = \omega_p \tau_v$.

We note that generally the parameters α_0 and k_0 are complex magnitudes and can be presented as

$$\alpha_0 = \text{Re}(\alpha_0) + i\text{Im}(\alpha_0) = \alpha_{01} + i\alpha_{02}, \quad (49)$$

$$k_0 = \text{Re}(k_0) + i\text{Im}(k_0) = k_{01} + ik_{02}. \quad (50)$$

We will call a real part of the wave number k_{01} that is equal to a number of the wavelengths fitting into a segment of the length of 2π , a coefficient of propagation. We will call an imaginary part of the wave number k_{02} that describes decay of the surface wave along the propagation direction, a longitudinal coefficient of decay. The coefficient of transverse decay will be understood by us as the real part of the parameter α_0 .

We introduce a notion of the length of propagation of the surface wave L that is a distance transmitted by the wave, which results in reduction of amplitudes of the vectors of the strengths of the electric and magnetic fields in e times. It follows from the expressions (36) and (37) that the propagation length is determined as a magnitude that is inverse to the imaginary part of the wave number k . By rating to the thickness of the semiconductor nanolayer, we obtain

$$L_0 = \frac{L}{a} = \frac{1}{\text{Im}(k_0)}. \quad (51)$$

Let us review the case of the degenerate electron gas ($\exp(\mu k_B T) \gg 1$) that corresponds to a high concentration, a low effective mass of the charge carriers and a low temperature. The equilibrium function of distribution takes the form of a stepped approximation:

$$f_l^{(0)}(\varepsilon_l) = \begin{cases} 1, & 0 < \varepsilon_l < \varepsilon_F, \\ 0, & \varepsilon_l > \varepsilon_F, \end{cases} \quad (52)$$

where ε_F — the Fermi energy.

The expressions for k_0 and α_0 take the form (47) and (48), respectively, in which the dimensionless conductivity Σ is determined as follows:

$$\Sigma = \frac{3}{4x_0 z_0} \sum_{l=1}^N \left(1 - \frac{l^2}{4x_0^2} \right) \left(1 - \chi \left(\frac{2x_0^2 z_0}{x_\lambda l} \right) \right), \quad (53)$$

$$N = \left[\frac{k_F}{k_1} \right] = [2x_0], \quad (54)$$

where N — the number of the subbands, the square brackets in the formula (54) denote an integral part, k_F — the wave vector of the charge carrier with the Fermi energy.

Let us review the case of the non-degenerate electron gas ($\exp(\mu k_B T) \ll 1$) that corresponds to the relatively low concentration, the large effective mass and the high

temperature. The equilibrium function of distribution of the charge carriers will be as follows

$$f_l^{(0)}(\varepsilon_l) = \exp((\mu - \varepsilon_l)/k_B T). \quad (55)$$

The expressions for k_0 and α_0 take the form (47) and (48), respectively, in which the parameter Σ is determined as follows:

$$\Sigma = \frac{1}{x_0 z_0} \sqrt{\frac{5}{2\pi}} \sum_{l=1}^{\infty} \exp\left(-\frac{5l^2}{8x_0^2}\right) \left(1 - \chi \left(\frac{2x_0^2 z_0}{x_\lambda l} \right) \right). \quad (56)$$

3. Analysis of results

Fig. 2 shows the dependences of the coefficient of propagation on the dimensionless frequency of the surface with the various permittivity of the insulating layers. The figure shows that the frequency dependence of the coefficient of propagation is practically monotonic. The figure exhibits a deviation from the linear dependence towards higher k_{01} with increase of the dimensionless frequency, which is typical for the law of dispersion of plasma oscillations [1]. With increase of the permittivity, the coefficient of propagation increases.

Fig. 3 shows the frequency dependences of the propagation length L_0 . It follows from the figure that increase of the permittivity of the insulating layers results in reduction of the propagation length. There are observed oscillations of the frequency dependences of the parameter L_0 , which have a more pronounced nature with increase of ε . A cause of origination of the oscillations is probably similar to appearance of the oscillations at the absorption spectra [24]. Oscillatory maximums of the parameter L_0 are observed at the frequencies which are multiples of a frequency

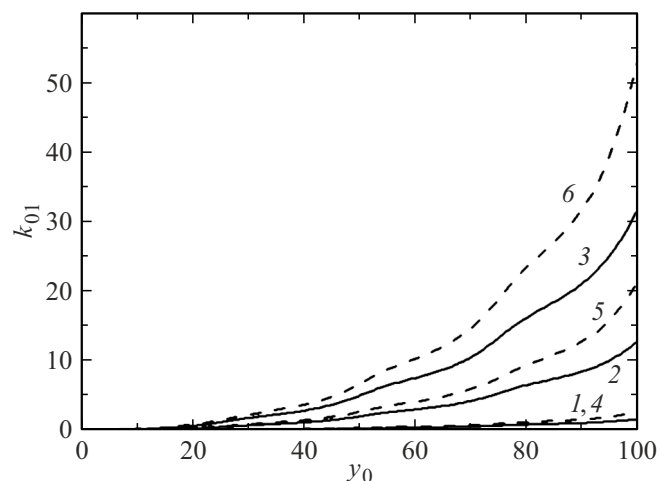


Figure 2. Dependences of the dimensionless coefficient of propagation k_{01} on the dimensionless frequency y_0 when $x_0 = 0.8$; $x_\lambda = 10$; $\rho = 0.005$; $y_p = 200$; $g_1 = 0$; $g_2 = 0.2$. The solid curves 1–3 and the dashed curves are constructed for the cases of the degenerate and the non-degenerate electron gas, respectively: 1, 4 — $\varepsilon = 1$; 2, 5 — $\varepsilon = 8$; 3, 6 — $\varepsilon = 20$.

of surface collisions of the charge carriers: the surface wave will decay more weakly, thereby contributing to more effective transfer of the wave energy along the nanolayer. The sharpest change of the dependence $L_0(y_0)$ is observed near the first maximum within the range of the dimensionless frequencies 25 – 30: the maximum value of the dimensionless length L_0 exceeds the minimum value thereof practically in five times. The effect related to the oscillations of the length of propagation of the wave can be used for designing plasmon waveguides designed to filter frequencies that correspond to minimum decay of the wave (plasmon filters). Fig. 3 shows that at the minimum decay in the nanolayers of the thickness of about tens of nanometers the surface wave can propagate for a typical distance of about several micrometers ($L_0 \approx 250$). For the most semiconductors, at the room temperature the mean free time of the charge carriers τ is within the range $10^{-12} - 10^{-13}$ s. The oscillation effects originate in the semiconductor nanolayers of the thickness of about 10 nm at the frequencies of about tens of terahertz. We note that in the case of the non-degenerate electron gas the oscillations of the frequency dependences of the propagation length are less pronounced as compared to the case of the degenerate gas due to the spread of thermal velocities of the charge carriers. The strongest difference between the solid and dashed curves is observed near the maximums of L_0 , thereby making it possible to use the surface plasma oscillations for determining a nature of degeneracy and the concentration of the charge carriers in the semiconductor nanolayer.

It follows from Fig. 3 that with decrease of the frequency the propagation length infinitely increases, i.e. there is no wave decay in a low-frequency limit. In fact, the theoretical model has limitations and it is necessary to

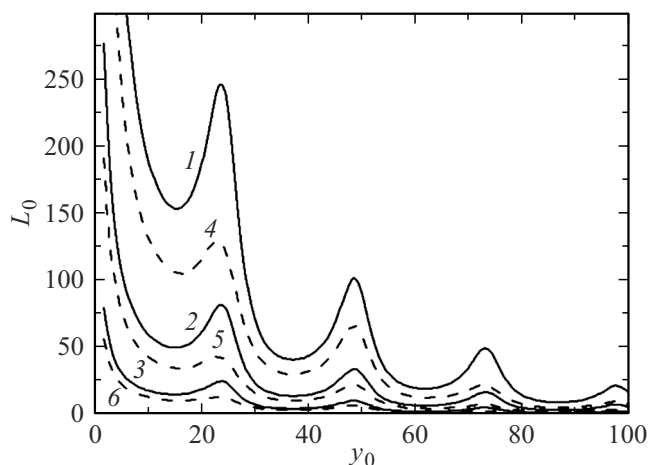


Figure 3. Dependences of the dimensionless propagation length L_0 on the dimensionless frequency y_0 when $x_0 = 0.8$; $x_\lambda = 10$; $\rho = 0.005$; $y_p = 200$; $g_1 = 0$; $g_2 = 0.2$. The solid curves 1–3 and the dashed curves are constructed for the cases of the degenerate and the non-degenerate electron gas, respectively: 1, 4 — $\varepsilon = 1$; 2, 5 — $\varepsilon = 3$; 3, 6 — $\varepsilon = 10$.

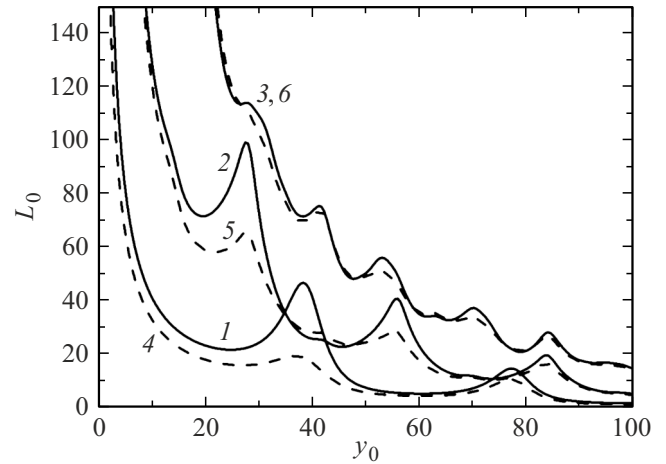


Figure 4. Dependences of the dimensionless propagation length L_0 on the dimensionless frequency y_0 when $x_\lambda = 10$; $\rho = 0.005$; $y_p = 200$; $\varepsilon = 3$; $g_1 = g_2 = 0.2$. The solid and dashed curves are constructed for the cases of the degenerate and the non-degenerate electron gas, respectively: 1, 4 — $x_0 = 0.9$; 2, 5 — $x_0 = 1.5$; 3, 6 — $x_0 = 3$.

specify an area of its applicability. At the low frequencies, the coefficient of propagation of the surface wave is small (Fig. 2), which corresponds to large sizes of areas of the nanolayer surface, where the positive (negative) charge is concentrated. In real conditions, there are limitations on the size of these areas and the propagation lengths as a result of thermal oscillations of the charge carriers. In the typical semiconductors, a diffusion length of the charge carriers, which defines a typical distance, within which the excessive concentration of the charge carriers decreases in e times, is within the range from tens to thousands of micrometers. The theoretical model constructed in the present study is applicable when the length of propagation of the surface wave does not exceed several micrometers (is much less than the diffusion length).

Fig. 4 has spectra of the length of propagation of the surface wave constructed at the various thicknesses of the semiconductor nanolayer. It follows from the figure that the period of oscillations quite strongly depends on the nanolayer thickness. With decrease of the thickness by $0.6\lambda_B$ (which corresponds to 6 nm), the period increases practically in 1.5 times. It indicates that the characteristics of the surface wave that propagates with minimum decay (the frequency, the amplitude, etc.) can be easily tuned to create the nanolayer of a certain thickness. With increase of the thickness, the oscillations become less pronounced and their amplitude decreases. The oscillations of the dependences $L_0(y_0)$ are observed in quite thin nanolayers, whose thickness does not exceed $3\lambda_B$.

Fig. 5 shows the frequency dependences of the propagation length with the various parameters of the surface roughness. The curves of Fig. 5 are constructed in a situation when the parameters of roughness of the upper and lower boundaries of the nanolayer are the same. In

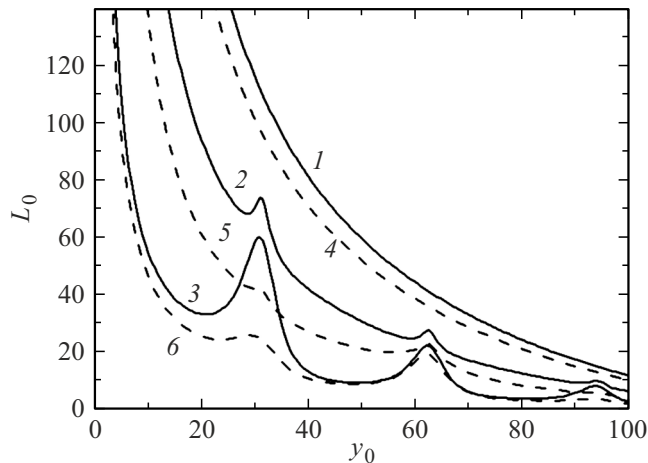


Figure 5. Dependences of the dimensionless propagation length L_0 on the dimensionless frequency y_0 when $x_\lambda = 10$; $\rho = 0.005$; $y_p = 200$; $\varepsilon = 3$; $x_0 = 1$, $g_2 = 0$. The solid and dashed curves are constructed for the cases of the degenerate and the non-degenerate electron gas, respectively: 1, 4 — $g_1 = 0$; 2, 5 — $g_1 = 0.07$; 3, 6 — $g_1 = 0.2$.

case of smooth surfaces, the oscillations of the spectra of the propagation length are not observed. It confirms a hypothesis that the oscillations occur not only as a result of the effects of dimensionless quantization, but with presence of surface scattering as well (the non-zero parameters g_1 and g_2). Variation of a surface irregularity degree results in the change of the amplitude of the oscillations of the dependences $L_0(y_0)$. This effect makes it possible to apply surface plasma oscillations for determining the surface roughness by measuring a typical distance that is transmitted by the wave at the minimum decay.

Fig. 6 compares spectra of the propagation length, which are constructed in case of two rough surfaces and in a situation when one nanolayer boundary is mirror-polished and the other is rough. The figures show that when one surface is rough and the other is smooth the period of the oscillations is in two times less than in the case of the same roughnesses (see the curves 1, 4 and 3, 6). It is related to the fact that in case of one mirror-polished surface the frequency of surface charge carrier scattering is in two times less than in the situation when both the surfaces are rough.

Fig. 7 shows the frequency dependences of the coefficient of propagation k_1 of the plasma oscillations which propagate along the GaAs layer of the thickness of 33 nm at the temperature of 1.85 K. Fig. 7 compares results obtained within the framework of the model with experimental data of the study [25]. A layer of the narrow-band GaAs semiconductor is placed between two wide-band $\text{Al}_x\text{Ga}_{1-x}\text{As}$ semiconductors ($0.01 < x < 0.03$) that function as barrier layers. According to results of measurements by means of the quantum Hall effect [25], the surface concentration of the charge carriers and mobility are $4 \cdot 10^9 \text{ cm}^{-2}$ and $9 \cdot 10^5 \text{ cm}^2/(\text{V}\cdot\text{s})$, respectively.

In order to compare the obtained results with the theoretical calculations, the following parameters were determined: the concentration of the charge carriers without taking into account the effects of dimensional quantization $3 \cdot 10^{16} \text{ cm}^{-3}$, the mean free path of the charge carriers $5.5 \mu\text{m}$. The best compliance is observed at the values of the parameters of the surface roughness $g_1 = 0.02$ and $g_2 = 0.09$, which corresponds to a mean square height of the surface 1.3 and 6 nm, respectively.

The theoretical calculations were carried out for symmetrical distribution of the charges at the layer boundaries. Another case, when the charges at the nanolayer boundaries are opposite in sign, requires additional consideration,

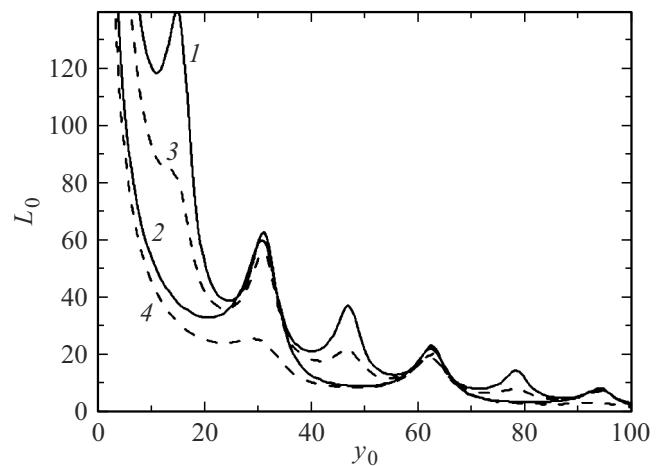


Figure 6. Dependences of the dimensionless propagation length L_0 on the dimensionless frequency y_0 when $x_\lambda = 10$; $\rho = 0.005$; $y_p = 200$; $\varepsilon = 3$; $x_0 = 1$, $g_2 = 0.2$. The solid and dashed curves are constructed for the cases of the degenerate and the non-degenerate electron gas, respectively: 1, 3 — $g_1 = 0$; 2, 4 — $g_1 = 0.2$.

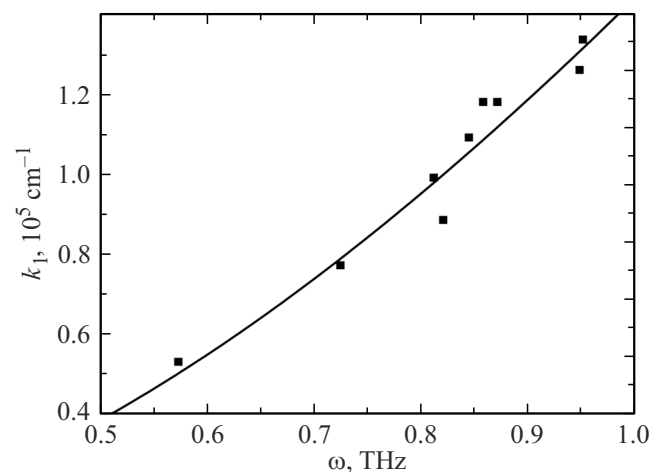


Figure 7. Dependences of the propagation parameter k_1 on the frequency ω of the GaAs layer of the thickness of 33 nm. The dots are experimental data of the study [21]. The solid curve is a theoretical calculation at the following parameters: $\lambda_{B0} = 67 \text{ nm}$; $\Lambda = 5 \mu\text{m}$; $\varepsilon = 12$; $g_1 = 0.02$; $g_2 = 0.09$.

although, it can be assumed that the coefficients of propagation and decay of the surface wave will significantly depend on the surface effects. It is related to the fact that with such distribution of the charge carriers at the layer boundaries the oscillations of the charge carriers will be affected by the transverse component of the electric field strength, whose value is determined by the charge at the layer boundaries. A size of near-surface areas, where the charge is concentrated, depends on the surface roughness. Therefore, surface charge carrier scattering will affect not only on their response from the side of the electric field, but on the field itself as well. It can be noted that in a limit case of a very thin layer there will be generated plasma oscillations that predominantly have symmetrical distribution of the charge at the nanolayer boundaries. This can be explained as follows. With a small thickness of the nanolayer, the areas at which the surface charge is concentrated will contact each other, i.e. the surface charges will be exchanged as a result of thermal fluctuations. If the signs of the charge at the opposite boundaries of the nanolayers are different (antisymmetric distribution), the absolute value of the charge at the boundaries will decrease with reduction of the thickness.

Conclusion

The analytical expressions are obtained for the coefficients of decay and propagation of the surface wave as a function of the dimensionless parameters: the thickness of the nanolayer, the frequency, the permittivity of the environment and the parameters of the surface roughness. It is shown that the dispersion curves have typical maximums. With decrease of the thickness, the maximum of the dispersion curve is shifted towards the low frequencies. This effect becomes significant at the thicknesses that are less than the de Broglie wavelength of the charge carriers. Oscillations of the spectra of a longitudinal decay parameter are detected and their period varies with the change of the thickness, while their amplitude varies with the change of the parameters of the surface roughness. The obtained expressions make it possible to calculate the thickness of the conducting nanolayer and the parameters of the surface roughness by the period, the amplitude and the form of oscillations of the decay spectra.

The results of the study can be applied in practice for designing the plasmon waveguides that effectively transfer the energy and have minimum decay.

Conflict of interest

The authors declare that they have no conflict of interest.

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