15

Non-contact suspension of the magnet with diamagnetic stabilization in horizontal orientation of magnetic moments

© K.A. Legostaev,[1,2] M.P. Volkov[2]

[1] Peter the Great Saint-Petersburg Polytechnic University,
195251 St. Petersburg, Russia
[2] Ioffe Institute,
194021 St. Petersburg, Russia
e-mail: legostaev.ka@spbstu.ru

Received November 26, 2024
Revised April 25, 2025
Accepted April 30, 2025

The study has considered a model of a non-contact suspension of the permanent magnet with diamagnetic stabilization of a position in the magnetic field that is created by two holding permanent magnets with horizontal orientation of the magnetic moments. The suspension mockup is created using NdFeB magnets for two variants of orientation of the magnetic moments in relation to an axis that connects centers of the holding magnets. It included measurement of dependences of interaction forces on a distance between the magnets for both the variants. It is found that the calculation satisfactorily describes experimental dependences of the interaction forces on a coordinate even at small distances compared to sizes of the magnets. The obtained expressions for the interaction forces can be used when designing and creating the multi-purpose magnetic suspension systems with an increased lifting force.

Keywords: non-contact suspension, permanent magnets, diamagnetic equilibrium stabilization, horizontal levitation system, dipole-dipole interaction

DOI: 10.61011/TP.2025.08.61744.429-24

Introduction

The non-contact magnetic suspensions are widely used in engineering, for example, as sensitive elements of measurement devices, device for microdisplacements and harvesters of mechanical oscillations [1]. A simple passive magnetic suspension is a vertical magnetic suspension, in which a strong permanent magnet or electromagnet creates a vertical force that compensate a weight of the small suspended permanent magnet, while the equilibrium position of the levitating permanent magnet is stabilized by using closely spaced plates made of a diamagnetic material, for example, pyrolytic graphite or bismuth [2]. The study [3] has proposed a modification of the magnetic suspension, in which the horizontal arrangement of the permanent magnets provides stable equilibrium of the suspended magnet along the vertical, while the horizontal position is stabilized by using the diamagnetic plates as well. As shown in the study [4], this horizontal suspension has a number of advantages as compared to the vertical variant. Based on the horizontal diamagnetic suspension, a number of technical devices was created, for example, the harvester of mechanical oscillations [5], the high-sensitivity force sensor [6] and the gas flow meter [7].

The study [8] has considered a possibility of increasing the lifting force of the vertical magnetic suspension by optimizing the holding magnetic system. The study [9] has analyzed interaction of the permanent magnets in the system of the vertical magnetic suspension when using pre-calculated values of the magnetic field in each point of the considered area. In the present study, magnetization of the axially magnetized cylindrical permanent magnets was calculated in an approximation of the single-layer cylindrical solenoid with the uniform surface density of the current.

It is also possible to optimize the magnetic system for the horizontal diamagnetic suspension; it requires to calculate the forces acting on the levitating magnet and their dependence on the mutual arrangement of the holding and levitating magnets. Thus, the study [10] has considered one of the variants of optimizing the magnetic system of the horizontal magnetic suspension, which is aimed at implementing a bistable magnetic system. It has calculated the forces acting between the magnets using calculation of the values of the magnetic field in each point of the considered area and the forces acting between the suspended magnet and the stabilizing diamagnetic plates.

The present study has considered two variants for arrangement of the holding magnets and the suspended magnet with the horizontally-aligned magnetic moments, calculated the forces acting on the suspended magnet by using an expression for the forces of interaction of the point magnetic dipoles for both the variants of arrangement of the magnets. The theoretical dependences have been compared with the experimental data obtained at assembled mockups of the magnetic suspension.

1. Calculation of the forces of interaction of the levitating and holding magnets with horizontal orientation of the magnetic moments

With horizontal orientation of the magnetic moments of the two magnets, it is sufficient to consider two basic variants of arrangement for calculation of the interaction force. The first variant means that the magnetic moments are in the same vertical plane that passes through the centers of the magnets, while the second variant means that the magnetic moments are perpendicular to this plane. In other variants of mutual orientation of the horizontally directed magnetic moments, the interaction force can be presented as superposition of the forces from the two basic variants.

1.1. First variant of arrangement of the permanent magnets

The first (standard) variant of the horizontal diamagnetic suspension is shown in Fig. 1, *a*. The figure shows the levitating magnet *1*, the holding magnets *2* and *3* (the magnetic moments of all the magnets are directed along the axis *ox*) and the stabilizing diamagnetic plates 4 and 5. The force acting on the levitating magnet by the holding magnet has a vertical component F_z and horizontal components F_x and F_y. The vertical components of the forces from the two holding magnets are summed up, thereby making it possible to balance the magnet, while their dependence on the coordinate makes it possible to ensure stable equilibrium along the vertical axis. The horizontal component of the fore F_y is absent due to the symmetry of the system, while with deviation from the symmetry axis a restoring force originates, as shown below. The horizontal components of the forces *0* from the holding magnets are directed in different directions and compensate each other when arranging the levitating magnet in the center of the system. When the levitating magnet deviates from the center, an uncompensated force originates, i.e. the position of the magnet along the horizontal axis *ox* is unstable. Introduction of the diamagnetic plates makes it possible to achieve stable equilibrium in this direction, too.

The dependences of the horizontal and vertical components of the forces acting on the levitating magnet on the respective coordinate can be calculated using simple formulas for interaction of the point magnetic dipoles [11]. When using these formulas, it is possible to obtain exact values of the interaction forces at great distances between the magnets, when their sizes can be neglected, but it results in inaccuracies with a small distance between the magnets. Thus, in the studies [12,13], the force of interaction of the magnets was calculated both by exact methods that take into account the sizes of the magnets (the model of surface currents, the model of fictitious magnetic charges) and by the formulas of dipole-dipole interaction. It was shown that the first two methods well describe the experimental data at all the distances between the magnets, while the calculation by the formulas of the dipole-dipole approximation coincides with the experiment at the quite large distances, resulting in a noticeable error at the small distances. At the same time, the error is below 10 %, if the distance between the magnets exceeds four typical sizes of the magnet (the diameters of the magnets in our case). In our problem, we use the regularly-shaped magnets (cylinders), the deviation of the levitating magnet from the symmetry axis is slight, and it can be expected that the forces of interaction of the magnets will be described by the formulas of dipole-dipole interaction at the lesser distances as well. In order to check this assumption, the present study has calculated the dependences of the forces of interaction on the distance and they were compared with the experimental dependences that were obtained on the created model of the magnetic suspension using the NdFeB magnets.

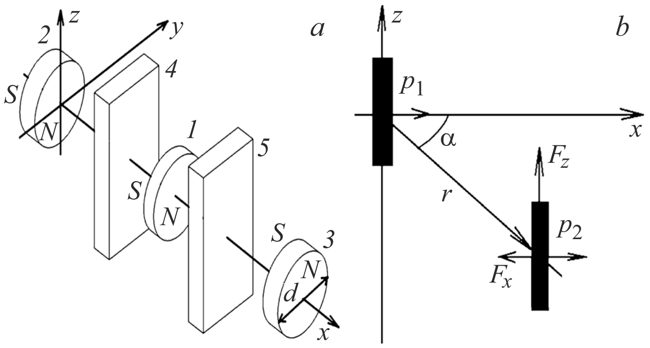

Figure 1. *a* — the diagram of the horizontal magnet levitation system for the first configuration, *b* — the diagram for calculation of the forces of interaction of the holding and levitating magnets.

The forces acting on the levitating magnet by the holding magnet will be calculated by the formula for interaction of the point magnetic dipoles [11]:

$$\mathbf{F} = \frac{3(\mathbf{p_1}\cdot\mathbf{r})}{r^5}\mathbf{p_2} + \frac{3(\mathbf{p_2}\cdot r)}{r^5}\mathbf{p_1} + \frac{3(\mathbf{p_1}\cdot\mathbf{p_2})}{r^5}\mathbf{r} - \frac{15(\mathbf{p_1}\cdot\mathbf{r})(\mathbf{p_2}\cdot\mathbf{r})}{r^7}\mathbf{r}. \tag{1}$$

Fig. 1, *b* shows a diagram for calculating the forces in the magnet system (the first variant of Fig. 1, *a*). In this diagram, the horizontally-aligned magnetic moments of the magnets are in the same vertical plane *oxz* and the force of their interaction is:

$$\mathbf{F} = \frac{3p_1p_2\cos\alpha}{r^4}\mathbf{i} + \frac{3p_1p_2\cos\alpha}{r^4}\mathbf{i} + \frac{3p_1p_2\cos\alpha}{r^4}\mathbf{i} + \frac{3p_1p_2\sin\alpha}{r^4}\mathbf{k} - \frac{15p_1p_2\cos^3\alpha}{r^4}\mathbf{i} - \frac{15p_1p_2\sin\alpha\cos^2\alpha}{r^4}\mathbf{k}$$
$$= \mathbf{i}\left(\frac{9p_1p_2\cos\alpha}{r^4} - \frac{15p_1p_2\cos^3\alpha}{r^4}\right) + \mathbf{k}\left(\frac{3p_1p_2\sin\alpha}{r^4} - \frac{15p_1p_2\sin\alpha\cos^2\alpha}{r^4}\right) = \mathbf{i}F_x + \mathbf{k}F_z.$$

Thus, the components of the forces are

$$F_z = \frac{3p_1p_2}{r^4}\sin\alpha(1-5\cos^2\alpha),$$

$$F_x = \frac{3p_1p_2}{r^4}\cos\alpha(3-5\cos^2\alpha).$$

The following replacements will be made:

$$r\cos\alpha = x,\quad r\sin\alpha = z,\quad r=\sqrt{x^2+z^2},$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+z^2}},\quad \sin\alpha = \frac{z}{\sqrt{x^2+z^2}}.$$

We obtain

$$F_z = \frac{3p_1p_2(z^3-4zx^2)}{(x^2+z^2)^{\frac{7}{2}}}, \tag{2}$$

$$F_x = \frac{3p_1p_2(3xz^2-2x^3)}{(x^2+z^2)^{\frac{7}{2}}}. \tag{3}$$

For analysis of the functional dependences of the components of the force on the distance, we will assume that the calculation is for two identical magnets of a cylindrical form $p_1 = p_2$.

The magnetic moment of the cylindrical magnet is equivalent to the magnetic moment of a single-layer solenoid with the same sizes and the density of the surface current **J**, which is determined by a material of the magnets [14]:

$$p = JhS, \tag{4}$$

where $S = \frac{\pi\cdot d^2}{4}$ — the cross-sectional area of the permanent magnet of the diameter d; h — the thickness of the permanent magnet; J — the surface density of the current, which is determined by the material of the magnet.

Then, it is possible to normalize the coordinates z and x in the formulas (2), (3) by the value of the diameter of the magnets d. The graph of the dependence of the reduced force F_z/A, where $A = (3/16)(\pi hJ)^2$, on the displacement z/d along the axis oz is shown in Fig. 2 for a number of the fixed values of deviation along the axis ox.

It is clear from Fig. 2 that with increase of the distance between the magnets along the axis oy the absolute values of the vertical component of the force are reduced, the maximum of the dependence is slightly displaced, but the form of the function F_z/A weakly varies. The function F_z is even in relation to the variable x, therefore, the forces from the two holding magnets (*2*, *3* in Fig. 1, *a*) on the levitating magnet are summed up, whence, it follows that the equilibrium condition is that F_z is equal to a half weight of the magnet. Let us assume that the vertical component of the force is equal to the half weight of the levitating magnet when $x = d$ in the points E_1 and E_2 (Fig. 2), wherein stable equilibrium will be observed only in the point E_2 at the negative values of z, since when the levitating magnet is displaced from this point, the restoring force originates. If the magnet is in the region from E_1 to E_2, then the force

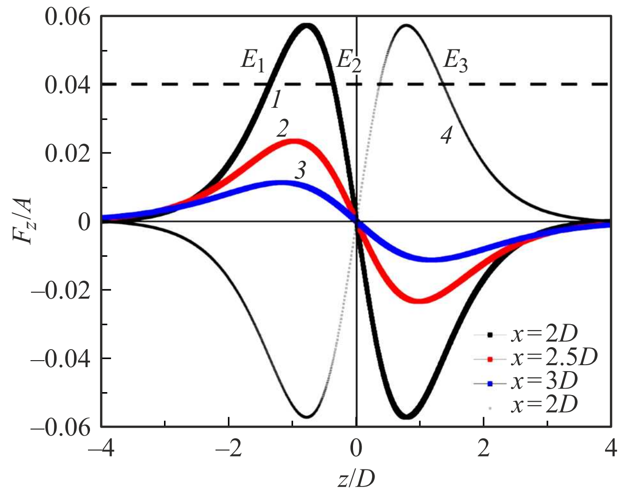

Figure 2. Dependences of the reduced force F_z/A on the displacement z/d at the fixed values $x = 2d$, $2.5d$, $3d$ for the first variant of arrangement of the magnets.

acting on it will be more than its weight and it will be displaced into the position E_2. With increase of the distance between the holding magnets, the points E_1 and E_2 become closer to each other and the maximum distance between the magnets, when the equilibrium is still possible, corresponds to the case when the points E_1 and E_2 coincide in the point of a local maximum.

Due to the symmetry, the expression for the component of the force $F_y(y)$ has the same functional form as for $F_z(z)$, and with slight deviation of the magnet from $y = 0$ the restoring force will originate, i.e. stable equilibrium is implemented along this axis.

The horizontal component of the force of interaction of the magnets F_x is an odd function of the distance between the magnets, therefore, in the center between the two holding magnets a total force acting on the levitating magnet must be zero. With slight deviation of the levitating magnet from the center for the distance δ, a small uncompensated force $F(\delta)$ originates:

$$F(\delta) = F_x(x+\delta) + F_x(-x+\delta), \tag{5}$$

where x is an initial distance between the holding and levitating magnets, δ is a displacement of the levitating magnet in relation to the equilibrium position along the axis x.

This small force is compensated by a force of repulsion from the diamagnetic materials, for example, the plates of pure bismuth or pyrolytic graphite. The small value of diamagnetic susceptibility of the bismuth and graphite plates used in the study makes it possible to obtain only a small potential well in the center between the plates, and achievement of full levitation of the suspended magnet (i.e. presence of noticeable clearance between the suspended magnet and the plates) requires exact tuning of the distance both between the levitating magnet and the holding magnets as well as between the levitating magnet and the diamagnetic plates.

We note that with antiparallel orientation of the magnetic moments of the holding magnets and the levitating magnet the dependence $F_z(z)$ (the dashed line of Fig. 2) also has a magnet weight balancing point, where stable equilibrium can be expected (the point E_3). But in this point, the opposite direction of the magnetic moment of the levitating magnet and the magnetic field from the holding magnets will result in origination of a force moment that tends to arrange the magnetic moments in parallel. The stationary suspension is not implemented for this case.

1.2. Second variant of arrangement of the permanent magnets

Fig. 3, *a* shows the second variant of arrangement of the magnets, when the moments are directed along the axis *oy*, while Fig. 3, *b* shows a diagram for calculating the forces of interaction F_x and F_z of the permanent magnets when the planes of the magnets are in the same vertical plane *oxz*, and the magnetic moments are oriented normally to this plane (the case when the levitating magnet is shifted along the axis *oy* will be considered separately). The designations in these figures are the same as in Fig. 1.

The forces of interaction F_x and F_z for the system of the permanent magnets of Fig. 3, *b* have also been calculated using the formula (1):

$$\mathbf{F} = \frac{3(\mathbf{p_1}\cdot\mathbf{r})}{r^5}\mathbf{p_2} + \frac{3(\mathbf{p_2}\cdot\mathbf{r})}{r^5}\mathbf{p_1} + \frac{3(\mathbf{p_1}\cdot\mathbf{p_2})}{r^5}\mathbf{r} - \frac{15(\mathbf{p_1}\cdot\mathbf{r})(\mathbf{p_2}\cdot\mathbf{r})}{r^7}\mathbf{r} = \frac{3p_1p_2}{r^4}(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{k}) = \mathbf{i}\left(\frac{3p_1p_2\cos\alpha}{r^4}\right) + \mathbf{k}\left(\frac{3p_1p_2\cos\alpha}{r^4}\right) = \mathbf{i}F_x + \mathbf{k}F_z.$$

Thus, the components of the forces are

$$F_z = \frac{3p_1p_2}{r^4}\sin\alpha, \quad F_x = \frac{3p_1p_2}{r^4}\cos\alpha.$$

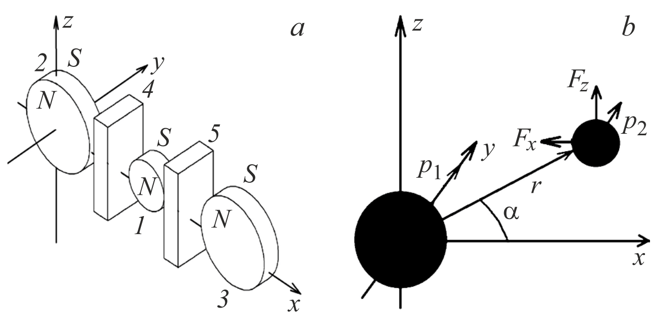

Figure 3. *a* — the diagram of the horizontal magnet levitation system for the second configuration; *b* — the diagram for calculation of the forces of interaction F_x and F_z of the holding and the levitating magnet.

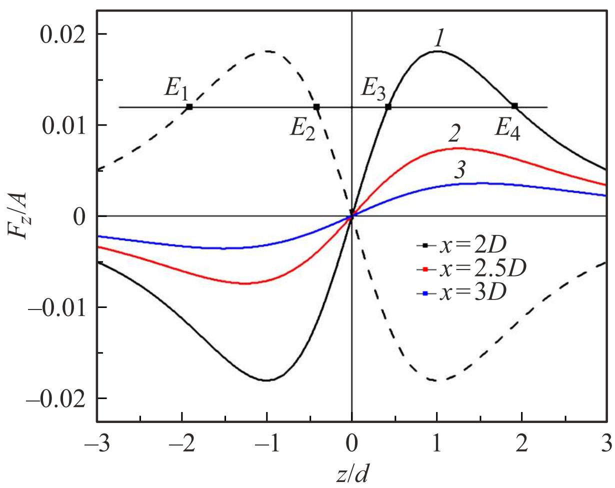

Figure 4. Dependences of the reduced force F_z/A on the displacement z at the fixed values $x = 2d$, $2.5d$, $3d$ for the second configuration.

The following replacements will be made:

$$r\cos\alpha = x, \quad r\sin\alpha = z, \quad r = \sqrt{x^2+z^2},$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+z^2}}, \quad \sin\alpha = \frac{z}{\sqrt{x^2+z^2}}.$$

We obtain

$$F_z = \frac{3p_1p_2z}{(x^2+z^2)^{\frac{5}{2}}}, \tag{6}$$

$$F_x = \frac{3p_1p_2x}{(x^2+z^2)^{\frac{5}{2}}}. \tag{7}$$

When analyzing the functional dependence of the components of the forces of interaction on the distance, the magnetic moments will be assumed to be equal to each other ($p_1 = p_2$) and the coordinates z and x in the formulas (6), (7) will be normalized by the value of the diameter of the magnets d (similar to the first calculation). The dependence of the reduced force F_z/A is graphically shown in Fig. 4 for the set of the fixed values of deviations of the magnets along the axis *ox*.

It is clear from Fig. 4 that with increase of the distance between the magnets along the axis *oy* , similar to the first configuration, the absolute values of the vertical component of the force are reduced, the maximum of the dependence is slightly displaced, and the form of the function F_z/A weakly varies. As in the first configuration, the function F_z is even in relation to the variable x, whence, it follows that the force acting on the levitating magnet will be a sum of the forces from the two holding magnets (*2*, *3* in Fig. 3, *a*), therefore, the condition of equilibrium will be that F_z is equal to the half weight of the magnet, which is shown in Fig. 4 by a horizontal line.

It could be assumed that stable equilibrium will be observed in the point E_4, but in this point the direction of the magnetic moment of the levitating magnet is opposite to the magnetic field of the holding magnets and a force moment originates to rotate the levitating magnet by 180°. The dependence of the force of interaction for antiparallel orientation of the magnetic moments is shown in Fig. 4 by a dashed line. Stable equilibrium will be observed in the point E_2 at the negative values of z, as restoring forces will originate during displacement from this point. If the magnet is in the region from E_1 to E_2, then the force acting on it will be more than its weight and it will be elevated into the position E_2. With increase of the distance between the holding magnets, the points E_1 and E_2 become closer to each other and the maximum distance between the magnets, when the equilibrium is still possible, corresponds to the case when the points E_1 and E_2 coincide in the point of a local maximum, which is also observed in the first configuration.

As in the first variant of arrangement of the magnets, the component F_x is an odd function of the distance between the magnets and in the center between the two holding magnets a total force acting on the levitating magnet will be zero. With slight deviation of the levitating magnet from the center for the distance δ, a small uncompensated force originates and it is balanced by a force from the diamagnetic plates.

The component $F_y(y)$ was calculated using the formula (1) by taking into account opposite orientation of the magnetic moments of the holding and levitating magnets. The diagram for calculating $F_y(y)$ is shown in Fig. 5, *a*.

$$\mathbf{F} = -\frac{3p_1p_2\sin\alpha}{r^4}\mathbf{j} - \frac{3p_1p_2\sin\alpha}{r^4}\mathbf{j} - \frac{3p_1p_2\sin\alpha}{r^4}\mathbf{j}$$

$$-\frac{3p_1p_2\cos\alpha}{r^4}\mathbf{i} - \frac{15p_1p_2\sin^3\alpha}{r^4}\mathbf{j} - \frac{15p_1p_2\cos\alpha\sin^2\alpha}{r^4}\mathbf{i}$$

$$= -\mathbf{j}\left(\frac{9p_1p_2\sin\alpha}{r^4} + \frac{15p_1p_2\sin^3\alpha}{r^4}\right)$$

$$-\mathbf{i}\left(\frac{3p_1p_2\cos\alpha}{r^4} + \frac{15p_1p_2\cos\alpha\sin^2\alpha}{r^4}\right).$$

In this case the component of the force F_x is similar to the component as per the formula (6) (it coincides with it when $\alpha = 90°$) and it will be zero in the middle between the tow holding magnets, while the uncompensated force that originates with slight deviation of the levitating magnet from the center must be balanced by a force from the diamagnetic plates. In this configuration of arrangement of the magnets, the center of the suspended magnet is at a greater distance from the diamagnetic plate than in the first configuration and it is much more difficult to achieve full levitation of the magnet (without contact of one of the plates and with noticeable clearance between the magnet and the plates). The fully non-contact suspension can be achieved when using a small-size suspended magnet and with the maximum possible distance between the holding magnets - when the force that originates with deviation of the magnet from the central position is the smallest.

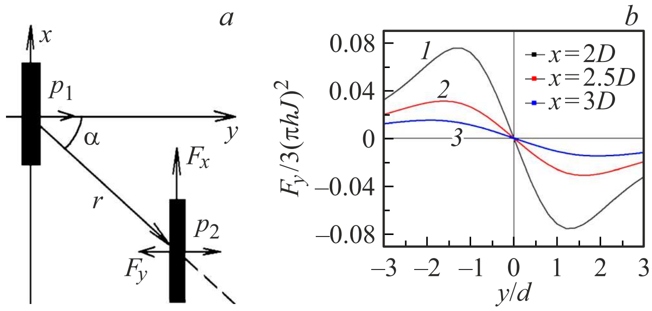

Figure 5. *a* — the diagram for calculation of the force of interaction; *b* — the dependences of the reduced forced F_y/A on the displacement y at the fixed values $x = 2d$, $2.5d$, $3d$ for the second configuration.

The component F_y is

$$F_y = -\frac{3p_1p_2}{r^4}\sin\alpha(3 + 5\cos^2\alpha).$$

Taking into account the previously-introduced replacements, we obtain a final dependence for the horizontal component F_y for the second case of orientation and a direction of the magnetic moments.

$$F_y = -\frac{3p_1p_2(8y^3 + 3yx^2)}{(x^2+y^2)^{\frac{7}{2}}}. \quad (8)$$

Fig. 5, *b* shows the dependence of the reduced force $F_y/3(\pi hJ)^2$ on the displacement y for the set of the fixed values of x. With displacement of the levitating magnet along the axis oy, a restoring force will originate, therefore, stable equilibrium will be observed along the axis oy.

2. Experimental procedure

The magnetic moment of the used permanent magnets was determined by measuring a force of interaction of the two identical magnets that are arranged on the same axis and have co-directional magnetic moments **p**, as a dependence on the distance between the magnets. Then, as per the formula (1) the force between the magnets is

$$F = \frac{6p^2}{r^4}. \quad (9)$$

Two identical NdFeB magnets of the diameter of 50 mm and the thickness of 5 mm were used in the experiment. The force of interaction of the magnet was measured using electronic balances. In order to exclude the influence of the magnets on the balances system, the balances had a styroform unit placed, whose upper part fixed one of the magnets with a vertically-aligned direction of the magnetic moment. The second magnet had the same orientation of the magnetic moment and was smoothly displaced in the vertical direction. Readings of the electronic balances were fixed in the dependence on the distance between the magnets.

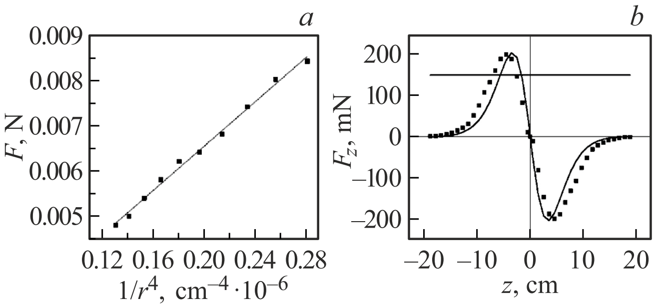

Figure 6. *a* — the dependence of the force of interaction of the two identical magnets with the co-directional magnetic moments on r^{-4}; *b* — the dependence of the vertical component of the force between the holding and levitating magnet on the vertical coordinate z when $x = 8.5$ cm for the first configuration (the dots are the experiment, the solid line is the calculation as per the formula (2), the horizontal straight line is the weight of the levitating magnet).

Another experiment included measurement of the force of interaction between the central magnet *1* and the side magnets *2* and *3* (Fig. 1, *a*). For this purpose, the central magnet *1* with the horizontally-aligned direction of the magnetic moment was placed on the styroform unit, while the side magnets *2* and *3* were smoothly displaced in the vertical direction while keeping the distance therebetween and variation of the force of interaction was recorded. This experiment was carried out for both the variants of arrangement and orientation of the magnetic moments.

The mockup of the horizontal diamagnetic suspension was manufactured using the NdFeB magnets and the bulk plates made of pyrolytic graphite as diamagnetic stabilizers of equilibrium. The holding magnets had a diameter of 50 mm and a thickness of 10 mm and were coaxially fixed so as to smoothly vary the distance between them. The levitating magnets were of two types: „a pill“ of the diameter of 5 mm and the thickness of 1 mm and two „discs“ of the diameters 12 and 25 mm and the thicknesses 1 and 2 mm.

3. Results and discussion

3.1. Measurements of the forces of interaction of the permanent magnets

Fig. 6, *a* shows a dependence of the force of interaction of the two permanent magnets of the diameter of 50 mm and the thickness of 5 mm with co-directional magnetic moments on the distance between them along the symmetry axis, wherein the dots mark results of the measurements. With the large distances between the magnets (more than 40 cm) the distance is well approximated by the dependence r^{-4} (marked by a solid line) and using the formula (4), it is possible to determine the value of the magnetic moment of this magnet, which turned out to be $0.16\,\mathrm{A{\cdot}m^2}$.

The magnetic field of the cylindrical permanent magnet with axial magnetization can be modelled by a field of the single-layer solenoid with the surface density of the current J, whose length and radius are equal to the length and the radius of the magnet. According to the formula (4), the surface density of the current of the studied magnet turns out to be $J = 7800$ A/m and it is assumed to be the same for all the permanent NdFeB magnets used by us. Using this value of J, we have obtained the values of the magnetic moments for each magnet: for the pill — $p = 1.5\,\mathrm{A{\cdot}m^2}$, for the discs of the diameter of 12 mm — $p = 8.8\,\mathrm{A{\cdot}m^2}$ of the diameter of 25 mm — $p = 77\,\mathrm{A{\cdot}m^2}$.

The dependences of the measured vertical component of the force F_z between the holding magnets (of the diameter of 50 mm and the thickness of 10 mm) and the levitating magnet (the diameter of 20 mm and the thickness of 5 mm), which are arranged at the distance of 8.5 cm, are shown in Fig. 6, *b* (the dots) with the magnetic moments directed along the axis ox. The dependence of the force F_z on the distance z (the formula (2)) for the magnetic moments $p_1 = 1600\ \mathrm{A{\cdot}m^2}$ and $p_2 = 30\,\mathrm{A{\cdot}m^2}$ when $y = 8.5$ cm is shown in Fig. 6, *b* by the solid line.

Fig. 6, *b* shows good compliance of the calculated and experimental dependences $F_z(z)$, although the distance between the holding and levitating magnets (8.5 cm) just in 1.7 times exceeds the diameter of the holding magnet (5 cm). It should be noted that this satisfactory compliance of the experimental and calculated values was obtained in the conditions when applicability of the formula for interaction of the point dipole moments seems slightly justified.

The horizontal line of Fig. 6, *b* shows the weight of the central magnet and the vertical component F_z is equal to the magnet weight when $z \sim -2$ cm, which corresponds to the point of stable equilibrium E_2 in Fig. 3. With increase of the distance between the holding magnets the vertical component of the force F_z sharply decreases, while with $x = 8.5$ cm the force maximum $F_{\max} = 200$ mN (Fig. 5, *b*), but when $x = 10.5$ cm — $F_{\max} \sim 50$ mN, which is significantly less than the magnet weight.

The second configuration of the magnetic moments has also had the dependence of the vertical component F_z between the holding and levitating magnets measured. For it, the same magnets were used as in the case of measuring the component F_z in the first configuration (the holding magnets: the diameter of 50 mm, the thickness of 10 mm; the levitating magnet: the diameter of 20 mm, the thickness of 5 m). The distance between the levitating and the holding magnet was 8.5 mm, too.

The dots of Fig. 7, *a* show the experimental dependence of the component F_z, while the solid line marks the dependence for the used magnets, which is obtained using the formula (6). It is clear from Fig. 7, *a* that there is good compliance between the experimental data the theoretical dependence, which again enables to make sure that it is valid to use the formulas for dipole-dipole interaction for describing the magnet systems based on the finite-size permanent magnets with the small distances between them.

Fig. 7, *a* shows the force of interaction F_z at the distance of 8.5 cm between the levitating and the holding magnet and

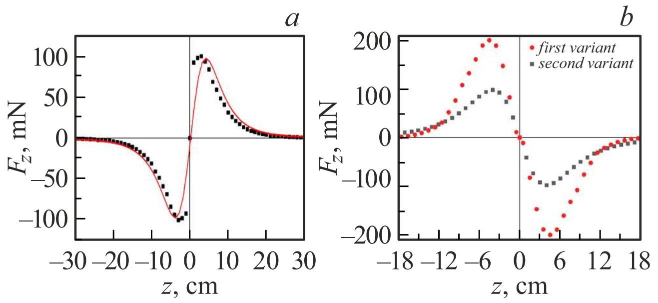

Figure 7. *a* — the dependence of the vertical component of the force between the holding and the levitating magnet on the vertical coordinate z when $x = 8.5$ cm for the second configuration (the dots are the experiment, the solid line is the calculation as per the formula (6); *b* — comparison of the experimental data for the two variants of orientations of the magnetic moments when the distance between the holding and levitating magnets is $x = 8.5$ cm.

its maximum value is less than the weight of the levitating magnet (150 mN). The point of stable equilibrium E_1 for this set of the magnets can be obtained provided that their magnetic moments are directed along the axis oy, when the levitating magnet and the holding magnets become closer to each other for the distances below 8.5 cm. Fig. 7, *b* compares the experimental data for both the variants of orientation and the direction of the magnets provided that the distance between the levitating and the holding magnet is 8.5 cm. It is clear that with the same distance between the magnets the vertical force for the second variant of arrangement and orientation of the magnets is less than for the first one, but the second orientation provides free access to the levitating magnet, which can be important for technical applications.

The maximum distance between the holding magnets and the levitating magnet $x_{\max}$, at which equilibrium of the levitating magnet is still possible, corresponds to the maximum point at the dependence $F_z(z)$ and was experimentally determined in the mockup of the magnetic suspension with smooth increase of the distance x up to loss of equilibrium of the levitating magnet. At this moment of time, each magnet had a value of deviation from the symmetry axis $z_{\max}$ recorded. Using the experimentally-determined value $x_{\max}$, the formula (2) was taken to determine the value of $z_{\max}$. The Table shows the characteristics of the experimentally-used magnets, the maximum distance between the holding and levitating magnets $x_{\max}$, the measured value $z_{\max}$ and the calculated values zmax that are calculated according to the formula (2). There is good compliance of the measured and calculated values of the magnitude $z_{\max}$.

3.2. Mockups of the magnetic suspensions

Fig. 8 shows the mockup of the magnetic suspension based on the first variant of arrangement and orientation of the holding magnet and the levitating magnet. Stable levitation is observed for all the magnets at the negative values of the coordinate z, as shown in the photo (Fig. 8) for „the disc“.

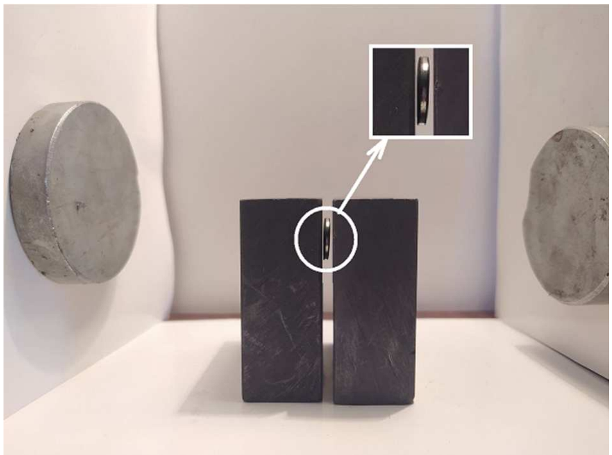

Figure 8. Mockup of the non-contact suspension system based on the first variant of arrangement and orientation of the holding magnets and the levitating magnet.

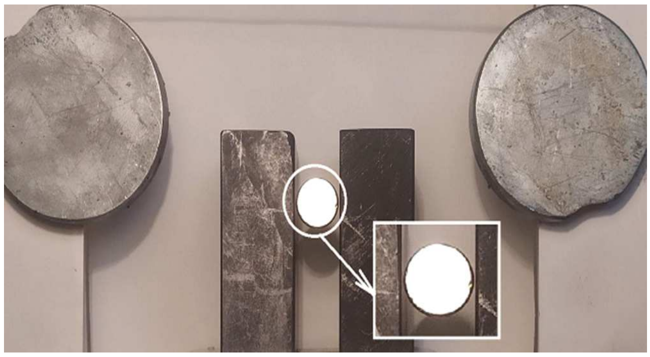

Figure 9. Mockup of the non-contact suspension system based on the second variant of arrangement and orientation of the holding magnets and the levitating magnet.

Fig. 9 shows the photo of the makeup of the non-contact suspension system based on the second variant of arrangement and orientation of the holding magnets and the levitating magnet, which is shown in Fig. 2. It included the magnets of the diameter of 50 mm and the thickness of 10 mm and diamagnetic screens made of pyrolytic graphite as holding permanent magnets and the disc-shaped levitating magnet of the diameter of 10 mm and the thickness of 2 mm. Stable levitation of the permanent magnet was also observed for this variant.

Conclusion

It is shown that in the considered systems with the horizontally-aligned magnetic moments of the permanent magnets stable equilibrium is realized for the two configurations of arrangement of the magnet with availability of diamagnetic stabilization: in the first one the magnetic moments of the holding magnets and the levitating magnet are in the same vertical plane, while in the second one the

Sizes, the magnetic moments and the parameters of equilibrium of the magnets

Magnets	Sizes of the magnets (the diameter, the thickness), mm	Magnetic moment, $A \cdot cm^2$	Measured value x_{max}, mm	Measured value z_{max}, mm	Calculated value z_{max}, mm
Pill	5; 1	1.5	93	−32	32.4
Disc 1	12; 1	8.8	88	−34	34.8
Disc 2	25; 2	77	90	−33	33.1

magnetic moments are normal to the plane that connects the centers of the magnets. The expressions for the forces between the levitating magnet and the holding magnets were calculated using the formulas for dipole-dipole interaction of the magnetic moments. The dependences of the forces of interaction on the distances, which were experimentally measured using the electronic balances, are well described by the obtained formulas even at the small distances between the interacting magnets. Thus, the calculations of the forces of interaction of the permanent magnets using the formulas of dipole-dipole interaction can be used for calculation and optimization of the magnetic non-contact suspension systems.

Both the considered configurations of arrangement of the magnets with horizontal orientation of the magnetic moments have the magnetic suspension mockups assembled, which have confirmed availability of stable levitation with diamagnetic stabilization of the position along one axis.

Acknowledgments

The authors would like to thank G.S. Volkova for her help in carrying out the calculations.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Q.H. Gao, H. Yan, H.X. Zou, W.B. Li. Science China Tech. Sc., **64** (1), 44 (2020). DOI: 10.1007/s11431-020-1550-1
[2] M.D. Simon, L.O. Heflinger, A.K. Geim. Am. J. Phys, **69** (6), 702 (2001). DOI: 10.1119/1.1375157
[3] E. Cazacu, I.V. Nemoianu. Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **55** (2), 153 (2010).
[4] E. Cazacu, I.V. Nemoianu. Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **56** (3), 249 (2011).
[5] S. Palagummi, J. Zou, F.G. Yuan. J. Vib. Acous., **137** (2015). DOI: 10.1115/1.4030665
[6] J. Abadie, E. Piat, S. Oster, M. Boukallel. Sensors and Actuators A: Phys., **173** (1), 227 (2012). DOI: 10.1016/j.sna.2011.09.025
[7] K. Zhang, Y. Su, J. Ding, Z. Zhang. IEEE Sens. J., **18** (17), 6978 (2018). DOI: 10.1109/JSEN.2018.2853680
[8] L. Liu, F.G. Yuan. J. Soun. Vib., **332** (2), 455 (2013). DOI: 10.1016/j.jsv.2012.08.004
[9] J.N. Ho, W.-Ch. Wang. AIP Advances, **8**, 085107 (2018). DOI: 10.1063/1.5039753
[10] S. Palagummi, F.G. Yuan. Sensors and Actuators A: Phys., **279**, 743 (2018). DOI: 10.1016/j.sna.2018.07.001
[11] L.D. Landau, E.M. Lifshits. *Teoreticheskaya fizika. Teoriya polya* (Fizmatlit, M., 2003), t. 2 (in Russian).
[12] M. González. Eur. J. Phys., **38** (2), (2017). DOI: 10.1088/1361-6404/38/2/025202
[13] Y. Zhang, Y. Leng, H. Zhang, X. Su, S. Sun, X. Chen, J. Xu. J. Intell. Manuf. Spec. Equip., **1**, 43 (2020). DOI: 10.1108/JIMSE-09-2020-009
[14] E.M. Parcell. *Elektrichestvo i magnetizm. Berkleevskii kurs fiziki* (Nauka, M., 1975), t. 2 (in Russian).

Translated by M.Shevelev