Boundary instability of a two-dimensional electron fluid

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It was shown previously that the current-carrying state of a Field Effect Transistor with asymmetric source and drain boundary conditions may become unstable against spontaneous generation of plasma waves [1]. By extending the analysis to the two-dimensional case we find that the dominant instability modes correspond to waves propagating in the direction perpendicular to the current and localized near the boundaries. This new type of instability should result in plasma turbulency with a broad frequency spectrum. More generally, it is shown that a similar instability might exist, when a strong enough current goes through a single boundary between the gated and ungated regions.

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It was shown previously [1] that the current-carrying state of a Field Effect Transistor may become unstable against spontaneous generation of plasma waves in the transistor channel, provided there is an asymmetry in the boundary conditions at the source and at the drain. An extreme case of such asymmetry is the ac short-curcuit condition at the source and the ac open curcuit at the drain. For submicron gate lengths the frequencies of the plasma oscillations belong to the terahertz range, thus the FET can, in principle, serve as a generator of terahertz radiation. The nonlinear properties of the electron fluid in the transistor channel can be also used for detection and frequency mixing in the terahertz domain [2].

Experimentally, both terahertz emission [3–5] and detection [6] in nanometric transistors were demonstrated. Fig. 1 presents experimental data [7] for a GaAlN/GaN HEMT at 4 K clearly showing the emission threshold at a certain source-drain voltage (or current) and a typical broad emission spectrum in the terahertz domain. Contrary to the prediction of Ref. [1], the spectrum depends neither on the gate length, nor on the gate voltage. Similar results for terahertz emission were obtained at room temperature [5].

It is not firmly established that the observed emission is indeed related to the instability predicted in [1] (see [5]). However, one cannot directly compare the theory with the experiments because the experimental geometry is very different from the one-dimensional model adopted in [1]. In the standard experimental situation, the width of the gate W is much larger than the gate length L, typically $W/L \sim 100$, see Fig. 2, left. Under such conditions the one-dimensional model, where the plasma density and velocity depend on the coordinate x only, is not appropriate, since obviously oblique plasma waves with a non-zero component of the wave vector in the y direction can propagate. In such a geometry, the gated region is not a resonator, but rather a waveguide with a continuous spectrum of plasma waves, see Fig. 2, right.

The purpose of this work is to extend the analysis of stability of the steady-state flow [1] to the more realistic geometry of Fig. 2. Since $W \gg L$, we will consider the limit of a strip, which is infinite in the y direction. It will be demonstrated that in such a geometry a new mode of instability dominates, which is localized near the gate boundaries. Moreover, a similar instability should exist in the limit $L \to \infty$, i.e. near a single boundary of a current-carrying two-dimensional plasma.

Within the hydrodynamic approach the electrons in a gated 2D channel can be described by the following

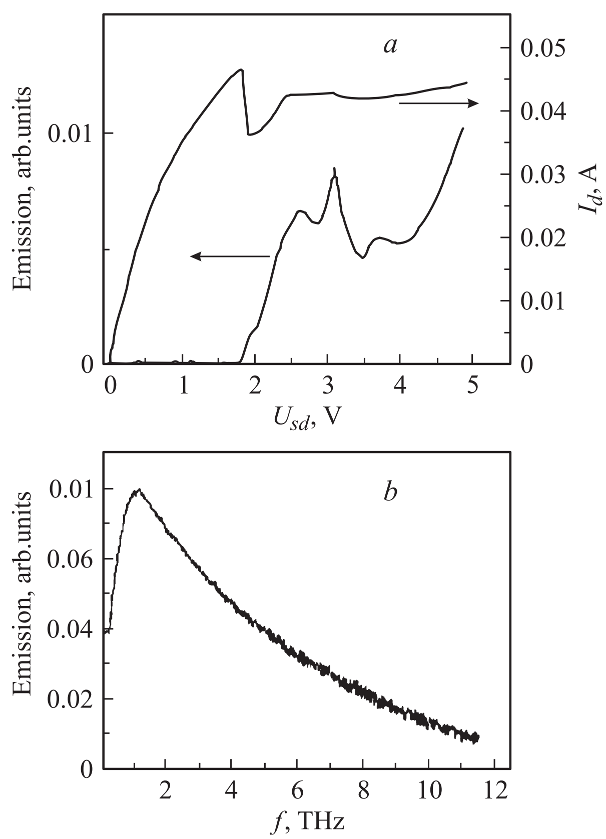

Figure 1. Experimental results for THz emission from a AlGaN/GaN HEMT at 4.2 K [7]. *a* — the drain current (right scale) and the emission intensity (left scale), as functions of the source-drain voltage, U_{sd}. Note the pronounced threshold for emission. *b* — the emission spectrum at $U_{sd} = 3$ V.

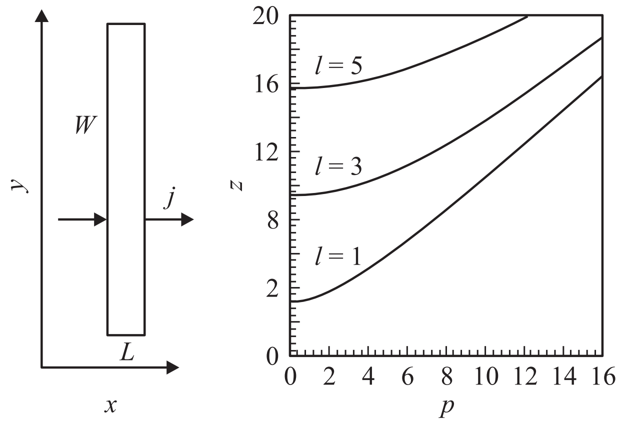

Figure 2. Left panel: the geometry of the gate. The width W is much greater than the length L. Right panel: the plasma wave spectrum in a strip, $z = 2L\omega/s$ is the dimensionless frequency, $p = 2qL/s$ is the dimensionless wave vector in the y direction, s is the plasma wave velocity.

equations [1]:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\frac{e}{m}\nabla U, \quad (1)$$

$$\frac{\partial U}{\partial t} + \nabla(U\mathbf{V}) = 0, \quad (2)$$

where $\mathbf{V}(\mathbf{r}, t)$ is the electron hydrodynamic velocity, $U(\mathbf{r}, t)$ is the gate-to-channel voltage swing, $\mathbf{r}$ is the vector in the 2D plane, e and m are the electron charge and the effective mass respectively. Equation (1) is the Euler equation, and Eq. (2) is, in fact, the continuity equation since the electron density in the channel, n, is related to the voltage swing, U, by the relation

$$en = CU, \quad (3)$$

where C is the gate to channel capacitance per unit area. This equation holds if the scale of the variation of the potential in the channel is large compared to the gate-to-channel separation d (the graduate channel approximation).

Collision processes give an additional term $-\mathbf{V}/\tau$ in the right-hand side of Eq. (1), where τ is the momentum relaxation time. In the following, this term will be neglected, however it should be understood that the instabilities studied below will practically exist only if the instability increment is greater than $1/\tau$, a condition that determines the instability threshold for the drift velocity, similar to the situation in the one-dimensional model [1].

We chose the x axis in the direction from source ($x = 0$) to drain ($x = L$) and, following Ref. 1, we impose the asymmetric boundary conditions of a fixed voltage at the source and a fixed current at the drain: $U = U_0$ at $x = 0$ and $j_x = j_0$ at $x = L$, where j_x is the x component of the current density. Because of Eq. (3), the latter condition can be rewritten as $(UV_x)_{x=L} = U_0 v_0$, where $v_0 = j_0/en$ is the electron drift velocity.

It was pointed out in [1] that Eqs (1) and (2) are identical to the equations describing the so called „shallow water" in conventional hydrodynamics [8], plasma waves in the channel being analogous to water waves in the case when the wavelength is much larger than the water depth. Furthermore, it was shown that the current-carrying steady state described by the stationary solution of Eqs (1), (2) with the above boundary conditions, $U = U_0$, $V_x = v_0$, is unstable against spontaneous generation of plasma waves with a growth increment given by

$$\gamma = \frac{s^2 - v_0^2}{2sL} \ln\left|\frac{s + v_0}{s - v_0}\right|, \quad (4)$$

where $s = (eU_0/m)^{1/2}$ is the plasma wave velocity.

This result followed from a one-dimensional analysis, e.g. small perturbations of the steady state were assumed to be independent of the coordinate y in the direction perpendicular to the current. As we shall see, the extension of the analysis to y-dependent perturbations not only gives corrections to Eq. (4) but, somewhat unexpectedly, gives a new mode of instability which always dominates.

We study the time dependence of small perturbations of the steady state. Accordingly we put $U = U_0 + (m/e)u$, $V_x = v_0 + v_x$, $V_y = v_y$ and we linearize Eqs (1), (2) with respect to the small quantities u, v_x, v_y.

The boundary conditions become:

$$u_{x=0} = 0, \; (v_0 u + s^2 v_x)_{x=L} = 0, \quad (5)$$

(zero ac voltage at the source and zero ac current at the drain). We look for the solutions of the linearized equations with $u, v_x, v_y \sim \exp(-i\omega t + ikx + iqy)$, where k and q are the components of the wave vector in the x and y directions respectively. This procedure gives

$$(\omega - kv_0)v_x - ku = 0, \quad (6)$$

$$(\omega - kv_0)v_y - qu = 0, \quad (7)$$

$$(\omega - kv_0)u - s^2(kv_x + qv_y) = 0. \quad (8)$$

The dispersion relation for the plasma waves follows:

$$(\omega - kv_0)^2 = s^2(k^2 + q^2), \quad (9)$$

the term kv_0 taking into account the Doppler shift due to the motion of the electron fluid. For given ω and q we find two values for the x-component of the wave vector, corresponding to oblique waves propagating downstream and upstream:

$$k_{1,2} = \frac{-\omega v_0 \pm s\sqrt{\omega^2 - (s^2 - v_0^2)q^2}}{s^2 - v_0^2}. \quad (10)$$

For $q = 0$ this reduces to $k_1 = \omega/(s + v_0)$, $k_2 = -\omega/(s - v_0)$. The general solution for u and v_x can be found using Eq. (6):

$$u = A\exp(ik_1x) + B\exp(ik_2x), \quad (11)$$

$$v_x = \frac{k_1}{\omega - k_1v_0}A\exp(ik_1x) + \frac{k_2}{\omega - k_2v_0}B\exp(ik_2x), \quad (12)$$

where A and B are constants.

The boundary conditions, Eq. (5), together with Eq. (10) give the relation

$$\exp\big(i(k_1 - k_2)L\big) = -\frac{\omega - k_1 v_0}{\omega - k_2 v_0}, \qquad (13)$$

which can be rewritten in the form:

$$\exp(i\sqrt{z^2 - p^2}) = \frac{\beta\sqrt{z^2 - p^2} - z}{\beta\sqrt{z^2 - p^2} + z}, \qquad (14)$$

where the dimensionless variables for frequency, wave vector, and drift velocity are introduced:

$$z = \frac{2sL}{s^2 - v_0^2}\,\omega, \quad p = \frac{2sL}{\sqrt{s^2 - v_0^2}}\,q, \quad \beta = \frac{v_0}{s}. \qquad (15)$$

Equations (14), (15) define the complex frequency $\omega = \omega' + i\gamma$ as a function of the drift velocity v_0 and the wave vector q. For $q = 0$ one obtains the previous one-dimensional result [1] with $\omega' = \pi l(s^2 - v_0^2)/(2sL)$, where l is an odd integer, and the increment $\omega'' = \gamma$ given by Eq. (4).

In the general case Eq. (14) can be solved only numerically. However, an analytical solution can be obtained for drift velocities small compared to the plasma wave velocity ($\beta \ll 1$). For $\beta = 0$ the solution of Eq. (14) is $z = (l^2 + p^2)^{1/2}$, or in dimensional units $\omega = s\big((\pi l/L)^2 + q^2\big)^{1/2}$, which represents the spectrum of plasma waves in an infinite strip with the assumed boundary conditions at $x = 0$ and $x = L$ (Fig. 2). The linear in β correction to this value is purely imaginary:

$$\gamma = \frac{v_0}{L}\frac{1}{1 + (qL/(\pi l))^2}. \qquad (16)$$

Thus, as q increases and becomes comparable to or larger than the quantized value of $k = \pi l/L$ for the l-th mode, the instability increment decreases from its value v_0/L given by Eq. (4) for $v_0 \ll s$. The correction to the real part of ω is of second order in β.

However, in addition to this predictable result, another solution of Eq. (14) exists, for which z (or ω) is purely imaginary. For $v_0 \ll s$ this solution can be found analytically by assuming that $|z| \ll p$ and $(z^2 - p^2)^{1/2} \approx ip$. This gives $z = i\beta p \tanh(p/2)$ or, in dimensional units $\omega' = 0$ and

$$\gamma = qv_0 \tanh(qL). \qquad (17)$$

For large qL this gives $\gamma = |q|v_0$, thus in contrast to the result given by Eq. (16), the growth increment of this new mode increases at large q, so that this mode of instability is the dominant one. The numerical solutions of Eq. (14) for $\beta = 0.5$ are presented in the Fig. 3, together with the approximate result for $\beta \ll 1$ given by Eq. (17).

It can be seen that for $qL \gg 1$ the new mode is localized near the boundaries at $x = 0$ and $x = L$ on a distance $\sim 1/q$. For example, in the case $\beta \ll 1$, $qL \gg 1$ we have $k_1 \approx -k_2 \approx iq$ (see Eqs (9), (10)). Thus the instability mode is formed by waves which are evanescent in the x direction.

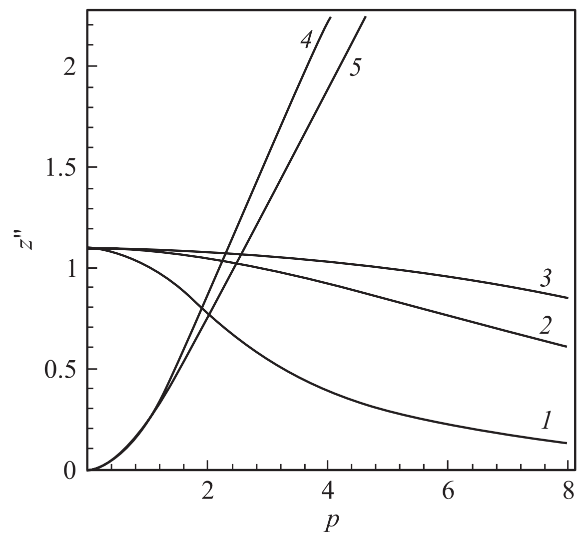

Figure 3. The instability increment z'' as a function of the transverse component of the wave vector p in dimensionless units for $\beta = v_0/s = 0.5$ (Eqs (14), (15), numerical calculation). *1, 2, 3* — for normal modes with $l = 1$, 3, and 5, respectively, *4* — for the new mode of instability, *5* — approximation given by Eq. (17).

Since for large $|q|L$ the growth increment for the new mode does not depend on L, and since in this case the mode is localized near the boundaries, it seems plausible that a similar instability of the steady-state flow should exist for a *single* boundary of an infinite (both in the y and the x directions) two dimensional current-carrying plasma. We now show that this is indeed the case.

Let a steady current with the drift velocity v_0 flow across the boundary ($x = 0$) of a semi-infinite sample situated at $x > 0$. The general boundary condition at $x = 0$ is defined by the impedance ξ relating the ac voltage and the ac current (compare with Eq. (5)):

$$u = \xi(v_0 u + s^2 v_x). \qquad (18)$$

The boundary condition at $x = \infty$ corresponds to the vanishing of the small perturbations, $u = v_x = v_y = 0$.

The impedance ξ will be considered as purely imaginary: $\xi = i\lambda/s$, where λ is the dimensionless parameter proportional to the effective capacitance. (The existence of a finite resistance, described by the real part of ξ, will obviously introduce damping of the initial perturbations and, if it is large enough, any instability will be supressed).

To insure the boundary condition at $x = \infty$, we now keep only one exponent in Eqs (11), (12), with the wave vector k, whose imaginary part is positive. These equations, together with Eq. (18), give:

$$\omega - kv_0 = \alpha sk, \qquad \alpha = \frac{i\lambda}{1 - i\lambda\beta}. \qquad (19)$$

Inserting this relation in Eq. (9), we find the value of the wave vector $k = i|q|(1 - \alpha^2)^{-1/2}$, where the sign of the square root should be chosen so that its real part be positive.

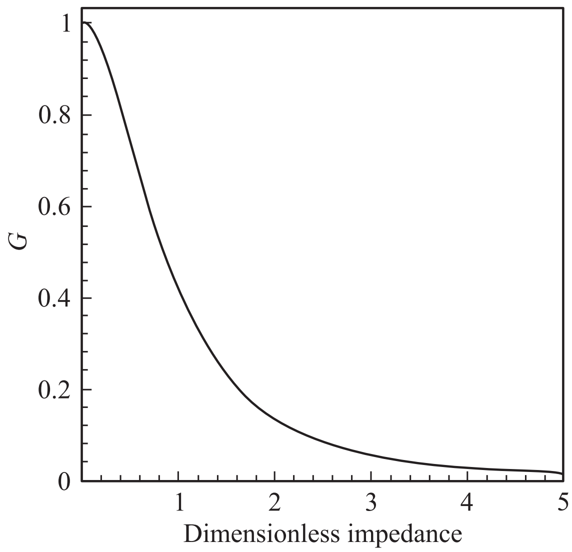

Figure 4. The coefficient G in Eq. (20) for $\beta = v_0/s = 0.5$ as a function of the dimensionless parameter λ defining the boundary condition at $x = 0$ (the boundary impedance is presented as $\xi = i\lambda/s$). For large λ, $G \sim 1/\lambda^3$.

Finally, from Eq. (19) one finds ω. Its imaginary part γ defines the instability increment:

$$\gamma = G|q|v_0, \qquad G = \frac{1}{\beta}\,\mathrm{Re}\Big(\frac{\alpha+\beta}{\sqrt{1-\alpha^2}}\Big). \qquad (20)$$

Note, that the values of q in Eqs (17), (20) are limited by the condition $q < 1/d$, where d is the gate-to-channel separation. For larger q the graduate channel approximation, used in deriving Eqs (1), (2) breaks down.

The dimensionless coefficient G depends on the value of λ, defining the boundary impedance, and on the flow velocity v_0, see Eq. (19). For $\lambda \ll 1$, we have $G = 1$ and Eq. (20) coincides with Eq. (17) for large $|q|L$. With increasing λ the coefficient G decreases (see Fig. 4), reducing the instability increment, which however remains always positive.

Thus, if the condition $|q|v_0 > 1/\tau$ is satisfied, the current-carrying steady state is unstable against small perturbations, and the region of instability is localized near the boundary. This is similar to what one observes in a river, when the water flows with sufficient velocity across an abrupt step in the waterbed: waves with wave vectors perpendicular to the flow are excited, while the wave vectors in the direction of the flow are purely imaginary, which accounts for the localization of the turbulent region near the step.

Certainly, the linear theory cannot predict the outcome of this instability. However, since the spectrum of plasma waves is continuous, it seems likely that the instability will result in a turbulent motion of the electron fluid near the boundary of the gated region. The spectrum of the plasma oscillations should be broad, similar to what is observed in experiments (Fig. 1). The width of the spectrum is expected to be limited by the value $\omega_{\max} \sim s/d$, where d is the gate-to-channel separation (see above).

The present theory can be also applied to the ungated electron fluid (analogous to the „deep water" in conventional hydrodynamics). It was shown [9] that a one-dimensional instability similar to the one described in Ref. [1] should exist in the ungated case too, under appropriate boundary conditions. It can be easily shown, that the boundary instability considered here will also occur in the ungated region, if the drift velocity is directed *inside* this region, similar to the results given by Eqs (17), (20) for the gated electron fluid. Thus, at the boundary between the gated and ungated regions, the turbulence should always appear on the downstream side.

It appears that the above concept acounts for the most important experimental observations [3–5]: the sharp threshold for terahertz emission and the broad emission spectrum, which does not depend on the gate length, and only weakly depends on the gate potential. A possible check of the proposed explanation would be to isolate the emission coming from one gate edge and to verify that the emission intensity (and possibly its spectrum) depends on the direction of the the drift velocity.

On the theoretical side, the very difficult issue of the true conditions at the boundary between the ungated and gated regions should be elucidated. (From the hydrodynamical point of view this is the problem of what happens for a flow across the boundary between deep and shallow water). Also, the role of the viscosity of the electron fluid, which may supress the instability for large wave vectors q, remains to be understood.

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