An accurate polynomial-based analytical charge control model for AlGaN/GaN HEMT

© pu Jinrong, Sun Jiuxun¶, Zhang Da

Department of Applied Physics, University of Electronic Science and Technology of China,
610054 Chengdu, People's Republic of China

(Получена 13 января 2011 г. Принята к печати 14 февраля 2011 г.)

A new nonlinear expression of Fermi-level variation with two-dimensional electron gas density in a high electron mobility has been proposed. It was found that our expression has a better fit with the numerical results. And, an analytical expression for n_s in terms of the applied gate voltage is developed. Comparing with other previous approximations, the solutions of our expression has a better agreement with the exact numerical results over the entire range of interest. Besides, the solutions of our expression of n_s versus V_G are compared with the experimental data and shown to be in good agreement over a wide range of bias conditions.

1. Introduction

The basic structure for a high electron mobility transistor (HEMT) consists of two layers in which the material with the wider bandgap energy (in this case AlGAN) is doped and that with the narrow bandgap energy (in this case GaN) is undoped. Owing to the difference in the electron affinity of the two layers, the electrons of the ionized donors will transfer into the GaN to form a conducting layer. The potential well formed at the interface is usually narrow enough to have well defined quantized energy levels in the direction perpendicular to the heterointerface, and in many cases the electronic system can be treated as a two-dimensional electron gas (2DEG). The confinement of carriers in the 2DEG of the unintentionally doped GaN, and the spatial isolation from their parent impurity atoms on the AlGaN side reduce their scattering, increasing their mobility and enhance other characters of the device [1]. AlGaN/GaN high electron mobility transistors have shown great potential for high-voltage and high-power operation at microwave frequencies, due to their properties of high electron mobility, saturation velocity, thermal stability and breakdown electric fields [2]. These enhanced characteristics have prompted the development of some approximated models in recent decades.

In 1982, Delagebeaudeuf first suggested that if we want to model the electron gas charge density at the interface, the simultaneous solution of the electrostatics equation in the wide band-gap semiconductor and the Schrodinger equation for the quantum well at the interface are needed. Through a triangular well approximation, and assuming two subbands in the GaAs, it can be show that the carrier density n_s is given by [3]

$$n_s = DkT \ln\left\{\left[1+\exp\left(\frac{E_F-E_0}{kT}\right)\right]\left[1+\exp\left(\frac{E_F-E_1}{kT}\right)\right]\right\}, \quad (1)$$

where $D = 3.24 \cdot 10^{13}\,\mathrm{cm}^{-2}\mathrm{V}^{-1}$ is the interface density of states, T is the temperature, E_F is the Fermi level, $E_0 = \gamma_0 n_s^{2/3}$ and $E_1 = \gamma_1 n_s^{2/3}$ are the positions of the first two allowed energy levels in the triangular well with $\gamma_0 = 2.5 \cdot 10^{-12}$ and $\gamma_1 = 3.2 \cdot 10^{-12}$. An important limiting feature of this model was that the accuracy of the model is not so good because of Fermi-level variation with electron density in the quantum well was neglected in order to get an analytical style. From then on, a serious of approximations for Fermi level E_F with sheet carrier density n_s were made and a number of expressions were proposed, and these expressions were halpful in the development of an improved analytical model for the HEMT. What's more, in the present papers, we can also find that the charge control model is also playing a very important role in the research of HEMTs. So, we will pay more attention to the research of charge control model.

In this paper, we have proposed a new expression of n_s versus E_F, valid for work at different temperatures, which is show to be more accurate than others that proposed before from subthreshold to high conduction. And we get a analytical charge control model for AlGaN/GaN HEMT based on our new expression on n_s versus E_F. For an AlGaN/GaN system, the solutions of our expression on n_s versus V_G are compared with that experimental data and shown to be in good agreement over a wide range of bias conditions. In the heterojunction quantum well of AlGaN/GaN, for equation (1), $D = 4\pi m^*/h^2$, m^* is the electron effective mass ($m^* = 0.22m_0$), $\gamma_0 = 2.123 \cdot 10^{-12}$, $\gamma_1 = 3.734 \cdot 10^{-12}$ [4].

2. Our model for Fermi level versus 2DEG density

In this section, a new nonlinear expression of Fermi level variation with two-dimensional electron gas (2DEG) density in a high electron mobility is proposed and based this expression, an analytical expression for n_s versus V_G is developed. Many figures will be shown to illustrate the superiority of our models.

In the area of device modeling, various linear analytical models as well as nonlinear expressions of Fermi level versus 2DEG have been proposed since the early 1980's. In 1982, Drummond et al.[5] proposed a liner approximation

¶ E-mail: sjx@uestc.edu.cn

for the E_F versus n_s relation. In 1988, Kola [6] putted forward a nonlinear approximation and the expression can be written as

$$E_F = K_1 + K_2(n_s + K_3)^{1/2}. \quad (2)$$

At 300 K, $K_1 = -0.20829$, $K_2 = 0.3029 \cdot 10^{-8}$, and $K_3 = 0.9666 \cdot 10^{15}$. In 1988, Shey [7] and Ku [8] made another approximation and gave the nonlinear expression as

$$E_F = E_{F0} + \gamma n_s^{2/3}, \quad (3)$$

where $E_{F0} = -0.062\,\text{eV}$ and $\gamma = 0.385 \cdot 10^{-11}\,\text{eV} \cdot \text{m}^{4/3}$. This approximation is appropriate as long as the device is not operated in the deep subthreshold region, where the quantization effect is not important because the potential well broadens notably and the subbands are closely spaced.

Particularly, in 1993, DasGupta [9] propese a good nonlinear approximation and the expression can be given by:

$$E_F = k_1 + k_2 n_s^{1/2} + k_3 n_s. \quad (4)$$

For this expression, the values of k_1, k_2, and k_3, given by [9], is not appropriate for a wider range of values of n_s, so, we try to find the best interpolation points and finally choose $n_s = 0.05 \cdot 10^{12}$, $2.5 \cdot 10^{12}$ as well as $10 \cdot 10^{12}\,\text{cm}^{-2}$ to calculate the values again and get the better results:

$$k_1 = -0.108, \quad k_2 = 0.1488, \quad k_3 = 0.04 \quad \text{at } 300\,\text{K}.$$

$$k_1 = -0.0168, \quad k_2 = 0.08, \quad k_3 = 0.0532 \quad \text{at } 77\,\text{K}.$$

In 2002, Rashmi [10] deduced some approximate expression from (1) appropriating for different region respectively. Obviously, it is not convenient for apply. Although the approximation has improved a lot, but all of above analytical expressions for E_F versus n_s have been proposed so far have disadvantages more or less, such as can not appropriate for a large range of values of n_s, the results is not accurate enough for modeling and so on.

In our model, a new approximation for E_F versus n_s has been formulated and it can be expressed by the polynomial:

$$E_F = k_1 + (k_2 n_s + k_3 n_s^2)^{1/2}, \quad (5)$$

where k_1, k_2 and k_3 are undetermined parameters. It is evident that E_F has to be computed for three different values on n_s to get the values of k_1, k_2 and k_3. Through solving eq. (1), we can have:

$$E_F = kT \ln\left\{\frac{(A+B)}{2}\left[\left(1 + \frac{4AB(C-1)}{(A+B)^2}\right)^{1/2} - 1\right]\right\}, \quad (6)$$

where

$$A = \exp(E_0/kT),$$
$$B = \exp(E_1/kT),$$
$$C = \exp(n_s/DkT).$$

In our formulation, E_F was calculated for $n_s = 0.05 \cdot 10^{12}$, $1 \cdot 10^{12}$ and $10 \cdot 10^{12}\,\text{cm}^{-1}$, these particular values of n_s were chosen in order to cover the operation of the device from subthreshold to high conduction. Here, we must notice that the interpolation point of our model can be chosen more freely but Gupta's is limited to some certain points for getting a better result. For these values of n_s, we can get corresponding values of E_F from eq. (6).

After rearranging equation (5), we can get that:

$$(E_{F_1} - k_1)^2 = k_2 n_1 + k_3 n_1^2, \quad (7)$$

$$(E_{F_2} - k_1)^2 = k_2 n_2 + k_3 n_2^2, \quad (8)$$

$$(E_{F_3} - k_1)^2 = k_2 n_3 + k_3 n_3^2. \quad (9)$$

Then, to solve the simultaneous solutions (7)–(9), we can get that:

$$W_0 k_1^2 - 2W_1 k_1 + W_2 = 0. \quad (10)$$

Eq. (10) is a standard quadratic equation, and solving this equation we can have:

$$k_1 = W_0^{-1}\left(W_1 + \sqrt{W_1^2 - W_0 W_2}\right), \quad (11)$$

where

$$W_0 = n_2 n_3 (n_2 - n_3) + n_3 n_1 (n_3 - n_1) + n_1 n_2 (n_1 - n_2),$$

$$W_1 = n_2 n_3 (n_2 - n_3) E_{F_1} + n_3 n_1 (n_3 - n_1) E_{F_2} + n_1 n_2 (n_1 - n_2) E_{F_3},$$

$$W_2 = n_2 n_3 (n_2 - n_3) E_{F_1}^2 + n_3 n_1 (n_3 - n_1) E_{F_2}^2 + n_1 n_2 (n_1 - n_2) E_{F_3}^2. \quad (12)$$

Then, substituting the values of E_F and n_s respectively in eq. (12), we can get the value of k_1, later, substituting the value of k_1 into eqs (7) and (8), we can get the values of k_2 and k_3. In this way, we can get the analytical solutions of the undetermined parameters. The values of k_1, k_2 and k_3 at different temperatures are given by:

$$k_1 = -0.128, \quad k_2 = 0.06189, \quad k_3 = 0.00184 \text{ at } 300\,\text{K};$$

$$k_1 = -0.0338, \quad k_2 = 0.028, \quad k_3 = 0.00363 \quad \text{at } 77\,\text{K}.$$

Finally, we can use eqs (2), (3), (4), (5) to figure the curves n_s versus E_F of all the models have been introduced above.

3. Analysis of charge control

To obtain an exact charge control formulation of the 2DEG channel in HEMT structures, Poisson's equation and Schrodinger's equation would need to be self-consistently solved. Unfortunately, the physical calculations are too involved for use in analytic device modeling. However, an approximation approach based on the linear charge control can be applied [11].

To take into account of a metal–(Si-doped)AlGaN–(undoped AlGaN)—(undoped CaN) structure, a two-dimensional electron gas is formed at the interface due to the difference in the electron affinity of these layers. The amount of charge transfer across the interface is gotten by equating the charge

depleted from the AlGaN to the charge accumulated in the potential well. The charge depleted from the AlGaN is given by [3]:

$$n_{s_0} = \sqrt{\frac{2\varepsilon N_d}{q}(\Delta E_c - E_{F_2} - E_{F_i}) + N_d^2 d_i^2} - N_d d_i. \quad (13)$$

If the AlGaN layer is thin enough or a sufficiently large negative gate voltage is applied, the gate depletion and junction depletion regions will overlap, in which case (13) must be replaced by

$$n_s = \frac{\varepsilon}{qd}(V_G - V_{off} - E_F), \quad (14)$$

where ε is the permittivity, d is the total thickness of the AlGaN layer and V_{off} is the threshold voltage of the HEMT, given by:

$$V_{off} = \phi_b - \Delta E_c - \frac{qN_D d_d^2}{2\varepsilon} - \frac{d}{\varepsilon}\sigma, \quad (15)$$

in which ϕ_b is the barrier of the Schottky gate, ΔE_c is the conduction band discontinuity at the heterojunction, N_D is the doping concentration, d_d is the thickness of the doped AlGaN layer and σ is the polarization sheet charge density of heterojunction AlGaN/GaN. In our structures, we use a Ni Schottky barrier contact at the surface; ϕ_b and ΔE_c are considered as the functions of Al concentration and we used the following formulations for the physical properties of $Al_xGa_{1-x}N$ in our calculations:

Ni Schottky barrier[12]:

$$\phi_b = 1.3x + 0.84; \quad (16)$$

band discontinuity [13,14]:

$$\Delta E_c = 0.7[E_g(x) - E_g(0)], \quad (17)$$

where the band gap of $Al_xGa_{1-x}N$ is measured to be [15]

$$E_g(x) = xE_g(AlN) + (1-x)E_g(GaN) - x(1-x)1.0\,eV, \quad (18)$$

$$= x6.13\,eV + (1-x)3.42\,eV - x(1-x)1.0\,eV. \quad (19)$$

With the variation of the Al concentration, the amount of sheet charge density for the undoped $Al_xGa_{1-x}N$/GaN is calculated by the following set of linear interpolations between the physical parameters of GaN, and AlN [16]:

$$\sigma(x) = |P_{PE}(AlGaN) + P_{SP}(AlGaN) - P_{SP}(GaN)|, \quad (20)$$

$$\sigma(x) = \left|2\frac{a(0)-a(x)}{a(x)}\left[e_{31}(x) - e_{33}(x)\frac{c_{31}(x)}{c_{33}(x)}\right] \times P_{SP}(x) - P_{SP}(0)\right|, \quad (21)$$

where the lattice constant

$$a(x) = (-0.077x + 3.189)10^{-10}\,m, \quad (22)$$

the elastic constants

$$c_{13}(x) = (5x + 103),$$

$$c_{33}(x) = (-32x + 405), \quad (23)$$

the piezoelectric constants

$$e_{31}(x) = (-0.11x - 0.49)\,C/m^2,$$

$$e_{33}(x) = (0.73x + 0.73)\,C/m^2, \quad (24)$$

the spontaneous polarization

$$P_{SP}(x) = (-0.124x - 0.034)\,C/m^2. \quad (25)$$

Thus, using equations (22)–(25), we can get the values of equation (21). And, substituting equations (16)–(21) into equation (15), we can calculate the values of the threshold voltage (V_{off}) with the variation of Al concentration. Then, substituting our expression equation (5) into equation (14), we can have:

$$\frac{qd}{\varepsilon}n_s + V_{off} + k_1 - V_G = -(k_2 n_s + k_3 n_s^2)^{1/2}. \quad (26)$$

Taking square for equation (26) on both sides, we can get:

$$\left[\left(\frac{qd}{\varepsilon}\right)^2 - k_3\right]n_s^2 + \left(2V_{off}\frac{qd}{\varepsilon} + 2k_1\frac{qd}{\varepsilon} - 2V_G\frac{qd}{\varepsilon} - k_2\right)n_s + 2k_1V_{off} - 2V_{off}V_G + V_{off}^2 + k_1^2 - 2k_1V_G + V_G^2 = 0. \quad (27)$$

Supposing

$$A = \left(\frac{qd}{\varepsilon}\right)^2 - k_3, \quad (28)$$

$$B = 2V_{off}\frac{qd}{\varepsilon} + 2k_1\frac{qd}{\varepsilon} - k_2, \quad (29)$$

$$C = 2k_1V_{off} + V_{off}^2 + k_1^2. \quad (30)$$

Equation (27) can be rewritten by:

$$An_s^2 + \left(B - 2\frac{qd}{\varepsilon}V_G\right)n_s + C - 2V_{off}V_G - 2k_1V_G + V_G^2 = 0. \quad (31)$$

Equation (31) is a standard quadratic equation, and solving this equation we can have:

$$n_s = \frac{2\frac{qd}{\varepsilon}V_G - B - \left[\left(B - 2\frac{qd}{\varepsilon}V_G\right)^2 - 4A(C - 2V_{off}V_G - 2k_1V_G + V_G^2)\right]^{1/2}}{2A}. \quad (32)$$

If we consider $V_G - V_{off}$ as a unity, we can get the variation of n_s with $V_G - V_{off}$ obtained from eq. (26), and the formulation can be written as:

$$n_s = \frac{2\frac{qd}{\varepsilon}V - N - \left[\left(N - 2\frac{qd}{\varepsilon}V\right)^2 + 4M(2k_1V - V^2 - k_1^2)\right]^{1/2}}{2M}, \quad (33)$$

where

$$M = \left(\frac{qd}{\varepsilon}\right)^2 - k_3,$$

$$N = 2k_1\frac{qd}{\varepsilon} - k_2,$$

$$V = V_G - V_{off}.$$

In section 2, we find Gupta's model for 2DEG versus Fermi level is the best except our model, so we will make a comparison between our charge control model and Gupta's [9]. Both of them are based the expression of Fermi level versus 2DEG themselves. And, the discussion will be made in next section.

We must notice that in this section the values of γ_0, γ_1 and D in eq. (1) have changed when we consider the AlGaN/GaN system. So we make interpolation for eqs (4) and (5) again to calculate the values of coefficients.

Eq. (32) provides an analytical expression for the variation of the sheet carrier concentration in the 2DEG as a function of the applied gate voltage. To verify our model, the 2DEG, with the variation of the applied voltage, has been computed using the physical data from Lee's paper [17] (with $d_d = 320 \cdot 10^{-10}$ m, $d = 350 \cdot 10^{-10}$ m, 25% Al). The comparison of the computed 2DEG with the experimental data will also be discussed in next section.

4. Results and discussion

Fig. 1, *a* and *b* show the variation of n_s with E_F obtained from all the approximations proposed previously (2)–(5) and compare the results with the exact curve of equation (6) at 300 K. We can find that Kola's approximation (2) is the best compared with linear approximation and as well as Shey's (3) in Fig. 1, *a* and the error figure also gave this result. The variation of n_s with E_F of our expression is demonstrated by Fig. 1, *b*, where the curves of approximations of Gupta (4) and Shey (3) are compared with the exact curve of equation (6) at 300 K. It can be observed that the curve of our expression has the best agreement with the exact curve, from subthreshold region to high conduct region. This result also can be cretificated by the error figure. So, considering Fig. 1, *a* and *b* simultaneously, we can find that our formulation get the optimal agreement over a wider range of device operation compared with the others at 300 K. Fig. 2 compares the curve obtained from our expression with the exact curve of equation (6) and also the curves of the linear approximation and Gupta's approximation (4) at 77 K. We must note that Kola [6] and Shey [7] have not proposed any approximations valid at 77 K. Fig. 2 also certificates that our expression incorporates the exact relationship for n_s versus E_F more accurately and get the optimal agreement over a wider range of values of n_s at 77 K. We can find from the error figure that our expression makes a very good approximation and the corresponding error curve almost keep pace with the „0" normal line over all the range. Figs 1 and 2 illustrate that

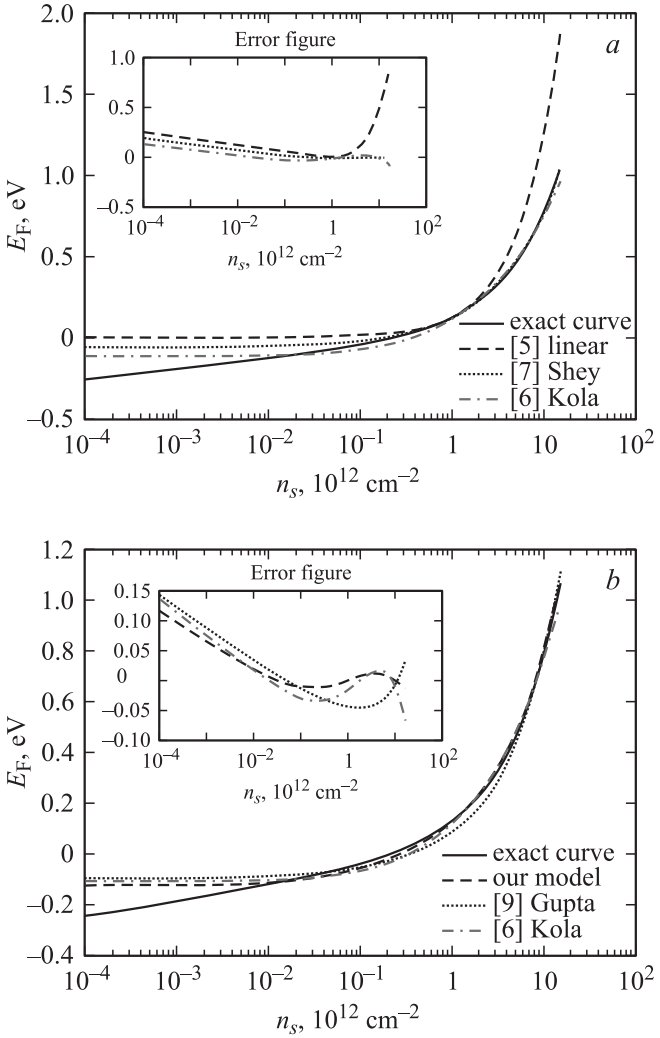

Fig. 1. Variation of the Fermi pootential (E_F) with sheet carrier concentration (n_s) at 300 K.

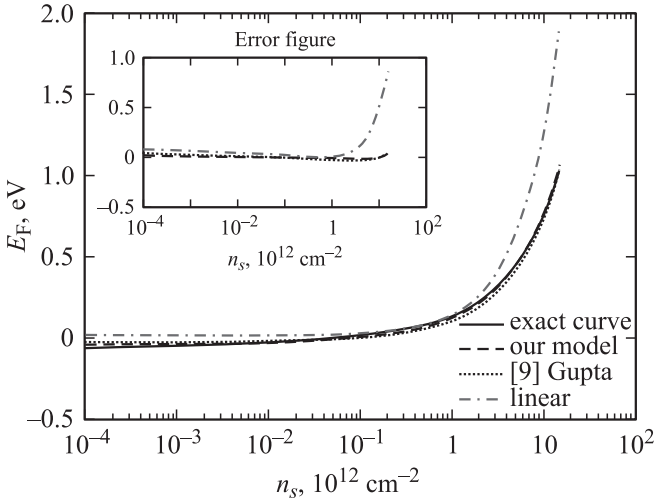

Fig. 2. Variation of the Fermi potential (E_F) with sheet carrier concentration (n_s) at 77 K: solid line (exact), dashed line (our model), dotted line [9], chain-dotted line (linear).

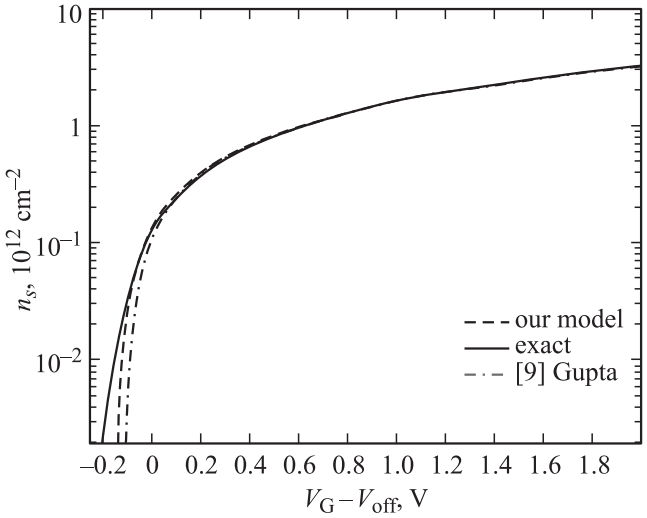

Fig. 3. Variation of sheet carrier concentration (n_s) with applied gate voltage ($V_G - V_{off}$) at 300 K: solid line (exact), dashed line (our model), chain-dotted line Gupta [9].

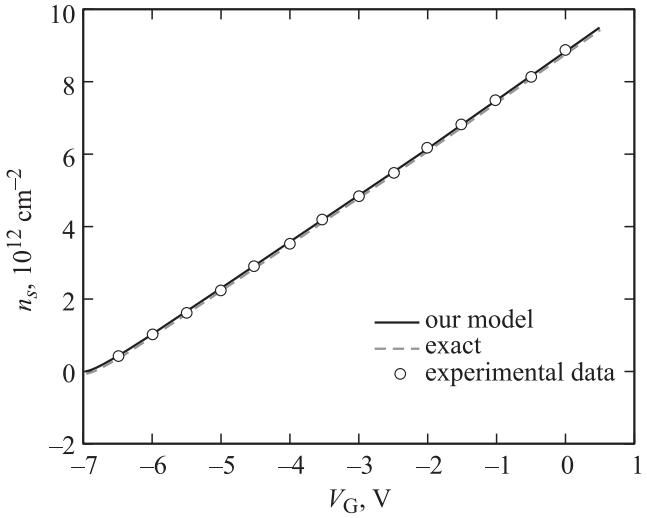

Fig. 4. Measured and simulated sheet carrier density versus the gate bias. The barrier thickness $d = 35$ nm and the threshold voltage $V_{off} = -6.78$ V.

the representing of our expression of Fermi level versus 2DEG for exact E_F versus n_s charecteristics is the best compared with other approximate functions in all regions of operation of interest at different temperatures. Especially, our expression is more accurate than other approximations in the subthreshold region.

Fig. 3 compares our charge control model (33) with Gupta's by substituting eq. (4) into eq. (14) by which can we get the variation of n_s with ($V_G - V_{off}$) of Gupta's. Besides, the exact numerical solution of eqs (6) and (14) is also shown in Fig. 3. Obviously, our curve has better agreement with the exact curve in compliance with Gupta's, and our curve is essentially coincident with the exact curve over all the range considered. The good agreement of our curve establishes that our expression (5) is a very good approximation for E_F versus n_s and is very useful and helpful for developing analytical models of HEMTs.

The calculated n_s versus V_G from our model (eq. (32)) is compared with those from the exact formulation (6), (14) and experimental data [17], as illustrated in Fig. 4. It can be seen that, our curve almost covers the exact curve and our results are very accurate from low voltage bias to very high bias. The comparison of our results with experimental data certificates the valid of our model and the useful of eq. (32) strongly. Eq. (32) provides an analytical expression for the variation of the sheet carrier concentration in the 2DEG as a function of the applied gate voltage. This analytical expression is not complicated and will be very useful for improving the analytical models and helpful for studying of other characteristics of HEMTs.

5. Conclusions and Summary

In section 2, a simple expression of the Fermi level variation with the sheet carrier concentration in the two-dimensional electron gas at the heterojunction of a HEMT is proposed. Figs 1 and 2 show the comparison of the fitted results of the linear approximation ([5]), some nonlinear approximations [6,7,9] and our expression with the exact numerical results. As the figures show, our expression (5) has the optimal agreement with the exact E_F versus n_s characteristics compared with other approximate functions in all regions of operation of interest at different temperatures. Especially, our expression is more accurate than other approximations in the subthreshold region. To further confirm the valid of our expression, the charge control model for AlGaN/GaN HEMT based on our new nonlinear expression of E_F versus n_s is developed in section 3. And, a comparison between our results and experimental data is carried, just like Fig. 4 shows. Referring to the schematic drawing of variation of sheet carrier concentration (n_s) with applied gate voltage ($V_G - V_{off}$) of Fig. 3, the curve of our model has a very good agreement with the exact curve, which certificated that our expression (5) is valid and the analytical expression (32) is valid and useful.

In conclusion, a new nonlinear expression for the Fermi-level variation with sheet carrier concentration of the two-dimensional electron gas has been presented. The expression gives a very good fit to the exact numerical solution at two different temperatures and is much better than all the conventional approximations. Particularly, our expression makes a large improvement in the subthreshold region. Based our expression, a analytical expression for the variation of the sheet carrier concentration with the applied gate voltage has been developed. It is found that our expression is more accurate than others proposed previously for analytical modeling of HEMTs. Besides, further certification is made by the comparison of our results with experimental data. Our expression of Fermi level versus 2DEG, as well as our analytical charge control

model, will be very helpful in modeling other characteristics of HEMTs for theiers higher degree of accuracy.

This work was supported by the Program for Excellent Talents of Sichuan Province of China under Grant No. 06ZQ026-010, Program for New Century Excellent Talents in University of China under Grant No. NCEN-05-0799, and Program for Excellent Talents of UESTC under Grant No. 23601008.

References

[1] H. Ahn, M.E. Nokali. IEEE Trans. Electron. Dev., **41**, 874 (1994).
[2] S.J. Pearton, J.C. Zolper, R.J. Shul. J. Appl. Phys., **86**, 1 (1999).
[3] D. Delagebeaudeuf, N.T. Linh. IEEE Trans. Electron. Dev., **29**, 955 (1982).
[4] Rashmi, A. Kranti, S. Haldar, R.S. Gupta. Solid-State Electron., **46**, 621 (2002).
[5] T.J. Drummond, H. Morkoc, K. Lee, M. Shur. IEEE Electron. Dev. Lett., **EDL-3**, 338 (1982).
[6] S. Kola, J.M. Golio, G.N. Maracas. Electron. Dev. Lett., **9**, 136 (1988).
[7] A.J. Shey, W.H. Ku. Electron. Dev. Lett., **9**, 624 (1988).
[8] A.J. Shey, W.H. Ku. IEEE Trans. Electron. Dev., **36**, 2299 (1989).
[9] N. DasGupta, A. DasGupta. Solid-State Electron., **36**, 201 (1993).
[10] Rashmi, A. Kranti, S. Haldar. Solid-State Electron., **46**, 622 (2002).
[11] A.N. Khondker, A.F.M. Anwar, M.A. Islam. IEEE Trans. Electron. Dev., **ED-33**, 1825 (1986).
[12] L.S. Yu, D.J. Qiao, Q.J. Xing, S.S. Lau, K.S. Boutros, J.M. Redwing. Appl. Phys. Lett., **73**, 238 (1998).
[13] G. Martin, S. Strite, A. Botchkaev, A. Rockett. Appl. Phys. Lett., **65**, 610 (1994).
[14] G. Martin, A. Botchkarev. Appl. Phys. Lett., **68**, 2541 (1996).
[15] D. Brunner, H. Angerer, H. Angerer. Appl. Phys. Lett., **82**, 5090 (1997).
[16] O. Ambacher, B. Foutz, J. Smart. J. Appl. Phys., **87**, 334 (2000).
[17] J.A. Barrido, J.L. Sanchez-Rojas, A. Jumenez. Appl. Phys. Lett., **75**, 2407 (1999).

Редактор Т.А. Полянская